Fast 3D Modeling of Borehole Induction Measurements in Dipping and Anisotropic Formations using a Novel Approximation Technique

Guozhong Gao¹, Carlos Torres-Verdín¹, and Sheng Fang²

ABSTRACT

Macroscopic electrical anisotropy of rock formations can substantially impact estimates of fluid saturation performed with borehole electromagnetic (EM) measurements. Accurate and expedient numerical simulation of the EM response of electrically anisotropic and dipping rock formations remains an open challenge, especially in the presence of borehole and invasion effects.

This paper introduces a novel efficient 3D EM approximation based on a new integral equation formulation. The main objective of this approximation is to simulate the multi-component borehole EM response of electrically anisotropic rock formations. Firstly, the internal electrical field is expressed as the product of spatially smooth and rough components. The rough component is a scalar function of location, and is governed by the background electric field. A vectorial function of location is used to describe the smooth component of the internal electric field, here referred to as the polarization vector. Secondly, an integral equation is constructed to describe the polarization vector. Because of the smooth nature of the polarization vector, relatively few unknowns are needed to describe it, thereby making its solution extremely efficient. One of the main features of the new approximation is that it properly accounts for the coupling of EM fields necessary to simulate the response of electrically anisotropic rock formations.

Tests of accuracy and computer efficiency against 1D and 3D finite-difference simulations of the EM response of tri-axial induction tools show that the new approximation successfully competes with accurate finite-difference formulations, and provides superior accuracy to that of standard approximations. Numerical simulations involving more than $10^6$ discretization cells require only several minutes per frequency and instrument location when performed on a Silicon Graphics workstation with a 300 MHz, IP30 processor.

INTRODUCTION

Integral equations have been widely used to simulate EM scattering in geophysical prospecting and antenna design applications. A number of applications and developments of integral equations for the simulation of subsurface geophysical problems have been reported, including 3D EM scattering in the presence of complex geometrical structures (Hohmann, 1975, and 1983, Wannamaker, 1983, Xiong, 1992, Gao et al., 2002, Hursan et al., 2002, and Fang et al., 2003, among others).

Solution of EM scattering by integral equations includes two sequential steps. First, the spatial distribution of the electric field within scatterers is computed via a discretization scheme. Second, the internal scattering currents are “propagated” to receiver locations. It is often necessary to discretize the scatterers into a large number of cells depending on (a) frequency, (b) electrical conductivity contrast, (c) size of the scatterers, and (d) proximity of the source and/or the receiver to the scatterers. This discretization gives rise to a full complex linear system of equations whose solution yields the spatial distribution of the internal electric field.
Requirements of computer memory increase quadratically with a linear increase in the number of discretization cells. Moreover, the need to solve a large, full, and complex linear system of equations places significant constraints on the applicability of 3D integral equation methods.

There are several numerical strategies used to overcome the difficulties associated with integral equation formulations of EM scattering. Fang et al. (2003) recently reported one such strategy. Their simulation approach makes explicit use of the symmetry properties of Toeplitz matrices. Fang et al.’s (2003) algorithm also applies a suitable combination of BiCGSTAB(l) (Bi-Conjugate Gradient STABlized (l)) (Sleijpen and Fokkema, 1993) and the FFT (Fast Fourier Transform) to iteratively solve the linear system of equations. The latter method is a natural extension of the widely used CG-FFT (Conjugate Gradient-Fast Fourier Transform) strategy (Catedra et al., 1995) to compute EM fields. Despite these significant improvements, integral equation methods are still impractical for routine use in the interpretation of borehole EM data. An alternative approach is to develop an approximate solution. Several approximations to the integral equation formulation have been proposed in the past. These include Born (1933), Extended Born (Habashy et al., 1993 and Torres-Verdin and Habashy, 1996, and Zhdanov, 2002). However, none of the integral equation approximations published to date has been formulated to approach the simulation of 3D EM scattering in the presence of electrically anisotropic media. Developing such an approximation is the main thrust of this paper.

Very recently, a novel approximation technique was introduced that has the ability to accurately and efficiently model borehole EM scattering in the presence of anisotropic rock formations (Gao et al., 2003). The present paper describes further developments performed in the implementation, testing, and benchmarking of Gao et al.’s (2003) novel integral equation approximation. This approximation attempts to synthesize the spatial variability of the secondary electric currents within a scatterer in two manners. First, a multiplicative term is introduced to “capture” the spatial variability of the secondary electric currents due to the close proximity of the EM source to the scatterer. A second multiplicative term is used to synthesize spatial variations in the phase and polarization of the secondary electric currents due to spatial variations in electrical conductivity, including those due to electrical anisotropy. It is shown that for borehole logging applications the latter multiplicative term is spatially smoother than the first term and hence can be described with fewer discretization blocks. Moreover, the accuracy of the proposed approximation depends on both the choice of the background model and the spatial distribution and number of discretization blocks.

This paper is organized as follows: First the integral equation method is introduced together with the theory and physical intuition behind the new approximation. Subsequently, technical details are provided concerning the choice of the background conductivity value. A section is also included to assess the influence of the spatial block discretization constructed within EM scatterers. Simulation examples are used to compare the accuracy of the new approximation against alternative integral equation approximations, i.e. Born, and Extended Born. Finally, several examples are provided to illustrate the performance of the new approximation in the presence of finite-size boreholes, mud-filtrate invasion, and electrical anisotropy of rock formations. These examples assume EM sources and receivers in the form of trial-axial multi-component borehole logging instruments.

THEORY OF INTEGRAL EQUATION MODELING

Assume an EM source that exhibits a time harmonic dependence of the type $e^{-i\omega t}$, where $t$ is time and $\omega$ is angular frequency. The magnetic permeability of the medium equals that of free space, $\mu_0$. Thus, the integral equation for electric and magnetic fields is in general given by (Hohmann, 1975)

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \int_G \mathbf{G}(r, r_0) \cdot \Delta \sigma(r_0) \cdot \mathbf{E}(r_0) d\mathbf{r}_0, \quad (1)$$

and

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_0(\mathbf{r}) + \int_G \mathbf{H}(r, r_0) \cdot \Delta \sigma(r_0) \cdot \mathbf{E}(r_0) d\mathbf{r}_0, \quad (2)$$

where $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ are the electric and magnetic field vectors, respectively, at the measurement location $\mathbf{r}$. The quantities $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{H}_0(\mathbf{r})$ in the above equations are the electric and magnetic field vectors, respectively, associated with a homogeneous, unbounded, and isotropic background medium of dielectric constant $\varepsilon_0$ and ohmic conductivity $\sigma_0$. Accordingly, the background complex conductivity is given by $\sigma_0 = \sigma_0' - i\omega\varepsilon_0$, and the wavenumber, $k_0$, of the background medium is given by $k_0^2 = \omega^2\mu_0\varepsilon_0$.

At low frequencies, the expression for the background wavenumber simplifies to $k_0^2 = i\omega\mu_0\sigma_0'$.

The electric Green’s tensor included in equation (1) can be expressed in a closed form

$$\mathbf{G}(r, r_0) = \left[ i\omega\mu_0 \sigma_0' + \frac{1}{\sigma_0} \nabla \nabla \right] g(r, r_0), \quad (3)$$

where the scalar function $g(r, r_0)$ satisfies the wave equation.
\[ \nabla^2 g(r, r_0) + k^2 g(r, r_0) = -\delta(r - r_0), \tag{4} \]

and whose solution can be explicitly written as

\[ g(r, r_0) = \frac{e^{ik|r-r_0|}}{4\pi|r-r_0|}. \tag{5} \]

The magnetic Green’s tensor is related to the electric Green’s tensor through the expression

\[ G(r, r_0) = \frac{1}{io\mu_0} \nabla \times E(r, r_0). \tag{6} \]

Finally, the tensor \( \Delta \sigma = \sigma - \alpha_0 \hat{1} = \Delta\sigma' - io\mu_0 \Delta\varepsilon, \alpha_0 \hat{1} \) is the complex conductivity contrast within scatterers, with \( \Delta\varepsilon = \varepsilon_s - \varepsilon_0, \Delta\sigma' = \sigma_s' - \sigma_0', \) and the symbol \( \hat{1} \) identifies the unity dyad.

Equations (1) and (2) are Fredholm integral equations of the second kind. A solution of these equations can be obtained using the method of moments (MoM). Traditional implementations of the MoM yield a full matrix equation, which normally involves the following difficulties for large-scale numerical simulation problems: (a) matrix filling time is substantial, (b) very large memory storage requirements, and (c) time-consuming solution of the complex linear system of equations. For large 3D scatterers, often the solution to EM scattering cannot be approached with a naïve implementation of the MoM. For instance, in a modeling example involving 1 million discretization cells, 0.2 CPU seconds are needed to compute 10,000 entries (each entry is a 3 by 3 tensor) of the MoM linear-system matrix.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Matrix filling time and computer storage associated with the assumption of 1 million discretization cells, and 0.2 CPU seconds needed to compute 10,000 entries (each entry is a 3 by 3 tensor) of the MoM linear-system matrix.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix filling time</td>
<td>231 days</td>
</tr>
<tr>
<td>Memory storage</td>
<td>67,054 Gigabytes (single complex precision)</td>
</tr>
</tbody>
</table>

A NEW EM SCATTERING APPROXIMATION

The EM scattering approximation reported in this paper is based on a new formulation of the integral equation (1). In so doing, the total electric field vector within each discretization cell is expressed as the product of a scalar function of the background field (spatially rough component) and a polarization vector (spatially smooth component), namely,

\[ E(r) = d(r)e_b(r), \tag{7} \]

where

\[ e_b(r) = (E_b^*(r) \cdot E_b(r))^{\alpha}, \tag{8} \]

and

\[ d(r) = \begin{pmatrix} \bar{d}_x(r) \\ \bar{d}_y(r) \\ \bar{d}_z(r) \end{pmatrix}. \tag{9} \]

The scalar component of the product in equation (7) is used to synthesize the relative spatial changes in magnitude of the electric field, whereas the vector component in the same equation is used to synthesize relative spatial changes in the polarization, phase, and to a less extent, of the amplitude of the electric field. In equation (8), \( \alpha \in [0,1] \) is a parameter controlling the spatial fluctuations of \( e_b \) and the spatial smoothness of \( d \). When \( \alpha = 0 \), the scalar function \( e_b \) becomes spatially constant and equal to one. In turn, this choice causes the vector function \( d \) to be identical to \( E_b \), thereby leading to the original integral equation. For the numerical examples described in this paper we adopt the choice \( \alpha = 1/2 \).

Substitution of equation (7) into equation (1) yields

\[ e_b(r)d(r) = E_b(r) + \int \bar{G} (r, r_0) \cdot \Delta\sigma(r_0) \cdot e_b(r_0)d(r_0)dr_0, \tag{10} \]

or, alternatively,
The new approximation stems directly from this last integral equation. In the above expressions, $e_b$ embodies relative changes in the magnitude of the internal electric field due to the proximity of the EM source. The larger the distance from the EM source to the scatterer, the less significant the spatial changes of $e_b$ within the scatterer. In the far field, one would expect $e_b$ to be spatially constant within the scatterer.

The spatial smoothness criterion necessary to accurately describe vector $d$ depends, to some extent, on the proximity of the EM receiver to the scatterer. Equation (1) shows that the simulation of EM scattering at the receiver location is performed by propagating the internal scattering electrical currents to the EM receiver location. In this case, the propagator is given by the electric Green’s tensor,

$$d(r) = d_b(r) + \int G\left(\mathbf{r}, \mathbf{r}_0\right) \cdot \Delta \sigma(\mathbf{r}_0) \cdot \frac{e_b(\mathbf{r}_0)}{e_b(\mathbf{r})} d(\mathbf{r}_0) d\mathbf{r}_0,$$

(11)

where

$$d_b(r) = E_b(r) / e_b(r).$$

(12)

The criterion adopted in this paper to control the degree of discretizing the scatterer into a collection of blocks, each block consisting of several cells. This procedure assumes that within each block the $d$ vector is constant, whereas the scalar function is assumed variable within a block but constant within a cell. Because of the choice of a uniform spatial discretization grid, all cells exhibit the same shape and size. The spatial distribution and size of blocks, however, can be chosen in a more flexible manner. It is only required that blocks be built to conform to cell boundaries. Finally, the $d$ vector associated with a given block is solved via equation (11). Such a procedure gives rise to an over-determined (rectangular) complex linear system of equations for the unknown vector $d$ within all of the discretization blocks. The rectangular, over-determined nature of the linear system of equations is due to the fact that the number of blocks is, by construction, smaller than the number of cells. Following a procedure described in the Appendix, the rectangular linear system is reduced to a $3Nx3N$ linear system of equations where $N$ is the number of cells. This reduction of the size of the linear system substantially decreases memory storage and CPU time requirements.

Additional savings in computer storage and CPU execution time are obtained with the use of a uniform spatial discretization scheme and a Toeplitz matrix formulation. When using uniform discretization grids, a Toeplitz matrix is constructed for each discretization block. Matrix vector multiplications are further accelerated using the FFT. Interested readers are referred to Fang et al., (2003) for technical details on the implementation of the FFT to solve block Toeplitz linear systems.

We remark that the spatial discretization of blocks and cells adopted in this paper is of a Cartesian type. Moreover, in an effort to properly model the borehole, the Cartesian block discretization is chosen with orthogonal axes conformal to the axis of the borehole (and hence conformal to the axis of the logging instrument). Cell locations and distances between a given cell and the axis of the borehole are measured perpendicular to the borehole axis. For the case of a non-conformal distribution of conductivity such as, for instance, dipping anisotropic beds, a conductivity averaging technique is used to assign a tensorial electrical conductivity to a specific cell. In this paper, we adopt the material averaging technique described by Wang and Fang (2001) to assign an electrical conductivity tensor to a particular cell.

**ON THE CHOICE OF A VALUE OF BACKGROUND ELECTRICAL CONDUCTIVITY**

As emphasized above, the integral equation approximation introduced in this paper makes use of a Green’s tensor defined over a homogeneous and isotropic unbounded medium. The choice of the simplest possible Green’s tensor is made to limit the complexity of the numerical computations associated with the integral equation solution.
Another important reason for this choice is that the Green's tensor associated with a homogeneous and isotropic unbounded medium remains space shift-invariant in all three Cartesian coordinates. It is also emphasized that the new approximation involves two conformal spatial discretization volumes. The first one is constructed using fine cells to describe the spatial variability of the scalar term $e_b$. In turn, the specific value of $e_b$ assigned to a given cell depends on the assumed background model. The value of background conductivity should be selected to provide the largest possible accuracy within the practical limits of the approximation.

The background conductivity should be some compromise between the contribution of small and large conductivity values in the rock formation model, for example by minimizing the difference between the minimum and maximum formation conductivity values using some weighted metric. Extensive numerical experiments suggested that the geometrical average of the minimum and maximum formation conductivity values provided optimal results for the examples considered in this paper. This geometrical average is given by

$$\sigma_b = \sqrt{\sigma_{\text{min}} \cdot \sigma_{\text{max}}}.$$  \hspace{1cm} (13)

The variables $\sigma_{\text{min}}$ and $\sigma_{\text{max}}$ in equation (13) are the minimum and maximum, respectively, of all the conductivity values considered in the numerical simulation.

Equation (13) is also suggested by studies in the theory of effective composite media involving the electrical conductivity of two-dimensional composites. It can be shown that a symmetric mixture of two components exhibits an effective conductivity given by the geometric average of the conductivities of the constituent materials. Alternative procedures could exist to choose an optimal background conductivity. These could include weighted averages of the conductivity distribution, where the weights would be determined by (a) proximity to the source(s), (b) proximity to the receiver(s), and (c) block volume. Yet another variation of equation (13) could be constructed with averages of electrical conductivity or resistivity taken along orthogonal or arbitrary directions. The latter possibility is enticing but we choose not to explore it in the present publication.

Quite obviously, the choice of background conductivity other than that of the borehole conductivity causes the borehole itself to become part of the anomalous conductivity region. Because of this, memory and CPU requirements increase when computing the internal electric field. Despite such difficulties, numerical experiments show that the choice of background conductivity different from that of the borehole does not substantially compromise the efficiency of the simulation algorithm. The small sacrifice in computer efficiency is drastically outweighed by the gain in numerical accuracy. Moreover, the implementation of the integral equation algorithm described in this paper makes use of 3D FFTs that can only be implemented on a uniform and spatially continuous discretization grid. The discretization does include the borehole region, and therefore the algorithm does not explicitly enforce a choice of background conductivity equal to the borehole conductivity.

**SENSITIVITY TO THE CHOICE OF SPATIAL DISCRETIZATION**

As emphasized above, there are two levels of spatial discretization involved in the computation of the integral equation approximation described in this paper. A fine cell structure is first constructed to describe the spatial variations within the scatterer of the scalar factor $e_b$ contained in equations (10) or (11). The relative spatial variations of this factor are primarily controlled by the proximity of the EM source to the scatterers. On the other hand, a relatively larger conformal block structure is constructed to describe the spatial variations of vector $d$. A given block in the discretization scheme of vector $d$ is composed of several cells used to spatially discretize the scalar factor $e_b$. The specific choice of block and cell structure can have a significant influence on the performance of the approximation.

The strategy chosen in this paper to construct block structures is one in which small blocks are placed in close proximity to the borehole, the transmitter(s), and/or the receiver(s). Small discretization blocks are required near receivers because an accurate representation of the polarization vector is needed in those blocks to properly account for the relative large influence of the dyadic Green’s tensor when propagating the internal electric field to receiver locations. Larger blocks are used to discretize the remaining spatial regions in the scattering rock formations.

Figure 1 shows the model used in this paper to study the effect of various block structures on the accuracy of the newly developed scattering approximation. Simulation results are also intended to assess the accuracy and efficiency of the approximation in the presence of various rock formation properties, presence of a borehole at various relative dip angles, and presence of mud-filtrate invasion. This model was adapted from an example proposed by Wang and Fang (2001). It consists of 5 horizontal layers in which the top and bottom layers are isotropic and exhibit an electrical resistivity of 50 $\Omega\cdot$m. The third layer is a 50 $\Omega\cdot$m isotropic layer of thickness equal to 12.0 ft. Finally, the second and fourth layers are electrically anisotropic with a horizontal resistivity of 3 $\Omega\cdot$m and a vertical resistivity of 15 $\Omega\cdot$m. These two layers are 2.0 ft and 10.0 ft thick, respectively.
Mud-filterate invasion may also be present within these last two layers, with an invasion length equal to 36 in, and with the resistivity in the invaded zone equal to 3 \(\text{\textohm} \cdot \text{m}\). The diameter of the borehole is equal to 8.0 in. and its resistivity is equal to 1 \(\text{\textohm} \cdot \text{m}\).

Simulation results are computed for borehole deviations of 0° and 60° for two cases of rock formation model: First, the formation is assumed to have no invasion and no borehole, i.e. to consist of a 1D stack of layers. The second model does assume a borehole and invasion, with the parameters described in the preceding paragraph. We compare simulation results with those obtained using a 1D code (identified as “1D” in the corresponding figures) and the 3D finite difference simulation algorithm (identified as “3D FDM” in the figures) developed by Wang and Fang (2001). The 3D FDM simulation results reported in this paper have been validated and benchmarked for accuracy by Wang and Fang (2001). In the descriptions and figures below, the identifier “3DIE Appr.” is used to designate simulation results obtained with the new approximation developed in this paper.

Figure 2 shows the borehole induction instrument assumed in the numerical simulations. It consists of one transmitter and two receivers moving in tandem along the borehole axis. Transmitter and receivers can be oriented in either of the \(x\), \(y\), or \(z\) directions. The spacing between the transmitter and the first receiver is 1.0 m (\(L_1\)), whereas the spacing between the transmitter and the second receiver is 1.60 m (\(L_2\)). It is further assumed that the instrument measurement is a linear combination of the response measured by the first magnetic receiver (\(H_1\)) and the second magnetic receiver (\(H_2\)), given by the formula

\[
H = H_2 - \frac{L_1}{L_2} H_1. \tag{14}
\]

Moreover, the numerical simulations reported in this paper consider only the imaginary component of the variable \(H\) in equation (14). This choice is made because of the availability of only the imaginary components of 1D and 3D FDM simulation results. According to our observations, the real component of the same variable approaches zero at low frequencies (of the order of 25 KHz) and becomes approximately equal to its imaginary counterpart at high frequencies (of the order of 250 KHz).

In this section, attention is focused on a model with invasion and a borehole deviated at 60°; the operating frequency is 220KHz. Figure 3 describes the simulated \(H_{zz}\) field component, i.e. the vertical magnetic field component due to a vertical magnetic source. This figure describes simulation results obtained using 8, 216, 1000, and 2400 discretization blocks, together with the corresponding results obtained with a 3D finite-difference code. In all of the above cases the number of discretization cells is 640,000. Figures 4 and 5 show the \(H_{xx}\) and \(H_{yy}\) components, respectively, simulated...
for the same formation model. These figures suggest that 1000 discretization blocks already cause the approximation to be as accurate as the 3D finite-difference code. Usage of 2400 blocks only provides a minor improvement in accuracy over that of 1000 blocks. The same figures indicate that usage of only 8 blocks already produces the basic behavioral features of the magnetic field components $H_{xx}$, $H_{yy}$, and $H_{zz}$, thereby lending credence to the validity of the new approximation.

The above exercises are not intended to guide the choice of the number of discretization blocks necessary to accurately simulate the EM response of a specific rock formation model. However, they do confirm that only a few discretization blocks are needed to reach an acceptable degree of accuracy. We remark that the minimum number of discretization blocks that can be used with the algorithm described in this paper is eight. This restriction comes from the fact that the discretization blocks are laid out symmetrically in all three directions with respect to the borehole axis. Therefore, the minimal structure that can be used for block discretization is the one with one block per octant.

**ASSESSMENT OF ACCURACY WITH RESPECT TO ALTERNATIVE APPROXIMATION TECHNIQUES**

In the past, several types of integral equation approximations have been developed to approach large-scale EM simulation problems. The objective of this section is to assess the accuracy and efficiency of the new integral equation approximation in comparison with Born and extended Born approximations. The reader is referred to Gao et al. (2003) for the same formation model. These figures suggest that 1000 discretization blocks already cause the approximation to be as accurate as the 3D finite-difference code. Usage of 2400 blocks only provides a minor improvement in accuracy over that of 1000 blocks. The same figures indicate that usage of only 8 blocks already produces the basic behavioral features of the magnetic field components $H_{xx}$, $H_{yy}$, and $H_{zz}$, thereby lending credence to the validity of the new approximation.

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for a similar comparison with the quasi-linear approximation of Zhdanov and Fang (1996).

Although similar comparisons of the above approximations have been reported by a number of authors, including Habashy et al., (1993), and Zhdanov and Fang (1996), none of the previous comparisons were performed in the context of electrically anisotropic media. It can be readily shown that the Born Approximation cannot reproduce the coupling of EM fields in the presence of electrically anisotropic media. On the other hand, the Extended Born approximation does account for some of the coupling of EM fields but its accuracy is compromised when the source is close to the scatterer (Torres-Verdín and Habashy, 2001, and Gao et al., 2003). Finally, it has been shown that the scalar and diagonal quasi-linear approximations of Zhdanov and Fang (1996) cannot account for the coupling of EM fields in the presence of electrically anisotropic media because of the existence of null components in the background electric field (Gao et al., 2003).

The rock formation model considered for this study is the same one used in the previous section. Simulation results are identified as follows: the label “Born” is used to designate simulations obtained with the first-order Born approximation, and the label “ExBorn” is used to designate results obtained with the Extended Born approximation. Again, 2400 blocks are used for the computation of the new approximation. Figures 6, 7, and 8 compare simulation results for the three approximations of the $H_{zz}$, $H_{xx}$, and $H_{yy}$...
field components, respectively. Simulation results summarized in these figures indicate a superior performance of the new approximation with respect to the Born and extended Born approximations.

### NUMERICAL EXAMPLES

Additional rock formation models and probing frequencies have been considered to further assess the accuracy and efficiency of the new approximation. These include: (a) a 1D formation that has no borehole and no invasion, with the source direction deviated at an angle of 0° and 60°, (b) a 3D formation with invasion and a borehole deviated at an angle of 0° and 60°. The probing frequencies considered in the simulations are 20 KHz and 220 KHz. Although the main purpose of this paper is to assess the accuracy and efficiency of the new approximation in the simulation of borehole EM logging measurements, some petrophysical comments are provided when interpreting the simulation examples. The intent of such comments is to assess the physical consistency of the approximation.

The spatial discretization grid constructed for the simulations reported in this paper consists of 80 cells in the x-direction, 80 cells in the y-direction, and 100 cells in the z-direction. Cell sizes are kept uniform and equal to 0.1 m. In total, 2400 blocks are used for the discretization of the models. Figures 9 through 10 show simulation results \( (H_{zz}, H_{sx}, \) and \( H_{sy} ) \) obtained with the new approximation assuming a 1D rock formation and a borehole deviation of 0°. Simulation results for two frequencies are compared to those obtained with the 1D code. The comparisons confirm the accuracy of the new approximation. Figures 9 and 11 indicate a substantial sensitivity of \( H_{zz} \) to the presence of deviation, as do Figures 10 and 12 for \( H_{sx} \). An even greater sensitivity to deviation can be observed by comparing the simulated \( H_{yy} \) components.

Figures 14 and 15 show simulation results \( (H_{zz}, H_{sx}, \) and \( H_{sy} ) \) obtained with the new approximation assuming a 3D rock formation. Simulation results for two frequencies are compared against those obtained with the 3D FDM code. The comparisons confirm the accuracy of the new approxi-
information for a 3D rock formation that includes borehole, invasion, and deviation. The influence of a borehole and invasion can be clearly observed on the behavior of the simulated magnetic field components $H_{zz}$ and $H_{xx}$.

Figures 16 through 18 show simulation results ($H_{zz}$, $H_{xx}$, and $H_{yy}$) obtained with the new approximation assuming a 3D rock formation (including both a borehole and invasion) and a deviation of 60°. Simulation results for two frequencies are compared against those obtained with the 3D FDM code. The comparisons confirm the accuracy of the new approximation for a 3D rock formation that includes borehole, invasion, and a 60° deviation. A comparison of Figures 11 and 16 provides evidence of the sensitivity of the simulated magnetic field components $H_{zz}$ to the presence of both a borehole and invasion. By contrast, a comparison of Figures 12 and 17 shows that $H_{xx}$ almost has no sensitivity to the presence of a borehole and/or invasion.

The above simulation exercises consistently show that the newly developed approximation yields accurate results in the presence of complex 3D anisotropy models for the two probing frequencies considered in this paper (20 KHz and 220 KHz). Simulation of EM fields for one single borehole location required approximately 3-4 minutes on a SGI OCTANE workstation (operating with a 300 MHz IP30 processor). By contrast, depending on the size of the spatial discretization grid, it takes anywhere from 20 minutes to 1 hour of CPU time to simulate one EM borehole location with the full-wave integral equation code developed by Fang et al., 2003. The latter integral equation code was...
developed using similar algorithmic strategies to those used in the development of the new integral equation approximation.

CONCLUSIONS

This paper describes a new EM scattering approximation introduced to substantially reduce computation times in the simulation of borehole induction responses of 3D anisotropic rock formations. The approximation makes use of a simple scalar-vectorial product to synthesize the spatial smoothness properties of EM scattering currents. Additional computer efficiency for the approximation is achieved with the use of uniform discretization grids. Numerical simulations and comparisons against 1D and 3D finite-difference codes indicate that the new approximation remains accurate within the frequency range of borehole induction instruments. Numerical experiments and benchmark comparisons also indicate that the new approximation remains accurate in the presence of a borehole, mud-filtrate invasion, and dipping and electrically anisotropic rock formations.

It was shown that the accuracy of the new approximation depends on the choice of both the background conductivity and the spatial block structure used for discretization. A criterion was described to select the background conductivity. Likewise, it was shown that only a relatively small number of spatial discretization blocks are needed to perform accurate simulations of borehole EM measurements.

Future work is envisioned for the construction of an optimal spatial block structure in an iterative manner. Such a procedure will commence with a sparse block structure that will be iteratively refined until no further improvement is
observed in the EM fields computed at the receiver locations.

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NOMENCLATURE

\( \sigma' \) ohmic conductivity
\( \varepsilon_0 \) electrical permittivity of free space
\( \varepsilon_r \) dielectric constant
\( \mu_0 \) magnetic permeability of free space
\( f \) frequency
\( \omega \) angular frequency \((2\pi f)\)
\( i = \sqrt{-1} \)
\( t \) time
\( e^{-i\omega t} \) time convention
\( \sigma = \sigma' - i\omega\varepsilon_0 \).
\( \mathbf{r} = (x,y,z) \) Cartesian coordinates, equal to \( \mathbf{x} + y\mathbf{y} + z\mathbf{z} \) denotes a 3x3 tensor
\( H_{xx} \) magnetic field generated in \( x \)-direction by a \( x \)-directed source (the second \( x \) represents the source direction).

REFERENCES

Born, M., 1933, Optics: Springer-Verlag, New York.

**FIG. 17** Comparison of the \( H_{xx} \) field component (imaginary part) simulated with the new approximation and a 3D-FDM code assuming a 3D formation that includes both a borehole and invasion. The borehole deviation is 60°. Simulation results are shown for probing frequencies of 20 KHz and 220 KHz.

**FIG. 18** Comparison of the \( H_{yy} \) field component (imaginary part) simulated with the new approximation and a 3D-FDM code assuming a 3D formation that includes both a borehole and invasion. The borehole deviation is 60°. Simulation results are shown for probing frequencies of 20 KHz and 220 KHz.


**APPENDIX**

**ALGORITHMIC IMPLEMENTATION OF THE NEW EM SCATTERING APPROXIMATION**

We first divide the scattering domain into \( N \) blocks, with \( V_n \) being the spatial region occupied by the \( n \)-th block, and

\[
\mathbf{d}(\mathbf{r}) = \sum_{n=1}^{N} \mathbf{d}_n P_n(\mathbf{r}), \tag{A.1}
\]

where

\[
P_n(\mathbf{r}) = \begin{cases} 1 & \mathbf{r} \in V_n \\ 0 & \text{elsewhere} \end{cases} \tag{A.2}
\]

Substituting equation (A.1) into equation (10) yields

\[
e_b(\mathbf{r}) \mathbf{d}(\mathbf{r}) - \sum_{n=1}^{N} \int_{V_n} \mathbf{G}(\mathbf{r}, \mathbf{r}_0) \cdot e_b(\mathbf{r}_0) \Delta \sigma(\mathbf{r}_0) d\mathbf{r}_0 \mathbf{d}_n = \mathbf{E}_b(\mathbf{r}). \tag{A.3}
\]

Because the conductivity tensor is constant within a given block one can rewrite equation (A.3) as

\[
e_b(\mathbf{r}) \mathbf{d}(\mathbf{r}) - \sum_{n=1}^{N} \int_{V_n} \mathbf{G}(\mathbf{r}, \mathbf{r}_0) \cdot e_b(\mathbf{r}_0) d\mathbf{r}_0 \Delta \sigma_0 \mathbf{d}_n = \mathbf{E}_b(\mathbf{r}). \tag{A.4}
\]

We now divide block \( V_n \) into \( P_n \) cells and proceed to match the incident fields at each cell location, \( \mathbf{r}_m \). Equation (A.4) becomes

\[
e_b(\mathbf{r}_m) \mathbf{d}(\mathbf{r}_m) - \sum_{n=1}^{N} \left[ \sum_{p=1}^{P_n} \int_{V_p} \mathbf{G}(\mathbf{r}_m, \mathbf{r}_0) \cdot d\mathbf{r}_0 e_b^p \right] \Delta \sigma_0 \mathbf{d}_n = \mathbf{E}_b(\mathbf{r}_m). \tag{A.5}
\]

For each cell, we define

\[
\mathbf{G} = \int_{V_p} \mathbf{G}(\mathbf{r}_m, \mathbf{r}_0) d\mathbf{r}_0 = \begin{bmatrix} G_{x x} & G_{x y} & G_{x z} \\ G_{y x} & G_{y y} & G_{y z} \\ G_{z x} & G_{z y} & G_{z z} \end{bmatrix}, \tag{A.6}
\]

and

\[
\mathbf{B} = \mathbf{G} e_b^p. \tag{A.7}
\]

Equation (A.4) then becomes

\[
e_b(\mathbf{r}_m) \mathbf{d}(\mathbf{r}_m) - \sum_{n=1}^{N} \left[ \sum_{p=1}^{P_n} B_{e_b}^p \Delta \sigma_0 \mathbf{d}_n \right] = \mathbf{E}_b(\mathbf{r}_m). \tag{A.8}
\]

Using matrix notation, equation (A.7) can be written as

\[
(A_{3M \times N} - C_{3M \times N} S_{3M \times N}) \mathbf{d}_{3M \times 1} = R_{3M \times 1}, \tag{A.9}
\]

where \( M \) is the number of cells and
matrix $S$ is the one described by Wang and Fang (2001). Finally,
\[ R = (E_{b_{11}}, E_{b_{12}}, E_{b_{13}}, \cdots, E_{b_{8n}}, E_{b_{8m}}, E_{b_{8n}})^T. \] (A.15)

To solve the over-determined complex linear system of equations represented by equation (A.9), we pre-multiply both sides of equation (A.9) by matrix $A^*$ to obtain

where matrix $A^*$ is the transpose conjugate of matrix $A$. Because the matrices $A^* A$, $A^* C$, and $A^* R$ are all independent of conductivity, they can be stored in hard-disk memory prior to performing the computations. Specifically, when the conductivity distribution changes with a change of location of the induction-logging instrument, it is only necessary to construct a new conductivity matrix. The remaining matrices included in equation (A.16) will not change with a change in instrument location.

It is pointed out that equation (A.16) is different from the least-squares solution of the over-determined complex linear system of equations described by equation (A.9). The way to obtain a least-squares solution of the over-determined linear system (A.9) is to pre-multiply both sides of the linear system by the matrix $(A - CS)^*$. However, we remark that such an operation may involve substantial computer resources. The rationale for using equation (A.16) instead of the standard least-squares solution is as follows. From inspection of equations (A.6), (A.7) and (A.13) one can conclude that, in general, the entries of matrix $C$ are much larger than those of matrix $CS$. One can easily show that the entries of matrix $C$ involve the entries of matrix $A$ multiplied by values derived from the Green’s tensor that, in turn, are customarily much smaller than 1. In view of the above, equation (A.16) remains an accurate and expedient alternative to the least-squares solution of equation (A.9). Extensive numerical experiments have confirmed the practical validity of equation (A.16).
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