Numerical Simulation of Mud-Filtrate Invasion in Deviated Wells

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Summary
Understanding the spatial distribution of fluids in the near-borehole region caused by mud-filtrate invasion is necessary for the accurate petrophysical interpretation of wireline measurements acquired in deviated wells. This paper provides a sorely needed petrophysical and fluid-flow template that can be used to integrate several wireline measurements into a unique model of petrophysical parameters in the near-borehole region of deviated wells.

We simulate numerically the physics of mud-filtrate invasion in vertical, horizontal, and highly deviated overbalanced wells. The numerical algorithm is adapted from a general 3D multiphase-fluid-flow simulator that is widely used in large-scale reservoir applications. Checks of numerical consistency and accuracy are performed against one commercial reservoir simulator. Emphasis is placed on describing the influence of mudcake buildup on the mud-filtration process. We approach the latter problem by introducing an effective-flow-rate function that describes the evolution in time of the rate of invasion of mud filtrate into rock formations. Parametric representations of the flow-rate function are derived on the basis of previously published laboratory experiments of mud circulation.

A sensitivity analysis quantifies the influence of several geometrical and petrophysical parameters on the spatial distribution of mud-filtrate invasion away from the borehole wall. These parameters include relative permeability, capillary pressure, permeability anisotropy, dipping layers, and degree of hydraulic communication between adjacent layers. Our simulations reveal the character of invasion profiles in complex geometrical environments taking place under realistic petrophysical conditions. We show that standard pistonlike descriptions of mud-filtrate invasion, commonly used in well-log interpretation, can lead to inaccurate interpretations of wireline measurements. An example is presented of the use of our simulation technique by calculating the sensitivity of borehole electromagnetic induction measurements to specific conditions of mud-filtrate invasion in a vertical well.

Introduction
Mud-filtrate invasion takes place in permeable rock formations penetrated by a well that is hydraulically overbalanced by mud circulation. This condition is of interest in numerous oilfield applications including drilling engineering, reservoir simulation, reservoir stimulation, and well-log interpretation. Electrical, electromagnetic, acoustic, and nuclear logging instrument responses are all influenced by the spatial distribution of fluids in the vicinity of the borehole resulting from the invasion of mud filtrate.

The invasion of mud filtrate into permeable rock formations is responsible for the development of mudcake on the borehole wall (solids deposition), as well as for the displacement of existing in-situ fluids laterally away from the borehole. Standard procedures used for the interpretation of well-logging measurements often conceive of the invasion of mud filtrate as a radial sequence of pistonlike fluid-saturation fronts. There have been a handful of techniques put forth to simulate (numerically and in the laboratory) the physics of mud-filtrate invasion. Drilling variables such as mud density and chemistry, mud circulation pressure, and time of filtration may all significantly affect the spatial extent of mud-filtrate invasion. In-situ rock formation properties such as porosity, absolute permeability, relative permeability, pore pressure, shale chemistry, capillary pressure, and residual fluid saturations also play important roles in controlling both the dynamic formation of mudcake and the time evolution of the invasion process.

Simple 1D models of mud-filtrate invasion exist based on the assumptions of a vertical well and a horizontal and infinitely thick rock formation. These models have been derived by the enforcement of mass-balance conditions and at best assume a homogeneous and isotropic spatial distribution of porosity and permeability while neglecting capillary forces and relative permeability effects. To date, there are no available numerical algorithms capable of simulating the physics of mud-filtrate invasion in rock formations comprising multiple hydraulically connected beds, nor are there algorithms that can simulate the geometrical complexity associated with deviated or horizontal wells.

In this paper, we introduce a general numerical algorithm to simulate the physics of mud-filtrate invasion in vertical and deviated boreholes. This algorithm is adapted from an existing 3D multiphase-fluid-flow simulator widely used to replicate the behavior of large-scale hydrocarbon reservoirs. The simulator, developed by the U. of Texas at Austin, is commercially referred to as UTCHEM. A detailed description of UTCHEM’s multiphase, multicomponent, and multichemical-species fluid-flow model is presented in the UTCHEM technical documentation.

One of the salient technical problems often considered in mud-filtrate-invasion studies is the description of mudcake buildup. The development and thickening of mudcake on the borehole wall causes the permeability of mudcake to decrease monotonically. In turn, this causes the flow rate of mud filtrate to decrease toward a low steady-state value. Both chemical and fluid mechanical processes determine (a) the rate at which solids accumulate on the borehole wall and (b) the thickness and effective permeability of the mudcake. Complications may arise as a result of periodic retrieval of drillpipe for bit changes. This can cause scraping of mudcake, resulting in secondary mudcake buildup and, hence, secondary mud-filtrate invasion.

Recently, and on the basis of laboratory experiments of mud circulation, Dewan and Chenevert reported a methodology to predict the time evolution of mudcake buildup as well as the effective petrophysical properties of mudcake. Dewan and Chenevert’s description is based entirely on six mud-filtrate parameters, all of which can be determined from a standard mud-filtrate test. As expected, Dewan and Chenevert’s work predicts a monotonically decreasing rate of mud-filtrate invasion as a function of time. Moreover, Dewan and Chenevert emphasize the fact that the rate of mudcake thickening remains practically unaffected by the petrophysical properties of the invaded rock formation. Borrowing from these results, in our simulations of mud-filtrate invasion we have completely avoided the problem of modeling mudcake buildup. Instead, we have chosen to model the effect of mudcake buildup on the invasion profile by defining an equivalent time-dependent flow rate of mud-filtrate invasion.

With an equivalent flow-rate function in hand, we have adapted UTCHEM to perform the remaining task of simulating the invasion of mud filtrate into permeable formations. Results from this experience are presented here as an attempt to quantify in a systematic manner the effects of a wide range of geometrical and petrophysical parameters on the invasion phenomenon. We visu-
alize the numerical simulations of mud-filtrate invasion in the form of spatial cross sections of water saturation. Our study considers the influence of mud composition, in-situ fluid saturation, relative permeability, permeability anisotropy, capillary pressure, gravity segregation, and hydraulic communication between adjacent layers. We also consider the geometrical influence imposed by horizontal and highly deviated wells. In all cases, the simulated spatial distributions of mud-filtrate invasion substantially depart from the standard notion of a radial sequence of pistonlike fluid-saturation fronts.

We believe that our 3D numerical simulator of mud-filtrate invasion will be useful as a working template to integrate all the available wireline measurements into a consistent and effective interpretation of petrophysical parameters in the vicinity of the borehole wall.

Mudcake Parameters and Effective Flow Rate of Mud-Filtrate Invasion

In general, the mechanical and petrophysical properties associated with mudcake can be characterized with six parameters: reference porosity, reference permeability, “pressure-up” compressibility index, “pressure-down” compressibility index, zero-pressure shear strength, and erosion friction factor.1,15 All these parameters can in principle be determined from an appropriately designed test sequence of mud filtration. In addition, mud has two parameters: the filtrate viscosity and the solids content. All eight parameters are necessary to calculate the flow rate of mud-filtrate invasion into permeable rock formations.

In the case of a static mud-filtrate invasion, the flow rate of invasion can be calculated from the total volume of mud-filtrate invasion during a given period of time. The equations describing the time evolution of both the fluid volume and the flow rate of invasion are given by:

\[ V(t) = C \sqrt{t} \]  \hspace{1cm} (1)

and

\[ q(t) = \frac{dV(t)}{dr} = \frac{C}{2} \frac{1}{\sqrt{t}} \]  \hspace{1cm} (2)

respectively, where \( t \) is time after the onset of invasion and \( C \) is a constant determined by the eight parameters listed previously. Fig. 1 is a geometrical description of the basic benchmark model considered in our simulations of mud-filtrate invasion. Geometrical and fluid-flow properties associated with the various components of this model are summarized in Table 1. Mud properties are summarized in Table 2. The model consists of one central layer of thickness equal to 0.61 m and permeability equal to 300 md, bordered by two layers of thickness equal to 0.61 m and permeability equal to 100 md. Impermeable slabs of infinite thickness terminate the model above and below the three permeable horizontal layers.

A finite-difference invasion simulator, referred to here as INVADE, was developed on the basis of the solution of fluid-flow partial-differential equations and boundary conditions for immiscible radial flow and coupled mudcake growth. INVADE was built as a specialized version of UTCHEM. In the process of invasion, the overbalance pressure is maintained as a boundary condition. The flow rate of mud filtrate across the wellbore is calculated from the petrophysical properties of the formation coupled with the model of mudcake growth. An additional feature of INVADE is that the simulations can take into account the process of salt mixing between mud filtrate and connate water. This feature becomes extremely important in the assessment of wireline electrical resistivity logs. Fig. 2 is a plot of the evolution of the flow-rate function during 2 days after the onset of invasion. In this figure, the borehole is assumed vertical, and flow-rate functions are calculated for each of the horizontal layers shown in Fig. 1. The two curves exhibit the expected behavior of a monotonically decreasing flow rate with increasing time caused by progressive mudcake thickening and hardening. As shown in Fig. 3, the volume of mud-filtrate invasion per formation thickness reaches 0.12 m^3/m for each layer. The dash-dot curve shown in Fig. 2 describes an equivalent constant-flow-rate function responsible for the same accumulated vol-
ume of invading fluid as that of the variable function during the 2-day interval. The equivalent constant flow rate is calculated with the equation
\[
q_e = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} q(t)dt
\]  
where \( q_e \) is the equivalent constant flow rate, \( q(t) \) is the flow rate of invasion, \( t \) is time, \( t_1 \) is invasion start time, and \( t_2 \) is invasion stop time. In this case, the equivalent constant flow rate is equal to 0.06 m³/d/m.

In the remaining sections of this paper, we make use of the flow-rate functions shown in Fig. 2 to perform a sensitivity study of mud-filtrate invasion. A slight variation of these functions will be introduced for the case of horizontal wells. Additional numerical simulations for specific mud properties and mud-circulation conditions will require the estimation of a different set of flow-rate functions from the corresponding six mud parameters introduced by Dewan and Chenevert.¹

**Checks of Accuracy and Internal Consistency of the Simulation Algorithm**

Unless otherwise noted, our sensitivity studies are based on the assumption of (a) a freshwater-based mud, (b) a borehole radius of 0.1 m, and (c) in-situ fluid saturation consisting of oil and irreducible water. Nominal relative permeability and capillary pressure curves used in our study to characterize the three permeable layers of Fig. 1 are shown in Figs. 4 through 6. Notice that we restrict ourselves to water/oil relative permeability curves because of our assumption of water-based mud. In the subsections to follow, we consider a number of geometrical and petrophysical variations to the basic model shown in Fig. 1, and we evaluate their influence on the mud-filtrate-invasion process. Simulations are performed that make use of the effective flow rates of mud-filtrate invasion shown in Fig. 2. These flow rates are entered into UTCHEM to perform the numerical simulations in a way similar to that of a standard water-injection process.

**2D Grid Validation for Vertical-Well Models.**
We performed extensive numerical checks to verify the accuracy and internal consistency of the simulation algorithm. The results are shown in Figs. 2 through 5. The consistency of the algorithm is demonstrated by the close agreement between the calculated and measured results. In the remaining sections of this paper, we use the validated algorithm to perform a detailed sensitivity study of the mud-filtrate invasion process.

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**Fig. 2**—Time evolution of the flow rate of mud-filtrate invasion. Comparison between constant and monotonically decreasing rates of mud-filtrate invasion across the horizontal layers shown in Fig. 1.

**Fig. 3**—Time evolution of the accumulated volume of mud filtrate during the process of invasion. The two curves identify invasion volumes across the horizontal layers shown in Fig. 1.

**Fig. 4**—Nominal set of oil/water relative permeability curves used in the sensitivity analysis of mud-filtrate invasion. The solid and dashed curves describe relative permeabilities as a function of water saturation for water and oil fractions, respectively.

**Fig. 5**—Alternative set of oil/water relative permeability curves. The solid and dashed curves describe relative permeabilities as a function of water saturation for water and oil fractions, respectively (compare to Fig. 4).
consistency of the numerical simulations. The first test consisted of performing a study of the convergence of the simulations with respect to a decrease in the step size used in the spatial-discretization scheme. Convergence of the ensuing numerical results was observed, with a decrease in the radial step size used in the finite-difference mesh. On the basis of these results, we adopted a radial logarithmic discretization scheme consisting of 60 radial grid steps. UTCHEM treats the outer boundary of the radial grid as a flow boundary. Therefore, the radial location of the outer boundary will not affect the simulation results provided that the invasion front is within that boundary. In the numerical simulations considered in this paper, the outer boundary is located 6.1 m away from the borehole wall.

3D Grid Validation for Horizontal-Well Models. A special discretization scheme was necessary to simulate mud-filtrate invasion in horizontal wells. The procedure we followed consisted of constructing 3D Cartesian grids with relatively small spatial steps in the vicinity of the borehole wall. In addition, nonhomogeneous normal-flow boundary conditions were enforced on grid cells interfacing the borehole wall. This allowed us to impose an integrated normal-flow condition consistent with the prescribed flow rate for mud-filtrate invasion. We validated such an approach by performing simulations of invasion into a homogeneous and isotropic 300-md formation and by disregarding the gravity effects of the flow from the borehole into the formation. Numerical simulations for this model were then compared against an equivalent vertical-borehole model calculated exclusively with a 2D grid. Fig. 7 shows the results of such a comparison. The two curves in that figure describe water-saturation profiles simulated radially away from the borehole wall for hypothetical vertical and horizontal wells, respectively, assuming the same rock formation properties. The corresponding radial profiles of water pressure are shown in Fig. 8. Despite some discrepancies between the two simulation results, especially in the radial profiles of water pressure (the maximum saturation difference is 0.03, and the maximum pressure difference is 0.2 psi), the 3D simulations performed with a horizontal well exhibit a high degree of similarity with the 2D simulations performed with the equivalent vertical well, thus giving credence to our 3D spatial-discretization approach.

Numerical Simulations Performed With ECLIPSE-100. An additional assessment of the numerical accuracy and internal consistency of our 3D spatial discretization was performed with the commercial simulator ECLIPSE-100. This study required the construction of 2D and 3D Cartesian finite-difference grids for input into ECLIPSE-100, similar to the UTCHEM grids described previously. Saturation and pressure simulations obtained with ECLIPSE-100 for the cases of a vertical and a horizontal well are shown in Figs. 9 and 10, respectively. As with the numerical simulations performed with UTCHEM, the radial profiles of water pressure exhibit only minor discrepancies, whereas the water-saturation profiles tend to separate with increasing values of distance away from the borehole wall. We observe a maximum saturation difference of 0.03 and a maximum pressure difference of 0.4 psi. More importantly, the ECLIPSE-100 results overlap extremely well with the corresponding UTCHEM results, hence giving further credence to our numerical simulation approach. We remark also that the availability of a 3D simulation model provides the added flexibility to simulate spatial variations of the flow rate of mud filtrate along the borehole wall. Such an option is explored in a subsequent section of this paper.

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Numerical Sensitivity Studies of Mud-Filtrate Invasion

The simulation results presented in this section consider several variations about the basic benchmark geometrical model described in Fig. 1. We assume a mud composed of fresh water and rock formations saturated with oil and irreducible water (see Table 1). Results of the simulations of mud-filtrate invasion are shown in the form of spatial cross sections of water saturation away from the borehole wall. Table 3 is a summary of the test cases considered in the numerical simulation of the process of mud-filtrate invasion.

Sensitivity Studies With a Vertical-Well Model. Case 1: Constant Rate of Mud-Filtrate Invasion. Fig. 11 shows the corresponding cross section (radial vs. vertical directions) of water saturation simulated under the assumption of a constant-flow-rate function with 2 days' invasion (see Fig. 2). Moreover, the simulations were performed using the relative permeability and capillary pressure curves shown in Figs. 4 and 6, respectively. Hydraulic communication in the vertical direction (vertical crossflow) was allowed between adjacent layers.

Case 2: Monotonically Decreasing Rate of Mud-Filtrate Invasion. This test case differs from Case 1 in that a variable rate of mud-filtrate invasion was enforced in the simulation of invasion.

Case 3: Enhanced Vertical Permeability. We consider the vertical permeability one-tenth of the base horizontal permeability \( k_v = 0.1 k_h \). A small value of \( k_v \) results in a decreased rate of crossflow between adjacent layers. This effect can be identified from the cross section shown in Fig. 13, especially when compared to the cross section in Fig. 12.

Case 4: Sensitivity to a Perturbation in the Water/Oil Relative Permeability Curves. We maintain the same model conditions described for Case 2 except that a change is made in the oil/water relative permeability curves entered into the simulations. The alternative set of oil/water relative permeability curves is shown in Fig. 5. Fig. 14 displays the ensuing cross section of water saturation. A marked enhancement in the radial extent of invasion is observed as a result of the change in relative permeability curves.

<table>
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<tr>
<th>Case No.</th>
<th>Well Type</th>
<th>Grid Type</th>
<th>Layer Count</th>
<th>Total Injection Volume (m³/m²)</th>
<th>( k_v/k_h )</th>
</tr>
</thead>
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<tr>
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<td>Vertical</td>
<td>2D Radial</td>
<td>3</td>
<td>0.121</td>
<td>Constant Flow Rate</td>
</tr>
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<td>2D Radial</td>
<td>3</td>
<td>0.121</td>
<td>1</td>
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<tr>
<td>3</td>
<td>Vertical</td>
<td>2D Radial</td>
<td>3</td>
<td>0.121</td>
<td>0.1</td>
</tr>
<tr>
<td>4</td>
<td>Vertical</td>
<td>2D Radial</td>
<td>3</td>
<td>0.121</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Vertical</td>
<td>2D Radial</td>
<td>3</td>
<td>0.121</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Horizontal</td>
<td>3D Cartesian</td>
<td>1</td>
<td>0.121</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Horizontal</td>
<td>3D Cartesian</td>
<td>1</td>
<td>0.121</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>Horizontal</td>
<td>3D Cartesian</td>
<td>3</td>
<td>0.121</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Deviated</td>
<td>3D Cartesian</td>
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<td>0.121</td>
<td>0.1, 1</td>
</tr>
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<td>10</td>
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<td>3D Cartesian</td>
<td>3</td>
<td>0.121</td>
<td>1</td>
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<tr>
<td>11</td>
<td>Deviated</td>
<td>3D Cartesian</td>
<td>6</td>
<td>0.121</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 9—Comparison of the numerical simulations of water saturation performed with ECLIPSE-100 for equivalent horizontal and vertical wells. The curves describe radial profiles of water saturation away from the borehole wall assuming a homogeneous formation and a time of invasion of 2 days.

Fig. 10—Comparison of the numerical simulations of water pressure performed with ECLIPSE-100 for equivalent horizontal and vertical wells. The curves describe radial profiles of water saturation away from the borehole wall assuming a homogeneous formation and a time of invasion of 2 days.

Fig. 11 shows the corresponding cross section of water saturation. The distribution of water saturation exhibits a smoother spatial decline away from the borehole wall than is shown in Fig. 11. Unless otherwise noted, we adopt the use of a variable rate of mud-filtrate invasion for the simulations described in subsequent test cases.

Fig. 12—Case 1: spatial cross section of water saturation. Example of a vertical well and a time-constant rate of mud-filtrate invasion. The time of invasion is 2 days.
Case 5: Sensitivity to Null Capillary Pressure. We return to Case 2 to study the influence of capillary pressure on the time evolution of the invasion process. Fig. 15 shows the cross section of water saturation obtained assuming that the rock’s capillary pressure is zero for all values of water saturation. This cross section exhibits a relatively sharp transition between the water and oil phases saturating the permeable formation and is reminiscent of the pistonlike saturation profiles often assumed in well-log interpretation. Moreover, the radial extent of the invasion process is relatively shallow, and there is almost no evidence of crossflow between adjacent layers.

Sensitivity Studies With a Horizontal-Well Model. Case 6: Homogeneous and Isotropic Formation. As explained earlier, the numerical simulation of mud-filtrate invasion in horizontal and deviated wells requires a 3D spatial-discretization scheme. Fig. 16 is a cross section of water saturation resulting from the simulation of filtrate invasion in a horizontal well, assuming density for water and oil equal to 1 and 0.85 g/cm³, respectively, and a time of invasion of 2 days. This cross section is taken to coincide with a plane perpendicular to the borehole axis. Because of symmetry, only one half of such a plane is shown in Fig. 16. The invaded rock formation is assumed to be homogeneous and isotropic, but its permeability is not high enough to exhibit gravity-segregation effects on the distribution of fluids around the borehole wall. We remark that the cross section shown in Fig. 16 exhibits a high degree of azimuthal symmetry around the borehole axis.

Case 7: Angle-Dependent Rate of Mud-Filtrate Invasion. Depending on the rate of mud circulation, and because of gravity forces, mudcake has a known tendency to thicken more rapidly on the “low” side of a horizontal wellbore than elsewhere on its wall. It is therefore expected that the rate of mud filtration will vary as a function of the angle around the borehole wall. We have made an attempt to model this behavior by introducing the angle-dependent rate of filtrate invasion shown in Fig. 17. The borehole is divided into 28 angular segments, with the flow rate for each angular segment given by

\[
q(\alpha, t) = \frac{1}{N} \cdot q(t) + \sin \left( \frac{\pi}{2} \left( \frac{\alpha}{90} - 1 \right) \right) \cdot q_a, \quad \ldots \ldots \ldots (4)
\]
where \( \alpha \) is the angle (degrees) between the borehole segment and the downward direction, \( t \) is time, \( N \) is the total number of angular segments, \( q(t) \) is the flow rate of invasion, \( q(\alpha, t) \) is the flow rate for the angular segment with angle equal to \( \alpha \), and \( q_\alpha \) is the flow-rate difference between the 180° and 90° segments. This dependence on vertical location is, of course, to be superimposed to the assumed time behavior shown in Fig. 2.

The corresponding simulation of mud-filtrate invasion yields a cross section of water saturation slightly different from that shown for Case 6. A plot of the difference between the distributions of water saturation with and without the inclusion of an angle-dependent flow rate is shown in Fig. 18. As expected, we observe both an excess and a deficit of water saturation above and below the borehole, respectively, as a consequence of the angle-dependent rate of invasion described in Eq. 4.

**Case 8: A High-Permeability Layer Above the Well.** A horizontal slab of permeability equal to 3,000 md is positioned just above the horizontal well, over the penetrated rock formation. The contrast of permeability between the drilled formation and the slab is 1:10. Fig. 19 shows the corresponding cross section of water saturation resulting from the invasion process. This cross section shows a clear tendency of the mud filtrate to flow toward the high-permeability slab despite the force of gravity.

**Sensitivity Studies With a Deviated-Well Model. Case 9: Basic Three-Layer Model.** We return to the basic three-layer model shown in Fig. 1. In this case, the well is assumed to penetrate the layered formations at an angle of 45°. Fig. 20 is a cross section taken along a vertical plane through the axis of the borehole that shows the simulated distribution of water saturation resulting from mud-filtrate invasion. This cross section clearly illustrates the influence of the most-permeable layer as well as that of the existing degree of hydraulic communication (crossflow) between adjacent layers.

**Case 10: Enhanced Horizontal Permeability.** A slight variation of Case 9 is pursued here to quantify the sensitivity of the

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**Fig. 16**—Case 6: spatial cross section of water saturation. Example of a horizontal well and a homogeneous and isotropic oil-bearing formation invaded with a water-based mud. The density of water and oil are equal to 1 and 0.85 g/cm³, respectively, and the time of invasion is 2 days.

**Fig. 17**—Angle-dependent rate of mud-filtrate invasion in a horizontal well. This flow-rate curve is used to study the effect of variable-size mudcake buildup around the wall of a horizontal borehole. Note that a maximum flow rate occurs at the highest vertical point along the borehole wall.

**Fig. 18**—Case 7: spatial cross section of the difference in water saturation between the cases of an angle-dependent and an angle-independent rate of mud-filtrate invasion. Example of a horizontal well and a water-based mud. The density of water and oil are equal to 1 and 0.85 g/cm³, respectively, and the time of invasion is 2 days.

**Fig. 19**—Case 8: spatial cross section of water saturation. Example of a horizontal well and a high-permeability layer above the well. The time of invasion is 2 days.
invasion process to a 10:1 enhancement in the horizontal permeability of the central layer. We observe a highly asymmetric spatial distribution of water saturation that develops as a result of the enhancement in horizontal permeability. The simulation results, shown in the cross section of Fig. 21, significantly depart from those of the isotropic case displayed in Fig. 20. We also observe a substantial reduction of the effect of crossflow between adjacent layers.

**Case 11: Multilayer Model.** A more general deviated-well model was constructed with a sequence of six horizontal layers. Different values of permeability, in the range of 100 to 800 md, were assigned to each of the layers to produce a highly structured model of mud-filtrate invasion. Fig. 22 shows the corresponding cross section of water saturation simulated with the same set of relative permeability and capillary pressure curves for all the layers. A uniform rate of mud-filtrate invasion also was assumed in the simulations irrespective of the permeabilities of the individual layers. Hydraulic communication between adjacent layers was enforced while performing the numerical simulations. The cross section in Fig. 22 sheds considerable light on the range of vertical variability one could expect in practical cases of fluid invasion into a vertically heterogeneous hydrocarbon reservoir. Most wireline-interpretation techniques dealing with deviated wells fail to incorporate this level of geometrical complexity and hence can be subject to sizable errors in their estimations of in-situ petrophysical parameters.

**Sensitivity of Wireline-Logging Instrument Response to the Spatial Distribution of Water Saturation**

Assuming clay-free porous rock formations, Archie’s equations can be readily used to transform the distributions of water saturation described previously into equivalent distributions of electrical resistivity. According to Archie, the bulk resistivity of a rock, $\rho_w$, is related to the resistivity of the brine in the rock, $\rho_r$, its porosity, $\phi$, and its water saturation, $S_w$, by

$$\frac{\rho_w}{\rho_r} = \frac{a \cdot \phi^n}{S_w^{m}}$$

where $a$ is the tortuosity/cementation factor and $m$ and $n$ are the cementation and saturation exponents, respectively. The resistivity of connate water included in this formula can be calculated from the water’s salinity and temperature.

We have made a simplistic attempt to perform one such transformation. Our aim was to quantify whether a commercial induction-logging instrument could capture some of the features of the mud-filtration models described in the previous sections. Similar analyses could be performed in conjunction with other wireline tools (e.g., sonic, nuclear, formation tester, etc.); however, the example of an induction tool seems ideal for the basic sensitivity study pursued in this paper. In related publications, Semmelbeck and Holditch,11 Phelps,4 and Ramakrishnan and Wilkinson12 have presented an in-depth analysis of the relationship borne by 1D mud-filtrate-invasion models and array-induction measurements. The spatial distributions for water saturation and salt concentration are obtained directly from the INVADE simulation results. In turn, salt concentrations are converted into equivalent values of connate water resistivity, $R_w$, using the formula

$$R_w(\tilde{r}) = \frac{3647.5}{C_w(\tilde{r})^{0.995}} + 39$$

where $T$ is temperature measured in degrees Centigrade, $C_w$ is salt concentration measured in ppm, and $\tilde{r}$ is the location of the observation point.

**Figs. 23 and 24** are spatial cross sections of the simulated distributions of salt concentration and formation resistivity, respectively, for Case 9. As described in Tables 1 and 2, the salt concentrations in the mud filtrate and in the formation are assumed equal to 43,900 ppm and 102,500 ppm, respectively. The process of salt mixing causes the electrical resistivity of connate water to experience considerable spatial variations in the vicinity of the borehole wall.

For completeness, **Figs. 25 and 26** show the simulated cross sections of salt concentration and formation resistivity, respectively, for Case 10. We remark that the spatial distribution of salt concentration has a considerable influence on the spatial distribution of electrical resistivity. This effect is often neglected in the estimation of in-situ hydrocarbon saturation by way of Archie’s equations. Parenthetically, a case study reported by Borah et al.19 makes use of the time-lapse physics of 1D mud-filtrate

Fig. 20—Case 9: spatial cross section of water saturation. Example of a deviated well. The angle of deviation is 45°, and the time of invasion is 2 days.

Fig. 21—Case 10: spatial cross section of water saturation. Example of a deviated well and of an enhanced horizontal permeability. The angle of deviation is 45°, and the time of invasion is 2 days.

Fig. 22—Case 11: spatial cross section of water saturation. Example of a deviated well intersected by six horizontal layers. Hydraulic communication exists between adjacent layers. The angle of deviation is 45°, and the time of invasion is 2 days.
invasion to jointly interpret induction and nuclear logs acquired in a vertical well.

**Sensitivity Studies of Time-Lapse Behavior.** We study the time-lapse behavior of the process of invasion using as an example the case of one vertical well and one permeable layer shouldered by impermeable shale layers. Petrophysical and fluid-flow parameters associated with the various components of this time-lapse case are summarized in Table 4. A value of 43,900 ppm was assumed for the salt concentration in the mud. This value of salt concentration is equivalent to an electrical resistivity of 0.1 ohm-m at formation temperature. The thickness of the formation was assumed to be 1.8 m, and the salt concentration of connate water was made equal to 102,500 ppm (equivalent to an electrical resistivity of 80 ohm-m).

Figs. 27 and 28 are radial profiles away from the borehole wall of the cross sections of water saturation and salt concentration, respectively, simulated as a function of time after the onset of invasion, from 0.5 to 2 days. The radial profiles are taken through the center of the permeable layer. Fig. 29 shows the radial profile of electrical resistivity calculated from the radial profiles of water saturation and salt concentration displayed in Figs. 27 and 28, respectively. At initial conditions, the resistivity of the formation is 80 ohm-m. After 2 days of invasion, the invasion front has advanced to 0.95 m away from the borehole wall, and the resistivity for the invaded zone is approximately 2 to 3 ohm-m.

The spatial cross sections of electrical resistivity were used to simulate the response of a dual-induction borehole-logging instrument (DIT-B*). Plots of the numerically simulated wireline dual-induction logs are displayed in Fig. 30 as functions of depth (measured along the axis of the well) and for various times of invasion.

**TABLE 4—SUMMARY OF PETROPHYSICAL AND FLUID-FLOW PARAMETERS CONSIDERED IN THE NUMERICAL SIMULATION OF THE TIME-LAPSE INVASION CASE**

<table>
<thead>
<tr>
<th>Reservoir Properties</th>
<th>Horizontal permeability (md)</th>
<th>Vertical permeability (md)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1,000.0</td>
<td>100.0</td>
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<tr>
<td>Porosity</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Irreducible water saturation</td>
<td>0.10</td>
<td></td>
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<tr>
<td>Residual oil saturation</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Fluid Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>24.00</td>
<td></td>
</tr>
<tr>
<td>Number of permeable layers</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Thickness of layer (m)</td>
<td>1.80</td>
<td></td>
</tr>
<tr>
<td>Archie’s equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formation water salinity (ppm)</td>
<td>102,500.00</td>
<td></td>
</tr>
<tr>
<td>Mud filtrate salinity (ppm)</td>
<td>43,900.00</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>$R_w$ (ohm-m)</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Well</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wellbore radius (m)</td>
<td>0.10</td>
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</table>

**Fig. 23**—Case 9: spatial cross section of salt concentration. Example of a deviated well. The angle of deviation is 45°, and the time of invasion is 2 days.

**Fig. 24**—Case 9: spatial cross section of formation resistivity. Example of a deviated well. The angle of deviation is 45°, and the time of invasion is 2 days. Electrical resistivities are displayed with a logarithmic scale.

**Fig. 25**—Case 10: spatial cross section of salt concentration. Example of a deviated well and of an enhanced horizontal permeability. The angle of deviation is 45°, and the time of invasion is 2 days.

**Fig. 26**—Case 10: spatial cross section of formation resistivity. Example of a deviated well and an enhanced horizontal permeability. The angle of deviation is 45°, and the time of invasion is 2 days. Electrical resistivities are displayed with a logarithmic scale.
The deep induction log (ILD)* is sensitive to a deeper zone of investigation than the medium induction log (ILM)* and hence shows a higher resistivity reading for the same invasion time. As time progresses, the resistivity readings for both ILM and ILD decrease because of the advancing column of mud filtrate. After 0.5 days of invasion, the resistivity readings for ILM and ILD are 7.3 and 8.9 ohm-m, respectively. At 2 days of invasion, the resistivity readings for ILM and ILD decrease to 3.5 and 4.4 ohm-m, respectively. This exercise shows that induction-instrument readings can experience measurable variations before the process of invasion comes to a halt. It is also evident from Fig. 30 that accurate assessment of water saturation from deep induction logs for the example under consideration would require a sizable correction for the effect of invasion.

Pistonlike saturation/resistivity radial profiles of invasion are often assumed in the interpretation of borehole induction data. The simulation examples described in this paper show that the combination of water-saturation and salt-concentration profiles seldom leads to a pistonlike radial profile of electrical resistivity. Accurate interpretation of borehole induction data requires consideration of the time-dependent nature of mud-filtrate invasion and salt mixing. In an effort to illustrate this significant point, Fig. 31 shows an example of formation resistivities assuming pistonlike invasion profiles for both water saturation and salt concentration as a function of time of invasion. The electrical resistivity is 2.0 ohm-m in the near-wellbore region and 80 ohm-m in the virgin zone. Simulation of dual-induction instrument readings was performed for the radial resistivity profiles shown in Fig. 31, and the results were

* Mark of Schlumberger Wireline Services, Houston.
accurate and comprehensive fluid-flow models of filtrate invasion. It is envisioned that 3D petrophysical templates based on a radial sequence of pistonlike invasion is the exception rather than the norm. Initial variations of salt concentration could cause substantial biases to the spatial distribution of electrical resistivity. In turn, unaccounted spatial distribution of salt concentration resulting from the process of mud-filtrate invasion could significantly distort the spatial distribution of resistivity readings, ILM and ILD, as a function of time of invasion.

Conclusions

We have introduced a procedure to simulate numerically complex cases of mud-filtrate invasion in overbalanced vertical, horizontal, and highly deviated wells. This procedure consisted of (a) developing an effective time-dependent flow-rate function that could capture the effect of mudcake buildup and (b) adapting an existing 3D multiphase reservoir simulator to enforce boundary, initial, and source conditions specific to the physics of mud-filtrate invasion. Tests of accuracy and internal consistency lent excellent credence to our 3D simulations of mud-filtrate invasion in horizontal wells.

A systematic sensitivity analysis was carried out to quantify the influence of several petrophysical and geometrical parameters on the spatial distribution of mud-filtrate invasion away from the borehole wall. The sensitivity studies were summarized with cross sections of the spatial distribution of water saturation resulting from invasion. All our sensitivity studies adopted the reference model of a freshwater-based mud and porous rock formations saturated with both oil and irreducible water. The sensitivity studies showed that the mud-filtrate-invasion process is affected by the geometry of the permeable beds and by petrophysical parameters such as relative permeability, permeability anisotropy, capillary pressure, gravity segregation, effective porosity, and hydraulic communication (crossflow) between adjacent layers.

The simulations reported in this paper shed new light on the interpretation of wireline measurements in terms of petrophysical parameters in the vicinity of the borehole wall. It was found that the spatial distribution of salt concentration resulting from the process of mud-filtrate invasion could significantly distort the spatial distribution of electrical resistivity. In turn, unaccounted spatial variations of salt concentration could cause substantial biases in the estimation of in-situ hydrocarbon saturation.

We have found that a customary notion that regards invasion as a radial sequence of pistonlike invasion is the exception rather than the norm. It is envisioned that 3D petrophysical templates based on accurate and comprehensive fluid-flow models of filtrate invasion will provide modern quantitative ways to integrate a large variety of wireline measurements into estimates of rock formation properties. A basic example of the application of such a template was shown here for the interpretation of wireline measurements of electromagnetic induction.

Nomenclature

\[ a = \text{Archie’s tortuosity/cementation factor} \]
\[ C = \text{static filtration constant} \]
\[ C_s = \text{salt concentration measured in ppm} \]
\[ k = \text{formation permeability} \]
\[ k_H = \text{horizontal permeability} \]
\[ k_{ro} = \text{oil relative permeability} \]
\[ k_{rw} = \text{water relative permeability} \]
\[ k_v = \text{vertical permeability} \]
\[ m = \text{Archie’s cementation exponent} \]
\[ N = \text{total number of angular borehole segments} \]
\[ n = \text{Archie’s saturation exponent} \]
\[ q(\alpha,t) = \text{flow rate for a segment with an angle of } \alpha \]
\[ q_{ao} = \text{flow-rate difference between the 180° segment and the 90° segment} \]
\[ q_{f} = \text{flow rate for mud filtrate} \]
\[ R = \text{radial distance away from the wellbore} \]
\[ R_{150} = \text{bulk resistivity of a medium} \]
\[ R_{nw} = \text{electrical resistivity of connate water} \]
\[ r = \text{observation point} \]
\[ S_w = \text{water saturation} \]
\[ t = \text{time} \]
\[ t_1 = \text{starting time for invasion} \]
\[ t_2 = \text{stop time for invasion} \]
\[ T = \text{temperature measured in degrees Centigrade} \]
\[ V = \text{total filtrate volume} \]
\[ X = \text{location in the X-direction} \]
\[ Z = \text{vertical distance} \]
\[ \alpha = \text{angle (degree) between borehole segment and downward direction} \]
\[ \phi = \text{porosity} \]

Acknowledgments

We would like to express our deepest appreciation to Baker Atlas, Halliburton, and Schlumberger for funding this work through the
U. of Texas at Austin’s Center of Excellence in Formation Evaluation. Our gratitude goes to Guozhong Gao for simulating the induction-resistivity logs shown in this paper. We are obliged to David Kennedy and Alberto Mezzatesta for their thorough review of the first version of the manuscript and for their constructive technical and editorial criticism. Their feedback has significantly improved the quality of the final paper.

References


SI Metric Conversion Factors

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<th>Conversion Factor</th>
<th>Unit</th>
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<tr>
<td>cm</td>
<td>1.0* E – 01</td>
<td>m</td>
</tr>
<tr>
<td>°F</td>
<td>(°F – 32)/1.8</td>
<td>°C</td>
</tr>
<tr>
<td>ft</td>
<td>3.048*</td>
<td>m</td>
</tr>
<tr>
<td>ft³</td>
<td>2.831 × 10⁶</td>
<td>m³</td>
</tr>
<tr>
<td>in.</td>
<td>2.54*</td>
<td>cm</td>
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<tr>
<td>psi</td>
<td>6.894 × 10⁵</td>
<td>kPa</td>
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</tbody>
</table>

*Conversion factor is exact.

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