Estimation of Permeability from Borehole Array Induction Measurements: Application to the Petrophysical Appraisal of Tight Gas Sands

Jesús M. Salazar, Carlos Torres-Verdín, Faruk O. Alpak, Tarek M. Habashy, and James D. Klein

ABSTRACT

This paper describes the successful application of a new petrophysical inversion method to estimate permeability from borehole array induction measurements. We consider measurements acquired in a North Louisiana tight gas sand formation subject to water-base mud-filtrate invasion. The inversion methodology incorporates the physics of two-phase immiscible displacement and salt mixing between the invading water-base mud filtrate and connate water. Moreover, the invasion model honors the physics of mudcake growth as well as the petrophysical properties that govern the process of two-phase three-component flow. The outcome of the inversion is the absolute permeability for each flow subunit within a gas-bearing production zone.

Rock formations under consideration consist of low-permeability amalgamated sands. Array induction measurements exhibit significant vertical fluctuations within an individual fluid production unit. In view of this, the estimation of permeability is designed to consider the effect of the number of layers and of their thickness when describing a fluid production unit. We show how the progressive addition of flow subunits improves the match of array induction measurements within the limits of vertical resolution. Accurate reconstructions of layer-by-layer permeability are primarily constrained by the availability of a-priori information about time of invasion, rate of mud-filtrate invasion, overbalance pressure, capillary pressure, and relative permeability. Sensitivity analyses show that the estimated values of permeability properly reproduce the measured array induction logs even in the presence of small changes of relative permeability, capillary pressure, porosity, and Archie’s parameters. The estimated values of permeability agree well with those of core measurements acquired from other wells in the same gas-bearing formation.

Keywords: array induction, permeability, invasion, invasion modeling, tight gas sands

INTRODUCTION

Tight sands comprise important accumulations of natural gas. Similar to conventional oil- and gas-bearing formations, tight gas sands are associated with complex geological and petrophysical systems that include heterogeneities at all spatial scales. However, unlike conventional oil and gas reservoirs, tight gas sands usually exhibit unique gas storage and production characteristics. Effective produc-
The main goal of the study is to estimate absolute permeability by quantifying the influence of mud-filtrate invasion on borehole array induction logs. This process requires the availability of well logs, fluid properties, and core measurements. Core laboratory data are not available for the formation under analysis. Instead, we make use of laboratory measurements from nearby fields penetrating the same geological interval to calibrate the petrophysical properties calculated from well logs and other empirical relationships (S.A. Holditch Associates, 1988 and Luffel et al., 1991).

Core measurements are used to synthesize the petrophysical model necessary to simulate the physics of mud-filtrate invasion. Such a model is “calibrated” against the existing suite of wireline logs, especially array induction logs. Flow subunits defined from porosity and the initial guess of absolute permeability are taken as horizontal layers to simulate the process of mud-filtrate invasion with a two-dimensional chemical flow simulator that includes the effect of salt mixing between mud filtrate and connate water. We generate spatial distributions of electrical resistivity from the simulated cross-sections of water saturation and salt concentration using Archie’s equation. In turn, these spatial distributions are used to simulate array induction imager tool (AIT™) measurements acquired across the formation.

Numerical simulation of mud-filtrate invasion has been attempted previously to simulate borehole electromagnetic measurements and to estimate petrophysical properties. Semmelbeck and Holditch (1988) modeled conventional induction resistivity tools in synthetic low-permeability homogeneous formations. Tobola and Holditch (1991) and Yao and Holditch (1996) used a history-matching approach to estimate permeability based on the one-dimensional (radial) simulation of time-lapse array induction resistivity logs for water-base mud invading a gas-saturated low-permeability formation. Salazar et al. (2005) estimated permeability in several field examples using the concept of petrophysical flow units and resistivity matching of a vertically heterogeneous tight gas sand formation under the effect of water-base mud-filtrate invasion. Nonlinear inversion of borehole induction resistivity measurements has been applied successfully using the physics of water-base mud-filtrate invasion for the estimation of permeability from one-dimensional (radial) fractional flow distributions (Ramakrishnan and Wilkinson, 1999), as well as for the estimation of porosity and permeability in two-dimensional axisymmetric rock formations (Alpak et al., 2006 and Torres-Verdín et al., 2006).

In this paper, we estimate absolute permeability using the inversion algorithm developed by Alpak et al. (2006), where estimation of permeability for each layer in a two-dimensional rock formation is posed as a nonlinear minimization problem. We consider limiting values of rock and fluid properties to quantify the impact of the spatial distributions of water saturation and salt concentration on array induction measurements. Sensitivity analyses are also performed to assess the impact of the assumptions made about mud properties, mudcake growth, time of invasion, capillary pressure, relative permeability, and fluid viscosity on the estimated values of permeability.

The first stage of the study consists of defining rock types by relating geological framework, lithofacies, and petrology to porosity and permeability. The second stage of the work integrates the rock-type model with formation evaluation data to define reservoir compartments and flow units (Salazar et al., 2005). We integrate logs and core measurements to extend the rock-type model, and to compute continuous storage and flow capacity specific to a given production unit.

The third stage of the work quantifies the influence of mud-filtrate invasion on the spatial distribution of fluids in permeable rocks around the wellbore. Overbalance pressure along with the low porosity of the rock contributes to deep invasion of mud filtrate. In-situ gas saturation of the formation ranges from 40 to 60%, with the remaining pore space occupied by irreducible water saturation. The salinity of mud filtrate is between 1,500 to 3,800 ppm, whereas the salinity of the connate water is approximately 160 kppm. After 5 days of invasion, spatial distributions of electrical resistivity are calculated from the simulated spatial distributions of water saturation and salt concentration. Subsequently, we simulate array induction measurements and compare them against actual field measurements.

Simulations of mud-filtrate invasion continue with the only free parameter being average absolute permeability per flow unit. All the remaining petrophysical parameters required by the simulation are either calculated from well logs or extrapolated from the core measurements. We use a modified Timur-Tixier permeability equation (Balan et al., 1995) to compute initial values of absolute permeability. These initial permeability values are progressively adjusted in the simulation of mud-filtrate invasion until an acceptable match is reached between the measured and simulated array induction curves. In the final stage, we perform

---

1Mark of Schlumberger
TABLE 1  Summary of Archie’s parameters and rock and fluid properties used to estimate water saturation and porosity.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archie’s tortuosity factor $a$</td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>Archie’s cementation exponent $m$</td>
<td></td>
<td>1.95</td>
</tr>
<tr>
<td>Archie’s saturation exponent $n$</td>
<td></td>
<td>1.75</td>
</tr>
<tr>
<td>Con nate water resistivity @ 210°F</td>
<td>ohm m</td>
<td>0.02</td>
</tr>
<tr>
<td>Invasion water resistivity @ 210°F</td>
<td>ohm m</td>
<td>0.56</td>
</tr>
<tr>
<td>Matrix density</td>
<td>g/cm³</td>
<td>2.65</td>
</tr>
<tr>
<td>Shale density</td>
<td>g/cm³</td>
<td>2.68</td>
</tr>
<tr>
<td>Water density</td>
<td>g/cm³</td>
<td>1.00</td>
</tr>
<tr>
<td>Water-hydrocarbon density (mixture)</td>
<td>g/cm³</td>
<td>0.50</td>
</tr>
</tbody>
</table>

automatic inversion of permeability from the AIT™ apparent resistivity curves using as a starting model the results obtained from manual adjustment of layer permeability values.

GEOLOGICAL DESCRIPTION

The North Louisiana tight gas sand formation under analysis consists of very fine- to fine-grained sandstone, shale, and some sandy, fossiliferous oolitic limestone (Finley, 1984). Primary sediments originated from a major influx of terrigenous clastic deposits during the Early Cretaceous. Fluvial deposition by at least two major rivers was responsible for sediment accumulation. The rocks in this formation are texturally mature quartz arenites and subarkose sands (McGowen and Harris, 1984). Interpretation of a large area of the formation suggests global sediment deposition by a system of coalescing deltas prograding from the west, northwest, and north (Finley, 1984). In general, geological cross-sections through the formation show a thick, sand-dominated wedge of sediments mainly consisting of braided stream deposits. Such braided stream facies were reworked by marine transgression when the deltas entered the marine environment (Finley, 1984).

ASSESSMENT OF WATER SATURATION AND EFFECTIVE POROSITY

We focus the petrophysical analysis to a 20.5-ft thick interval. The formation consists of a relatively clean gas-saturated clastic sequence with connate water salt concentration in the order of 160 kppm. Therefore, we use Archie’s (1942) equation to compute water saturation without specific adjustments for the presence of shale. Water saturation is given by

$$S_w = \frac{R_w}{R_i} \phi^{-a}$$

where $\phi$ is effective porosity, $R_w$ is connate water resistivity, $R_i$ is true formation resistivity, $a$ is the tortuosity factor, and $m$ and $n$ are the cementation (lithology) and saturation exponents, respectively. Effective porosity is computed using a dual-fluid dual-mineral nonlinear model given by

$$\tilde{\rho}_s = \rho_s + \rho_w \left( 1 - \phi - C_{sh} \rho_{sh} \right)$$

where $\tilde{\rho}_s$ is bulk density, $\rho_{sh}$ is matrix (quartz) density, $\rho_{sh}$ is shale density, $\rho_1$ is mud filtrate (fresh water) density, $\rho_2$ is the density of the gas/mud-filtrate mixture estimated from log interpretation charts (Schlumberger, 1991), $R_{wxo}$ is the invaded zone water resistivity, $R_{xo}$ is the invaded zone resistivity, and $C_{sh}$ is volumetric shale concentration calculated with a linear transformation of the gamma-ray log.

Application of Thomas and Stieber’s (1975) methodology to log-computed volumetric shale concentration and porosity indicated that the dominant distribution of clay in the formation was in the form of dispersed shale. We computed an initial guess of effective porosity ($\phi_o$) from shale-corrected neutron and density porosity via the formula

$$\phi_o = \frac{1}{2} \left[ (\phi_n^{sh})^2 + (\phi_B^{sh})^2 \right]$$

where $\phi_n^{sh}$ and $\phi_B^{sh}$ are shale-corrected neutron and density porosity, respectively. Equation (2) is solved by minimizing the difference between measured bulk density ($\rho_b$) and bulk density computed with the initial guess of porosity, i.e.,

$$\min_{\phi_o} \sum (\rho_{bi} - \tilde{\rho}_{bi})^2, \quad \phi_o \in [0,1] .$$

The minimum of equation (4) yields the best estimate of effective porosity. Water saturation is computed with equation (1). Table 1 describes the petrophysical and fluid parameters used to apply this methodology. In order to assess the value of true formation resistivity devoid of invasion and shoulder-bed effects, we simulated the AIT apparent resistivity measurements assuming a horizontal layer undergoing piston-like invasion. The simulated AIT measurements were compared to field measurements and the values of formation electrical resistivity adjusted until reaching a good match between simulations and measurements. This strategy yielded reliable values of initial water saturation ($S_w$). Figure 1 is a plot of lithology, resistivity, and porosity logs across the depth interval under analysis.
Figure 2 shows the log of water saturation computed with Archie’s equation and resistivity values obtained with forward modeling of AIT apparent resistivity measurements. We believe that the relatively high values of calculated $S_w$ (40%-60%) are due to irreducible water saturation in the formation that could be due to presence of clay and microporosity. Fluid production in the well is mainly gas, hence confirming that the values of computed water saturation correspond to irreducible fluid.

### INITIAL MODEL FOR ABSOLUTE PERMEABILITY

A permeability value (initial guess) is necessary to initialize the simulation of the process of mud-filtrate invasion. Core measurements were only available from nearby gas fields that penetrated the same formation. Such measurements were used to construct the initial guess of permeability. Several approaches were considered to calculate the best initial estimate of permeability (Balan et al., 1995 and Haro, 2004). Sensitivity analysis indicated that the generalized Timur-Tixier equation properly reproduced the interplay between porosity, permeability, and irreducible water saturation of core measurements. Accordingly, the relationship between permeability ($k$, md), porosity ($\phi$, fraction), and irreducible water saturation ($S_{iwr}$, fraction) is given by

$$k = A \frac{\phi^B}{S_{iwr}^C},$$  \hspace{1cm} (5)\

where $A$, $B$, and $C$ are constants to be determined from a multilinear regression analysis.

Klinkenberg-corrected permeability and porosity were obtained from the available core measurements at 2,000 psi of overburden pressure. We used gas-water capillary pressure curves obtained from the same core measurements to estimate irreducible water saturation. This procedure yielded the following equation for permeability:

$$k = 0.04 \frac{\phi^{1.83}}{S_{iwr}^{2.3}}.$$  \hspace{1cm} (6)

The degree of confidence for this equation is in the order of 60% based on the correlation coefficient of the multilinear regression. We applied equation (6) to well logs using porosity from equation (2) and irreducible water saturation\(^1\) computed from (Dewan, 1983)

\(^1\)Equation (7) is only used as an input of equation (6) to compute the initial guess of permeability. Based on the explanation given in the previous section, initial water saturation is considered equal to irreducible water saturation. Therefore, the remainder of our analysis is carried out assuming such a saturation condition.
where $\phi_i$ is total porosity and $\phi_{sh}$ is total shale porosity. An alternative way to estimate $S_{wirr}$ is via the general relationship $S_{wirr} = \beta C_{sh} + \gamma / \phi_i$, where $\beta$ and $\gamma$ are constants to be determined. We used equation (7) since it does not require knowledge of additional empirical constants and $S_{wirr}$ can be obtained directly from well logs. Moreover, the objective of equation (6) is to obtain only an estimate of permeability that will be used as an initial guess for the inversion algorithm. Table 2 is a summary of the average petrophysical properties calculated for the interval under study. Figure 2 displays the results of the petrophysical analysis with the main curves rendered by the methodology explained above. Finally, Figure 3 is an example of the finite-difference grid used in the simulation of mud-filtrate invasion.

### Table 2: Summary of average petrophysical properties for the single-layer case.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>ft</td>
<td>20.5</td>
</tr>
<tr>
<td>Effective porosity fraction</td>
<td></td>
<td>0.143</td>
</tr>
<tr>
<td>Water saturation fraction</td>
<td></td>
<td>0.410</td>
</tr>
<tr>
<td>Shale concentration fraction</td>
<td></td>
<td>0.132</td>
</tr>
<tr>
<td>Absolute Permeability</td>
<td>md</td>
<td>1.502</td>
</tr>
</tbody>
</table>

\[
S_{wirr} = C_{sh} \frac{\phi_{sh}}{\phi_i}, \tag{7}
\]

### SIMULATION OF MUD-FILTRATE INVASION

We use a multiphase chemical flow simulator to calculate the flow rate of mud-filtrate invasion in a manner similar to that of a water injection process. The software used is a modified version of the code UTCHEM developed by The University of Texas at Austin (Delshad et al., 1996). This software is a finite-difference simulator referred to as INVADE (Wu et al., 2004 and Wu et al., 2005) that was specifically developed to solve the partial differential equations and boundary conditions of immiscible cylindrical flow coupled with mudcake growth.

#### Radial grid and vertical flow units

Simulation of mud-filtrate invasion is performed assuming cylindrical flow and permeability isotropy. A two-dimensional finite-difference grid is constructed with 106 logarithmically spaced radial nodes that include shorter radial steps in the near wellbore region. Rock properties are considered constant in the radial direction. In a single-layer case, the vertical grid consists of thin numerical layers to ensure high accuracy in the estimation of the flow rate of mud-filtrate invading the formation.

We make use of a Lorenz plot (Gunter et al., 1997) to identify individual flow units in the formation under analysis. Figure 4 shows one such plot, describing the relationship between cumulative porosity and cumulative permeability as a function of reservoir thickness. The variable nature of this plot indicates the presence of several individual flow subunits within the formation. Specifically, each segment with a constant slope in the plot identifies an individual flow subunit with constant petrophysical properties. Steep slopes in the same plot are associated with high values of flow capacity. Figure 5 is an example of the finite-difference grid used in the simulation of mud-filtrate invasion.

### Capillary pressure and relative permeability

We use Brooks-Corey (Corey, 1994) two-phase equations to assign water-gas capillary pressure curves to each petrophysical layer. Accordingly, the first imbibition cycle of the capillary pressure curve is given by

\[
P_c = P_c^o \sqrt{\frac{\phi}{k}} (1 - S_N)^{\epsilon_p}, \tag{8}
\]

where $P_c$ is capillary pressure, $P_c^o$ is the coefficient for capil-
lary pressure, $e_p$ is the pore-size distribution exponent, $\phi$ is porosity, $k$ is permeability, and $S_N$ is the normalized wetting phase saturation, given by

$$S_N = \frac{S_w - S_{wr}}{1 - S_{nr} - S_{nwr}}$$

(9)

where $S_{wr}$ and $S_{nwr}$ are the residual wetting and non-wetting phase saturations, respectively. Water-gas relative permeability curves in the saturated zone are also estimated via the Brooks-Corey equations, namely,

$$k_{rw} = k_{rw}^0 S_N^{e_w},$$

(10)

and

$$k_{rnw} = k_{rnw}^0 (1 - S_N)^{e_{nw}},$$

(11)

where $k_{rw}$ and $k_{rnw}$ are wetting and non-wetting relative permeabilities, $k_{rw}^0$ and $k_{rnw}^0$ are relative permeability end points, and $e_w$ and $e_{nw}$ are empirical exponents for each fluid phase. We estimated the three parameters included in the Brooks-Corey equation (exponents, coefficients and end points) from core measurements acquired in a nearby field from the same rock formation. Figure 6 is a graphical representation of capillary pressure and relative permeability curves calculated with the equations described above.

### INVADE results

At the outset, the simulation of mud-filtrate invasion is performed assuming a homogeneous flow unit. Petrophysical properties of this flow unit are described in Table 2, and Table 3 describes the corresponding mud filtrate, mudcake and formation fluids properties. The simulation of the process of mud-filtrate invasion requires accurate knowledge of the time during which the well was exposed to invasion. According to field reports, the interval under analysis was exposed to invasion for a time period between three and six days before the acquisition of resistivity logs.

Several simulation cases for three, four, five, and six days of invasion were considered to obtain the best estimate
of the rate of mud-filtrate invasion. Figure 7 describes the transient behavior of the flow rate of mud filtrate, mudcake thickness, and pressure across mudcake. The flow rate decays monotonically with time before reaching its steady-state value. In low permeability rock formations (~1 to 5 md), laboratory experiments (Dewan and Chenevert, 2001) and numerical simulations (Wu et al., 2005) have proved that despite the fact that the flow rate is relatively high at the onset of the invasion, it quickly tends to steady-state behavior after the mudcake is completely formed. The maximum mudcake thickness is reached approximately 15 hours after the onset of invasion. Once the mudcake is completely formed, the overbalance pressure stabilizes at approximately 800 psi and the flow rate becomes constant.

The computed flow rate is averaged over the entire simulation time and input as a time-constant value to simulate the process of invasion. For the case of a vertically heterogeneous flow unit the flow rate is estimated separately for each individual flow subunit (petrophysical layer). A five-day interval was chosen as the optimum time of invasion after studying simulation results for different time intervals. For a long time of invasion (more than three days), mudcake is already formed and the average flow rate is dominated by its early time behavior (less than 15 hours after the onset of invasion). Therefore, the error in the estimation of flow rate for extended invasion times is negligible.

**FORWARD MODELING AND INVERSION ALGORITHMS**

The forward problem consists of simulating the process of two-phase flow of water-base mud filtrate invading a partially gas-saturated formation. This problem is modeled as convective transport of aqueous and hydrocarbon phases, and components of water, hydrocarbon, and salt concentration (Alpak et al., 2003). Isothermal convective miscible transport is assumed for the salt component while diffusion between mud filtrate and connate water is neglected. Upper, lower, and external boundaries of the formation impose no-flow conditions. A constant flow rate, obtained as the time average of the flow rate yielded by INVADE (the constant line shown in the upper panel of Figure 7), is imposed at the borehole wall for each numerical layer as a fixed source condition. Modeling of multi-phase and multi-component fluid-flow is performed with the ECLIPSE® commercial finite-difference reservoir simulator.

Petrophysical properties such as porosity, initial water saturation, and initial estimate of permeability obtained from the petrophysical assessment, along with core-calibrated Brooks-Corey capillary pressure and relative permeability curves are the main rock properties input to the simulations (Table 2). In addition, mudcake properties, hydro-

---

**TABLE 3** Summary of mudcake, fluid, and formation properties used in the simulation of the process of mud-filtrate invasion.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mudcake reference permeability</td>
<td>md</td>
<td>0.03</td>
</tr>
<tr>
<td>Mudcake reference porosity</td>
<td>frac</td>
<td>0.30</td>
</tr>
<tr>
<td>Mud Solid Fraction</td>
<td>frac</td>
<td>0.06</td>
</tr>
<tr>
<td>Mudcake maximum thickness</td>
<td>cm</td>
<td>1.02</td>
</tr>
<tr>
<td>Mudcake compressibility exponent</td>
<td>frac</td>
<td>0.40</td>
</tr>
<tr>
<td>Mudcake exponent multiplier</td>
<td>frac</td>
<td>0.10</td>
</tr>
<tr>
<td>Mud hydrostatic pressure</td>
<td>psi</td>
<td>5,825</td>
</tr>
<tr>
<td>Initial formation pressure</td>
<td>psi</td>
<td>5,000</td>
</tr>
<tr>
<td>Mud filtrate viscosity</td>
<td>cp</td>
<td>1.00</td>
</tr>
<tr>
<td>Gas viscosity</td>
<td>cp</td>
<td>0.02</td>
</tr>
<tr>
<td>Mud filtrate density</td>
<td>g/cm³</td>
<td>1.00</td>
</tr>
<tr>
<td>Gas density</td>
<td>g/cm³</td>
<td>0.0016</td>
</tr>
<tr>
<td>Mud filtrate salt concentration</td>
<td>ppm</td>
<td>3,600</td>
</tr>
<tr>
<td>Connate water salt concentration</td>
<td>ppm</td>
<td>160,000</td>
</tr>
<tr>
<td>Wellbore radius</td>
<td>cm</td>
<td>10.26</td>
</tr>
<tr>
<td>Maximum invasion time</td>
<td>days</td>
<td>5.00</td>
</tr>
<tr>
<td>Formation Temperature</td>
<td>°F</td>
<td>210</td>
</tr>
<tr>
<td>Formation outer boundary</td>
<td>m</td>
<td>262</td>
</tr>
<tr>
<td>Residual water saturation</td>
<td>frac</td>
<td>0.40</td>
</tr>
<tr>
<td>Residual gas saturation</td>
<td>frac</td>
<td>0.10</td>
</tr>
</tbody>
</table>

---

1Mark of Schlumberger
static and formation pressure, and fluid properties are input to model the process of mud-filtrate invasion. Table 3 summarizes the formation, fluid, and mudcake properties used in the numerical simulations described in this paper.

The main outputs of the simulation are the spatial distributions of water saturation and salt concentration. Water saturation is transformed into electrical resistivity via the Archie (1942) equation. On the other hand, the spatial distribution of salt concentration is used to compute equivalent values of water resistivity \( R_w \) using the equation documented by Bigelow (1992) and Hallenburg (1998) with the corresponding Arps (1953) temperature conversion factor, namely,

\[
R_w = \left( 0.0123 + \frac{3647.5}{[NaCl]^{0.955}} \right) \left( 81.77 \left( \frac{1}{T + 6.77} \right) \right), \tag{12}
\]

where \( T \) is reservoir temperature in °F, and \([NaCl]\) is salt concentration in ppm. Equation (12) is an average approximation of the three equations documented by Worthington et al. (1990). They used least-squares regression to fit the values given on tables of brine conductivities as a function of specific values of NaCl-brine concentration. Equation (12) matches the three resistivity zones defined in Worthington et al.’s work with an error below 2% for salinity between 500 and 100,000 ppm. For salt concentration between 100,000 to 230,000 ppm the error is 2-10%. In the reservoir under consideration, salt concentration ranges between 1,500 and 160,000 ppm. Therefore, we adopted equation 12 to convert salt concentration to water resistivity.

### Array induction resistivity modeling

The next stage is the forward modeling of array induction measurements from the spatial distribution of formation resistivity. This requires the frequency-domain solution of Maxwell’s equations. We performed the numerical simulation with the SLDINV finite-difference algorithm advanced by Druskin et al. (1999). This software provides multi-frequency simulations of array induction measurements via the Spectral Lanczos Decomposition Method (Alpak et al., 2003).

### Inversion method for the estimation of permeability

Inversion of permeability from array induction measurements is posed as the minimization of a quadratic objective function subject to model constraints (Alpak et al., 2006). In this paper, we adopt the quadratic objective function

\[
C(x) = \frac{1}{2} \left[ \|\mu \|e(x)\|^2 - \chi^2 \right] + \|x - x_p\|^2. \tag{13}
\]

where \( \chi^2 \) is the target data misfit, and \( e(x) \) is the vector of data residuals constructed as the normalized difference between the measurements and their simulations, i.e.,

\[
\|e(x)\|^2 = \sum_{j=1}^M \left[ \frac{S_j(x)}{m_j} - 1 \right]^2. \tag{14}
\]

In the above expressions, \( M \) is the number of measurements, \( m_j \) denotes the \( j \)-th measurement, and \( S_j \) is the corresponding simulated measurement for the vector of unknown model parameters, \( x \). The latter vector is given by

\[
x = [x_1, \ldots, x_N]^T, \tag{15}
\]

where the subscript \( T \) represents the transpose and \( N \) is the number of unknown parameters, in this case layer-by-layer values of absolute permeability. In equation (13), the positive scalar factor \( \mu \) is used as a stabilization parameter (also called a Lagrange multiplier) to assign relative importance to the two additive terms of the objective function. Finally, the vector \( x_p \) contains reference values of layer permeability used to bias the search for the minimum of the quadratic objective function. In our case, vector \( x_p \) is constructed with the initial guess of layer permeability values rendered by log analysis (equation (6)).

In this paper, the measurement vector consists of values of AIT apparent resistivity sampled at 0.25-foot intervals. There are five apparent resistivities per sampling depth and a total of 277 sampling points in the depth interval from -25.75 ft to +43.25 ft. Consequently, the total number of input measurements is 1385 including measurements above and below the flow unit shown in Figure 1 (0-20.5 ft). Based on measurement noise considerations, the target misfit in equation (13) is set to 0.01.

We assume that the locations of layer boundaries are known from the previously described modified Lorenz plots. Moreover, the inverted model parameters are constrained to remain within their physical bounds using a nonlinear transformation (Habashy and Abubakar, 2004). Minimization of the objective function is performed with a Gauss-Newton method that enforces a backtracking line search algorithm along the descent direction. This minimization strategy guarantees a monotonic reduction of the data misfit from iteration to iteration. The choice of the Lagrange multiplier is adaptively linked to the condition number of the Hessian matrix of the quadratic objective function (Alpak et al., 2004).

### ESTIMATION OF PERMEABILITY: FIELD EXAMPLE

We consider the field example described in Figure 1 to test our inversion methodology. A homogeneous sin-
A single-layer case was used as initial model to calibrate the simulation of mud-filtrate invasion and the resistivity inversion. Sensitivity analysis was performed to assess the influence of several petrophysical properties on the inverted permeability until obtaining the best forward model representing the formation. Subsequently, a heterogeneous multi-layer case was considered to estimate layer-by-layer permeabilities within the fluid production unit.

**Base case: homogeneous production unit**

Initially, we perform the simulation assuming that the fluid production unit is a homogeneous and isotropic single-layer formation. To this end, initial water saturation was obtained from Archie's equation, porosity from a nonlinear two-fluid two-mineral model, and the initial permeability guess from the modified Timur-Tixier equation. The coefficient for capillary pressure, $P^c_o$, is set to 1.1 psi·darcy$^{1/2}$, and the exponent for capillary pressure, $e_p$, is equal to 1.8. The water end-point for relative permeability, $k^o_{rw}$, is set to 0.9 and for gas, $k^o_{rg}$, is set to 0.3. Finally, the water exponent, $e_w$, is set to 2.0 and the exponent for gas, $e_{nw}$, is set to 2.5. Figure 6 displays the water-gas capillary pressure and relative permeability curves. As observed from the relative permeability curves of Figure 6, critical water saturation$^1$, $S_{wcr}$, is on the order of 58% for this homogeneous-formation case.

$^1$Critical water saturation is here defined as the value of water saturation when relative permeability of the wetting phase is equal to that of the non-wetting phase.

Figure 8 shows spatial distributions (radial and vertical directions) of water saturation, salt concentration, and electrical resistivity within the rock formation yielded by the simulation of mud-filtrate invasion. The assumed time of invasion is five days. Invasion reaches a radial distance of approximately 7 ft from the wellbore. Such a relatively deep invasion substantially influences the array induction measurements. The cross-sections shown in Figure 8 are input to SLDMINV to simulate array induction imager tool (AIT) measurements with an initial guess of absolute permeability equal to 1.5 md. Simulation of AIT measurements is performed at every 0.25 ft vertically along the borehole. Results of such a simulation are shown in Figure 9 along with the gamma ray log. On average, the simulated deep apparent resistivity (AIT 90) follows a similar trend to that of the field data. Conversely, the shallowest apparent resistivity curve (AIT 10) follows the trend of the field data only along the cleanest interval of the reservoir (4-12 ft) which is also the one that exhibits the largest resistivity values. Intermediate apparent resistivity curves (AIT 20, AIT 30, and AIT 60) on average do not follow the same trend of the field data. A radially smoother spatial distribution of electrical resistivity is necessary to match the intermediate AIT apparent resistivity curves. Two of the most important parameters that control the shape of the radial profile of invasion are capillary pressure and relative permeability. Consequently, a sensitivity analysis is necessary to assess the influence of those two formation parameters on the simulated spatial distributions of electrical resistivity.

**FIG. 8** Results of the simulation of mud-filtrate invasion at five days after the onset of invasion. The cross-sections describe water saturation, salt concentration, and formation resistivity for the case of a vertically homogeneous fluid production unit.

**FIG. 9** Comparison of the simulated and measured array induction apparent resistivities (right-hand panel) for the base case. The left-hand panel shows the gamma-ray log.
Sensitivity of array induction measurements to capillary pressure and relative permeability

We performed simulations of array induction measurements after enforcing small perturbations to the water-gas capillary pressure and relative permeability curves. Small perturbations (±10%) were performed to the capillary pressure coefficient, $P_c^*$, as well as to the wetting phase exponent, $e_w$, for relative permeability. No significant changes were observed in either the spatial distributions of electrical resistivity or the simulated array induction measurements resulting from these perturbations. Thus, we proceeded to enforce yet larger perturbations (approximately -70% and +300%) of the same parameters. Figure 10 shows the simulated array induction measurements obtained for the cases when $P_c^* = 0.35$ psi.darcy$^{1/2}$ (low capillary pressure) and $P_c^* = 3.7$ psi.darcy$^{1/2}$ (high capillary pressure). Although such changes in capillarity are rather high, they scarcely affect the simulated apparent resistivity curves.

One of the best ways to observe the effect of changes in relative permeability is by modifying the critical water saturation ($S_w^{cr}$). Figure 11 shows the simulated array induction curves for the cases when $e_w = 20.0$ ($S_w^{cr} \approx 80\%$) and $e_w = 2.0$ ($S_w^{cr} \approx 35\%$), in that order. In the second case ($S_w^{cr} \approx 35\%$), it was necessary to change the values of irreducible and initial water saturation to achieve such an extreme shape in the relative permeability curves. For high values of critical water saturation we observe less splitting of the intermediate apparent resistivity curves. This observation indicates that the invasion front is relatively sharp and, therefore, does not entail a good agreement between the measured and simulated array induction apparent resistivities. A more pronounced splitting of the simulated apparent resistivity curves is observed for the case of low critical water saturation, thereby resulting in a better agreement between the measured and simulated array induction curves. However, we remark that for a gas-saturated reservoir such as the one considered in this paper, it is unlikely to encounter relative permeability curves with such a low value of $S_w^{cr}$. Therefore, the base case value ($e_w = 2.0, S_w^{cr} \approx 58\%$) was used to perform the inversion.

One important observation from this analysis is that large variations of relative permeability cause appreciable changes in the shape of the spatial distribution of electrical resistivity. Capillary pressure is also an important factor to be considered in the analysis. However, our sensitivity analysis shows that the effect of capillary pressure on the shape of the invasion profile is negligible compared to that of relative permeability. It is also pertinent to mention that neither of these two properties affects the radial length of invasion or the electrical resistivity in the near-borehole and virgin zones. Only the transition zone remains influ-

**Fig. 10** Numerical simulation of array induction measurements: sensitivity to capillary pressure. The left-hand panel shows simulation results obtained for the case of a rock formation whose capillary pressure is 70% lower than that of the base case. The right-hand panel shows simulation results obtained for the case of a rock formation whose capillary pressure is 300% higher than that of the base case.

**Fig. 11** Numerical simulation of array induction measurements: sensitivity to relative permeability. The left-hand panel describes simulations performed for a strongly water-wet sand with critical water saturation 45% higher than that of the base case. The right-hand panel shows simulation results obtained for the case of critical water saturation 35% lower than that of the base case.
Enforced by the changes of relative permeability and capillary pressure.

Sensitivity of array induction measurements to permeability, porosity, Archie’s parameters, and initial water saturation

We performed sensitivity analyses to the initial guess of absolute permeability \((k = 1.5 \text{ md})\) to assess its influence on the AIT apparent resistivity curves. Figures 12 and 13 show simulated apparent resistivity curves for the cases of 1 (base case, \(k = 1.5 \text{ md}\)), 10, 0.1, and 100 times the initial guess of permeability. We observe a significant influence of small values of absolute permeability on the simulated resistivity curves. Specifically, the shallow apparent resistivity (AIT 10) increases with high values of permeability while the deep apparent resistivity (AIT 90) slightly decreases. A larger splitting of the intermediate apparent resistivity curves is also observed for high values of permeability. Figures 12 and 13 also indicate that the shallow apparent resistivity curves remain the most sensitive to changes of absolute permeability. Therefore, we conclude that absolute permeability strongly affects the spatial distribution of electrical resistivity. This observation applies not only to the electrical resistivity of the transition zone, but also to the radial length of invasion.

Perturbations of porosity and Archie’s parameters produce a uniform parallel shift of the five simulated apparent resistivity curves when plotted on a logarithmic scale. This effect contrasts with that due to perturbations of permeability, where the shift of the simulated apparent resistivity is not uniform for all the AIT curves. For low values of cementation exponent \((m)\), the simulated values of apparent resistivity uniformly decrease with respect to those of the base case. A similar situation occurs either with a decrease of the saturation exponent \((n)\) or with an increase of porosity. For the problem at hand, we found that the simulated AIT apparent resistivity curves exhibited higher sensitivity to \(n\) than to either \(m\) or porosity. However, when the perturbations of \(m\), \(n\), and porosity were below 10% with respect to the values assumed for the base case, we observed no appreciable differences on the simulated apparent resistivity curves.

Similar to the case of Archie’s parameters, changes in initial water saturation entail a uniform parallel shift of the simulated AIT apparent resistivity curves. As expected from Archie’s equation, low values of initial water saturation yield high values of resistivity and high values of initial water saturation yield low values of resistivity. For changes below 10% of initial water saturation the simulated AIT apparent resistivity curves do not change appreciable with respect to those of the base case. Based on this analysis, we conclude that large variations of initial water saturation (>10%) have an important influence on the shape of the radial profile of invasion. Furthermore, the influence of initial water saturation on the radial shape of invasion is the largest in the virgin zone.

**Fig. 12** Numerical simulation of array induction measurements: sensitivity to the initial guess of permeability. The left- and right-hand panels describe apparent resistivity curves simulated for the cases of 1 and 10 times the permeability of the base case, respectively.

**Fig. 13** Numerical simulation of array induction measurements: sensitivity to the initial guess of permeability. The left- and right-hand panels describe apparent resistivity curves simulated for the cases of 0.1 and 100 times the permeability of the base case, respectively.
Sensitivity of the estimated permeability to mudcake properties and time of invasion

We performed additional sensitivity analyses to study the influence of various mudcake parameters on the estimated permeability. Variations of mudcake properties entail variations in the flow rate of mud-filtrate invasion. Mudcake permeability, maximum mudcake thickness, and mudcake compressibility exponent are the three parameters that primarily govern the flow rate of invasion. These parameters affect both the shape of the invasion profile and the radial length of invasion. The higher the mudcake permeability the higher the flow rate; the lower the mudcake thickness and compressibility exponent the higher the flow rate. Figure 14 describes the influence of mudcake permeability on the flow rate of invasion. We observed that variations above 20% in these properties entailed variations over 15% of the flow rate of invasion. In turn, these variations of flow rate entailed variations of one order of magnitude or less of the inverted values of formation permeability. Figure 15 displays the simulated AIT apparent resistivity curves for the base case assuming two values of mudcake permeability. Perturbations in other invasion parameters such as mudcake porosity, mud solid fraction, and mudcake exponent multiplier scarcely affected the calculated flow rate of mud-filtrate invasion.

Time of invasion was another important parameter considered in our sensitivity analysis. We found that the effect of time of invasion on the estimated values of permeability was similar to that associated with variations of mudcake permeability assuming a constant time of invasion. For differences of time of invasion of one day or less we observed variations of half an order of magnitude of the estimated permeability. Variations of the estimated values of permeability remained marginal for variations of the time of invasion of six hours or less.

Base-case permeability inversion

We first consider the base formation model. All petrophysical properties are assumed known with the exception of absolute permeability. The initial permeability guess ($k = 1.5$ md) entered to the inversion was obtained from the modified Timur-Tixier equation. Five Gauss-Newton iterations were necessary to achieve convergence of the minimization process, yielding a value of absolute permeability equal to 55.4 md. Figure 16 shows the simulated array induction measurements associated with the inverted value of absolute permeability. The simulated AIT apparent resistivity curves follow a trend similar to that of the measurements. A clear splitting of the intermediate curves is also noticeable. We observe a better agreement between the measured and simulated AIT apparent resistivity curves as a result of the inversion. The next stage of the analysis consists of improving the vertical agreement between the measured and simulated AIT apparent resistivity curves by progressively increasing the number of flow subunits in the rock formation. This procedure yields layer-by-layer estimates of absolute permeability.

![Figure 14](image1.png)  
**Fig. 14** Time evolution of the flow rate of mud-filtrate invasion for three values of mudcake permeability. The base case corresponds to $k_{mc} = 0.030$ md. In general, variations of $k_{mc}$ lower than 20% do not cause appreciable variations of flow rate.

![Figure 15](image2.png)  
**Fig. 15** Numerical simulation of array induction measurements: sensitivity to mudcake permeability. The left- and right-hand panels describe apparent resistivity curves simulated for the cases of variations of -17% and +17% the $k_{mc}$ of the base case, respectively.
General case: vertically heterogeneous flow unit

In order to account for vertical heterogeneities in the flow unit, we subdivided it into multiple horizontal layers. The modified stratigraphic Lorenz plot shown in Figure 4 was used to identify layer boundaries. This plot indicated that the maximum number of flow units was eight. Simulations of mud-filtrate invasion and array induction measurements were performed for cases with 2, 3, 4, 5, 6, 7, and 8 layers. In this paper we report inversion results only for the case of highest vertical resolution (8 layers).

Initial values of porosity, water saturation, and permeability were calculated in similar fashion to the base formation case and averaged across each flow subunit. Table 4 describes the thickness and petrophysical properties of each layer used in the description of the formation. Mudcake and mud filtrate properties remained the same as in the base case. However, flow rates of mud-filtrate invasion were computed specifically for each flow unit. The rate of mud-filtrate invasion is largely controlled by rock properties in low permeability formations (Wu et al, 2005). Because capillary pressure remains a function of irreducible water saturation, porosity, and permeability, the flow rate of mud-filtrate invasion is slightly different for each flow unit. Relative permeability, on the other hand, depends on irreducible water saturation and the Brooks-Corey parameters. Therefore, we assumed the same shape of relative permeability curves for all the layers but included layer-dependent variations of critical water saturation.

Figure 17 describes the spatial distributions of water saturation, salt concentration, and electrical resistivity resulting from the simulation of mud-filtrate invasion. The invasion front reaches radially deeper zones in the formation when permeability is high. Figure 18 compares the corresponding simulated AIT apparent resistivity curves with the measured curves. We simulated the AIT apparent resistivity curves shown in Figure 18 using the initial permeability guess. As observed, the match between measured and simulated AIT apparent resistivity curves is not acceptable. The latter observation indicates that the best estimate of absolute permeability has not been reached from forward mod-

<table>
<thead>
<tr>
<th>$h$, ft</th>
<th>$\phi$, frac.</th>
<th>$S_w$, frac</th>
<th>$k_w$, md</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.084</td>
<td>0.980</td>
<td>0.067</td>
</tr>
<tr>
<td>3.0</td>
<td>0.156</td>
<td>0.477</td>
<td>1.016</td>
</tr>
<tr>
<td>2.0</td>
<td>0.126</td>
<td>0.449</td>
<td>0.827</td>
</tr>
<tr>
<td>3.5</td>
<td>0.157</td>
<td>0.417</td>
<td>5.697</td>
</tr>
<tr>
<td>3.0</td>
<td>0.169</td>
<td>0.440</td>
<td>0.600</td>
</tr>
<tr>
<td>2.5</td>
<td>0.153</td>
<td>0.552</td>
<td>0.134</td>
</tr>
<tr>
<td>2.0</td>
<td>0.129</td>
<td>0.571</td>
<td>0.192</td>
</tr>
<tr>
<td>3.0</td>
<td>0.075</td>
<td>0.980</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Figure 16 Numerical simulation of array-induction measurements yielded by the inversion of absolute permeability. The left-hand panel is a cross-section of electrical resistivity and the right-hand panel compares the simulated and measured AIT apparent resistivity logs.

Figure 17 Cross-sections of water saturation, salt concentration, and electrical resistivity simulated for the case of a vertically heterogeneous flow unit. The time of invasion is five days. Layer permeabilities were assigned using initial permeability values obtained from petrophysical analysis (starting guess).
eling. Additional changes of permeability for each layer are needed to reach a better agreement between simulated and measured apparent resistivity curves.

**Estimation of layer-by-layer permeabilities**

We proceed to adjust the layer permeability values to improve the match between simulated and measured AIT apparent resistivity curves. This adjustment is guided by vertical variations of the resistivity curves. Simulation of mud-filtrate invasion is performed after each manual change followed by numerical simulation of array induction measurements. The process is repeated as many times as needed to reach an acceptable match between the simulated and measured AIT apparent resistivity curves. Figure 19 describes the spatial distributions of water saturation, salt concentration, and electrical resistivity simulated after several multiple changes of layer permeabilities.

Figure 20 compares the simulated and measured AIT apparent resistivity curves. We observe that the simulated deepest (AIT90) and shallowest (AIT10) apparent resistivity curves agree well with the corresponding measured curves. However, the agreement between the intermediate apparent resistivity curves is not acceptable. We conclude that automatic permeability inversion is needed to improve the agreement between simulations and measurements.

**General case of permeability inversion**

We implemented the inversion of layer permeabilities using as initial guesses the permeability values rendered by the manual adjustment process described in the previous section. The inversion was conducted in two steps. Initially, the permeability values yielded by the manual resistivity matching were taken as the starting guesses. Eight Gauss-Newton iterations were necessary to achieve convergence of the inversion. Permeability values obtained from
TABLE 5  Summary of permeability values inverted from array induction measurements for the case of a multi-layer formation. Different columns describe the percent change of permeability with respect to that of the initial guess.

<table>
<thead>
<tr>
<th>Resistivity</th>
<th>Matching</th>
<th>% of change</th>
<th>Inversion</th>
<th>% of change</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>(md)</td>
<td></td>
<td>(md)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.031</td>
<td>-54</td>
<td>1.23</td>
<td>1733</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>0.757</td>
<td>-25</td>
<td>18.63</td>
<td>1734</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>1.340</td>
<td>62</td>
<td>22.16</td>
<td>2580</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>2.295</td>
<td>-60</td>
<td>22.20</td>
<td>290</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>2.482</td>
<td>314</td>
<td>16.90</td>
<td>2719</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>0.124</td>
<td>-8</td>
<td>7.53</td>
<td>5502</td>
<td>Moderate</td>
<td></td>
</tr>
<tr>
<td>0.106</td>
<td>-45</td>
<td>2471.51</td>
<td>1285309</td>
<td>Very High</td>
<td></td>
</tr>
<tr>
<td>0.021</td>
<td>-50</td>
<td>82.90</td>
<td>197281</td>
<td>High</td>
<td></td>
</tr>
</tbody>
</table>

this inversion were used to re-compute the flow rate of mud filtrate with INVASE and then used as the new initial guesses for the subsequent inversion. Table 5 describes the values of layer permeability yielded by the second step of the inversion after 8 iterations along with the corresponding percentage change when compared to the Timur-Tixier permeability values. The deviation of the inverted permeabilities with respect to the initial guesses is as low as one-half order of magnitude and as high as four orders of magnitude. Table 5 gives a qualitative indication of the level of uncertainty of each estimated value of permeability (right-most column) based on the sensitivity of the simulated apparent resistivity curves to a small perturbation of permeability.

Figure 21 shows the spatial distribution of electrical resistivity yielded by the inversion, whereas Figure 22 compares the simulated and measured apparent resistivity curves. Even though most of the simulated AIT apparent resistivity curves (in several layers) agree well with the field curves, additional work is needed to assess more realistic values of absolute permeability for those layers that exhibit moderate to high levels of uncertainty. We remark that layers close to the top and base of the fluid production unit are highly influenced by shoulder-bed effects. Some of the differences between simulated and measured apparent resistivity curves could be due to azimuthal variations of electrical resistivity neglected by the simulation model.

Uncertainty analysis

Sensitivity analyses were carried out to assess the uncertainty of the estimated flow rate of mud-filtrate invasion. The inverted absolute permeability decreased by two orders of magnitude when the average flow rate was made equal to twice the value of the flow rate for the initial case. However, for 5 and 10 times the value of the initial flow rate, the inverted values of permeability remained the same as those obtained for the case of 2 times the initial flow rate. For the case at hand, it is very unlikely for such large errors to be made in the estimation of flow rate as the latter is mainly conditioned by formation permeability. We observed that
SUMMARY AND CONCLUSIONS

Using a challenging field example of tight gas sands, this paper considered the estimation of absolute permeabilities of rock formations from borehole array induction measurements. The estimation was based on simulation of the process of mud-filtrate invasion. This approach required detailed knowledge of borehole environmental variables, including overbalance pressure, temperature, mud properties, and time of invasion. Accurate simulation of the process of mud-filtrate invasion also required knowledge of fluid and rock-fluid properties such as porosity, permeability, relative permeability, capillary pressure, initial water saturation, salt concentration of connate water, and hydrocarbon density and viscosity, among others. Some of these properties were available from core laboratory measurements, while others were estimated from basic petrophysical analysis of well logs. To estimate permeability from array induction logs it is imperative that, except for permeability, all of the borehole environmental variables and rock-fluid properties be known with a good level of certainty. Uncertain knowledge of some of these properties requires that their influence on the estimation of permeability be quantified in a systematic manner.

The field study considered in this paper was approached with a thorough analysis of the role played by several borehole and rock-fluid properties on the time evolution of the process of mud-filtrate invasion. We found that, for the tight gas sand reservoir under analysis, salt mixing between mud filtrate and connate water, and relative permeability had the largest influence on the spatial distribution of electrical resistivity resulting from invasion. Rocks with high critical water saturation (strongly water wet) were associated with piston-like invasion fronts and hence did not show appreciable variations of the intermediate induction apparent resistivity curves. On the other hand, rocks with low critical water saturation were associated with spatially smooth spatial distributions of electrical resistivity. Such variations on the shape of the spatial distribution of electrical resistivity only occurred with a substantial change of relative permeability. Therefore, one can safely conclude that array induction measurements are not sensitive to small variations of relative permeability and capillary pressure. Conversely, the sensitivity analyses considered in this paper did indicate that absolute permeability had the largest effect on the measured array resistivity curves, especially on the shal-low-reading curves. This observation provided a solid physical background for the estimation of permeability.

We approached the estimation of permeability from induction logs by progressively increasing the number of individual flow subunits within a single production zone. Such a strategy allowed us to further refine the assumed values of porosity, capillary pressure and relative permeability by assessing their impact on the simulated array induction measurements, especially the average values and separation of the five AIT apparent resistivity curves.

Permeability values yielded by the inversion of array induction measurements were higher than those calculated with standard petrophysical formulas (see Table 5). However, as observed in Figure 3, these results were in the same range of core permeabilities for porosity values larger than 10%.

We remark that the methodology tested in this paper to estimate permeability was based on the use of apparent resistivity curves yielded by the processing of AIT induction measurements. It is likely that the resolution and accuracy of the estimated values of permeability would improve with the use of “raw” voltages instead of processed apparent resistivity curves. Likewise, it is possible that the resolution and accuracy of the estimated values of permeability would improve with the use of micro-resistivity measurements in addition to AIT measurements.

NOMENCLATURE

\(a\) Archie’s tortuosity factor
\(c_{nw}\) empirical exponents for \(k_{rmw}\) equation
\(c_p\) empirical exponents for \(P_c\) equation
\(c_w\) empirical exponents for \(k_{nw}\) equation
\(k_{rmw}\) non-wetting-phase relative permeability
\(k_w^0\) \(k_{rmw}\) end point
\(k_{nw}^0\) \(k_{nw}\) end point
\(m\) Archie’s cementation exponent
\(n\) Archie’s saturation exponent
\(P_c\) capillary pressure, (psi)
\(P_c^*\) coefficient for \(P_c\) equation, (psi.darcy\(^{1/2}\))
\(R_t\) true formation resistivity, (ohm m)
\(R_w\) connate water resistivity, (ohm m)
\(R_{so}\) invaded-zone resistivity, (ohm m)
\(R_{wzo}\) water invaded-zone resistivity, (ohm m)
\(S_{NW}\) normalized water saturation, (frac.)
\(S_w\) water saturation, (frac.)
\(S_{NW}\) irreducible water saturation, (frac.)
\(C_{sh}\) shale concentration, (frac.)
\([NaCl]\) equivalent salt concentration, (ppm)
\(\phi\) effective porosity, (frac.)
\(\phi_t\) total porosity, (frac.)
\( \phi_{sh} \) \text{ shale total porosity, (frac.)} \\
\( \phi^D_{sh} \) \text{ shale corrected density porosity, (frac.)} \\
\( \phi^N_{sh} \) \text{ shale corrected neutron porosity, (frac.)} \\
\( \rho_1, \rho_2 \) \text{ fluid density, (g/cm}^3\text{)} \\
\( \rho_b \) \text{ bulk density, (g/cm}^3\text{)} \\
\( \rho_{ma} \) \text{ matrix density, (g/cm}^3\text{)} \\
\( \rho_{sh} \) \text{ shale density, (g/cm}^3\text{)} \\

**ACKNOWLEDGEMENTS**

We are obliged to ConocoPhillips for permission to publish these results and for partial funding through a summer internship position offered to Jesús M. Salazar in 2004. A note of special gratitude goes to The University of Texas at Austin’s Bureau of Economic Geology for providing the core measurements. Partial funding for the work reported in this paper was provided by UT Austin’s Research Consortium on Formation Evaluation, jointly sponsored by Aramco, Baker Atlas, BP, British Gas, Chevron, ConocoPhillips, ENI E&P, ExxonMobil, Halliburton Energy Services, Hydro, Marathon, Mexican Institute for Petroleum, Occidental Oil and Gas Corporation, Petrobras, Schlumberger, Shell International E&P, Statoil, Total, and Weatherford.

**REFERENCES**


S. A. Holditch Associates, 1988, Special Core Analysis Report, Staged Field Experiment #2: Core Laboratories, Irving, TX.


Semmelbeck, M. E., and Holditch, S. A., 1988, The effect of


**ABOUT THE AUTHORS**

**Jesús M. Salazar** is a Research Assistant and a PhD Candidate in the Department of Petroleum and Geosystems Engineering at The University of Texas at Austin. He received a BS in Physics (with honors) from Universidad Central de Venezuela in 1998 and MSE in Petroleum Engineering from The University of Texas at Austin in 2004. From 1997 to 2002 he worked for PDVSA (Maracaibo, Venezuela) as a petrophysicist and well log analyst. Jesús has also worked as a summer intern for ConocoPhillips and Occidental Oil and Gas Corporation performing petrophysical interpretation and modeling. He was bestowed a student grant, 2002-2003, and three scholarship, 2003-2004, 2005-2006, and 2006-2007 by the SPWLA. His research interests include petrophysics, log analysis, inverse problems, and numerical reservoir simulation.

**Carlos Torres-Verdín** received a PhD in Engineering Geoscience from the University of California, Berkeley, in 1991. During 1991–1997 he held the position of Research Scientist with Schlumberger-Doll Research. From 1997–1999, he was Reservoir Specialist and Technology Champion with YPF (Buenos Aires, Argentina). Since 1999, he has been with the Department of Petroleum and Geosystems Engineering of The University of Texas at Austin, where he currently holds the position of Associate Professor. He conducts research on borehole geophysics, formation evaluation, and integrated reservoir characterization. Torres-Verdín has served as Guest Editor for *Radio Science*, and is currently a member of the Editorial Board of the *Journal of Electromagnetic Waves and Applications*, and an associate editor for *Petrophysics* (SPWLA) and the *SPE Journal*. He is co-recipient of the SPWLA’s 2003 and 2004 Best Paper (*Petrophysics*) Awards and is recipient of the 2006 SPWLA’s Distinguished Technical Achievement Award.

**Faruk O. Alpak** is a research reservoir engineer with Shell International E&P at the Bellaire Technology Center, Houston, Texas. His research interests include parallel reservoir-simulation techniques, computational fluid dynamics, uncertainty analysis, inverse problems, numerical optimization, and electromagnetic wave propagation. Alpak holds PhD and MSc degrees in Petroleum Engineering from The University of Texas at Austin.

**Tarek M. Habashy** received a PhD from MIT in 1983 in Electrical Engineering. During 1982-1983 he was a Research Associate in the Department of Electrical Engineering at MIT. Since September 1983, he has been with Schlumberger-Doll Research, and is now a Research Director of the Mathematics and Modeling Department. He is also a Scientific Advisor conducting research in electromagnetic logging tools, inverse scattering theory, and numerical analysis. He is the Editor of *Radio Science* and a member of the Editorial Boards of *Inverse Problems* and the *Journal of Electromagnetic Waves and Applications*. He is a Fellow of the Institute of Physics and IEEE.

**James D. Klein** works for ConocoPhillips as Principal Petrophysicist, after working two years for Phillips Petroleum Company and 17 years for ARCO. He obtained his BS degree in Geophysics in 1970 at the Colorado School of Mines, and MS and PhD degrees in Geophysics at the University of Utah in 1977 and 1980. During his career in petrophysics he has worked on a number of diverse reservoir characterization studies, and has developed expertise in a variety of logging technologies, including resistivity logging and modeling, through-casing resistivity, and electrical anisotropy. He has published approximately 20 papers on reservoir petrophysics and characterization, and has co-authored five patents.