ABSTRACT

We develop a procedure to simulate and detect the spatial distribution of mud-filtrate invasion in the near-wellbore region of horizontal wells. Examples are considered for the general theme of a water-base mud invading hydrocarbon-bearing rock formations. A commercial multiphase fluid flow simulator is used to quantify the sensitivity of mud-filtrate invasion to a large range of the rock's petrophysical parameters. Spatial distributions of fluid saturations rendered by the simulator are then converted into corresponding distributions of electrical conductivity. Subsequently, a finite-difference code is used to simulate the associated electromagnetic response of an array induction tool.

Several important conclusions can be drawn from our sensitivity study: In many practical cases, the rock's petrophysical parameters can have a strong imprint on the spatial distribution of fluid saturations, and in consequence, on the response of array induction tools. On the other hand, the response of array-induction tools may exhibit limited sensitivity to several of the complex geometrical features of the saturation distribution around a horizontal borehole. We have also found that simple parametric representations of the invasion process can be used to interpret quantitatively array induction measurements acquired in horizontal wells. Examples are shown of how array induction measurements could be used to provide accurate estimates of electric resistivity in the uninvaded region of rock formations.

INTRODUCTION

Determining the hydrocarbon content of porous rock formations is of significant interest for the economic assessment of reservoirs. Accurate determination of in-situ formation resistivity as a direct measure of initial oil saturation is one of the primary motivations for wireline logging. In addition to assessing reserves, it is imperative to determine the amount of recoverable oil. Mud-filtrate invasion can be regarded analogous to a waterflood. Consequently, the invaded zone resistivity, \( R_{\text{oi}} \) remains a direct indicator of residual oil saturation, \( S_{\text{or}} \). Both the saturation profile and the invasion geometry around a borehole are governed by the underlying physics of multiphase fluid flow (Ramakrishnan and Wilkinson, 1997 and 1999). From the log measurement viewpoint, the presence of an invaded region around the borehole can substantially affect the electromagnetic logs. Depending on the extent of invasion, and on the complexity of the invasion geometry, interpretation of electromagnetic logs can be nontrivial. Undoubtedly, the interpretation of resistivity tools in horizontal or highly deviated wells is much more challenging than in vertical wells.

There is no ideal logging tool that measures continuous resistivity profiles. However, tools are available with multiple depths of investigation. The Array Induction Tool (AIT*) provides 28 raw measurements (not all independent) that can be reduced into five resolution-matched channels. The response of multiarray induction tools has been previously analyzed by Anderson et al. (1999) for the cases of highly dipping formations and arbitrary 3D geometries in the presence of mud-filtrate invasion. However, the invasion geometries considered by that study were not explicitly derived from petrophysical and multiphase flow properties. Simulation of the near-wellbore dynamics using multiphase flow equations has been previously applied to the interpretation of induction logging by Zhang et al. (1999) and Ramakrishnan and Wilkinson, (1997 and 1999) for vertical well geometries.

The motivation for this paper is to quantify the sensitivity of array induction tools to the spatial distributions of near-wellbore fluids arising from mud-filtrate invasion. More specifically, we perform a systematic study to appraise the sensitivity of the AIT* to variations of multiphase
petrophysical and fluid properties in the presence of horizontal wells.

METHODOLOGY

The first step of our study is to describe the permeable medium and saturating fluids in terms of standard petrophysical parameters. Spatial distributions of fluid saturations are derived from the physics of mud-filtrate invasion posed in a three-dimensional (3D) Cartesian coordinate frame. Numerical simulations of the process of mud-filtrate invasion are performed using a commercial black-oil multiphase fluid flow simulator, namely, ECLIPSE 100*. Fluid saturation distributions around a horizontal wellbore are computed at a number of wireline logging times for various combinations of petrophysical and fluid properties. The simulated saturation fields are subsequently transformed into distributions of electrical resistivity using Archie’s law. In each case, we investigate a number of spatial smoothing approaches that provide a transformation of the detailed saturation field into an equivalent smoothed spatial distribution of resistivity. This smoothing procedure is designed to preserve the most dominant features of the saturation field obtained from fine-grid multiphase flow simulations; it is also designed to describe both the magnitude and geometry of the saturation field in terms of a small number of parameters in the resistivity domain. Various degrees of spatial smoothing are investigated for each simulation case. The work presented here becomes exceedingly valuable in the search of an economic procedure to directly infer petrophysical parameters from array induction data. Finally, AIT* measurements are simulated using a finite-difference spectral Lanczos decomposition method, SLDMINV (Druskin et al., 1999). Figure 1 is a flowchart that summarizes the steps considered in our simulation and sensitivity study.

SIMULATION OF MUD-FILTRATE INVASION

We specifically consider the case of water-base mud-filtrate invading a hydrocarbon-bearing formation. Mud-filtrate invasion is modeled as the isothermal simultaneous flow of water and oil phases (two-phase flow) in a permeable medium.

Simulation of isothermal multiphase fluid flow in permeable media requires of mass balance and transport equations as well as a constitutive equation of state. We disregard the presence of chemical reactions, rock/fluid mass transfer, and diffusive/dispersive transport. The mass balance equation for the $i$-th fluid phase (either water or oil) in 3D spatial coordinates is given by

$$\frac{\partial (\rho \phi S_i)}{\partial t} + \nabla \cdot (\rho \phi \vec{u}_i) = -q_{vi}, \quad i = 1, \ldots, k,$$

where $\rho$ is fluid density, $\vec{u}$ is fluid velocity, $\phi$ is porosity, $q_v$ is a source-sink term (the driving source term), $S_i$ is phase saturation, and $i, k$ are phase index and total number of phases, respectively. On the other hand, the transport equation is Darcy’s law, given by

$$\vec{u}_i = -\frac{k}{\mu_i} (\nabla p_i - \gamma_i \nabla D_z),$$

where $\vec{k}$ is the absolute permeability tensor of the porous medium, $k_r$ is the relative permeability, $\mu$ is the phase viscosity, $p$ is the phase pressure, $\gamma$ is the phase specific gravity, and $D_z$ is the vertical location below some reference level. Finally, the constituent equation follows from the equation of state. The assumption is also made that both fluid and rock compressibilities are constant over the pressure range of interest, given by

$$c_B = -\frac{1}{V} \frac{\partial V}{\partial p} \bigg|_T = \frac{1}{\rho} \frac{\partial \rho}{\partial p} \bigg|_T,$$

and

$$c_r = \frac{1}{\phi} \frac{\partial \phi}{\partial p} \bigg|_T.$$

The formation is initially fully saturated with liquid hydrocarbons at irreducible water saturation. We further assume that throughout the mud-filtrate invasion process all the available gas remains dissolved in the oleic phase; in other words, the oleic phase pressure stays above bubble point both spatially and temporally during invasion. The following set of two-phase flow equations is solved in space and time to simulate mud-filtrate invasion (Aziz and Settari, 1979):

$$\frac{\partial \left( \phi \frac{S_o}{B_o} \right)}{\partial t} - \nabla \cdot \left[ M_o \frac{\vec{v}}{k} \cdot (\vec{v} \cdot \vec{p} - \gamma_o \vec{v} D_z) \right] = 0,$$

and
subject to the saturation identity
\[ S_o + S_w = 1, \]
where
\[ M_o(S_w) = \frac{k_{ro}(S_w)}{\mu_o B_o}, \quad \gamma_o = \frac{\rho_o g}{g_c}, \]
and
\[ M_w(S_w) = \frac{k_{rw}(S_w)}{\mu_w B_w}, \quad \gamma_w = \frac{\rho_w g}{g_c}. \]

In Eq. (6), \( R^{sc}_{w} \) identifies the sink/source term for the water component at standard conditions, whereas in Eqs. (8) and (9), \( B \) identifies the formation volume factor for each phase, \( g \) corresponds to the gravitational acceleration, and \( g_c \) is the gravitational conversion factor. Finally, in Eq. (6), \( P_{cow} \) identifies the capillary pressure relationship, described as
\[ P_{cow}(S_w) = p - p_w, \]
where \( p \) corresponds to oleic and \( p_w \) to aqueous phase pressures, respectively. For cases where capillary pressure effects are non-negligible we employ an empirical relationship introduced by Sanchez-Bujanos (1996), namely,
\[ P_{cow} = P_c^o(1-S_{wirr}^o)(1-S_D)^{e_w}, \]
where \( P_c^o \) is a base capillary pressure for the permeable medium and \( e_p \) is a curvature exponent determined empirically from lab experiments. We adopt a deterministic power law to govern the dependency of relative permeability on water saturation. To that end, we define a dimensionless saturation,
\[ S_D = \frac{S_w - S_{wirr}}{1 - S_{or} - S_{wirr}}, \]
where \( S_{wirr} \) and \( S_{or} \) are irreducible water and residual oil saturations, respectively. Relative permeability functions are then given by
\[ k_{rw}(S_D) = k_{ro}^o(1 - S_D)^{e_o}, \]
and
\[ k_{rw}(S_D) = k_{ro}^o(1 - S_D)^{e_o}, \]
where \( k_{rw}^o \) and \( k_{ro}^o \) are the end-point values of the water-oil relative permeabilities, and \( e_w \) and \( e_o \) are water and oil saturation exponents, respectively (Lake, 1989).

**SIMULATION OF ARRAY INDUCTION DATA**

Three-dimensional modeling of induction tools requires solving Maxwell’s equations in a discrete setting either by finite elements or by finite differences. We make use of a 3D finite-difference formulation adapted for the modeling of multi-array induction tools. The simulation code uses the SLDINV algorithm (Druskin et al., 1999). This algorithm is based on the Spectral Lanczos Decomposition Method (SLDM) (Druskin and Knizhnerman, 1994), using Krylov subspaces generated from the inverse powers of the Maxwell operator. The frequency domain problem for Maxwell’s equations is considered in the SLDINV as follows:
\[ \nabla \times \mathbf{E} + i\omega \mu \mathbf{H} = 0, \]
and
\[ \nabla \times \mathbf{H} - (\mathbf{\sigma} + i\omega \mathbf{\varepsilon}) \mathbf{E} = \mathbf{J}, \]
wherein negligible displacement currents are assumed, i.e., the dielectric constant, \( \varepsilon = 0 \). In the above equations, \( \mathbf{E} \), \( \mathbf{H} \), and \( \mathbf{J} \) identify electric field, magnetic field, and electric source density, respectively. The symbols \( \mathbf{\sigma} = \mathbf{\sigma}(x, y, z) \) and \( \mu \) denote ohmic conductivity and magnetic permeability, respectively. Moreover, \( \omega \) is the angular frequency, and \( i \) is equal to \( \sqrt{-1} \). Substitution of Eq. (15) leads to an equation solely expressed in terms of electric field \( \mathbf{E} \), i.e.,
\[ \mathbf{\sigma}^{-1}\mu^{-1}\nabla \times \nabla \times \mathbf{E} + i\omega \mathbf{E} = i\omega \mathbf{\sigma}^{-1}\mathbf{J}. \]

The spatial matrix operator, \( A \), for this last equation is defined as
\[ A = \mathbf{\sigma}^{-1}\mu^{-1}\nabla \times \nabla \times, \]
whereas a source function \( \varphi \) is defined as
\[ \varphi = i\omega \mathbf{\sigma}^{-1}\mathbf{J}. \]

Equation (17) then becomes
\[ (A + i\omega I)\mathbf{E} = \varphi, \]
where $I$ denotes the identity matrix. Boundary conditions for the aforedescribed electromagnetic problem are given by

$$E \times n \big|_{\text{bdy}} = 0,$$  \hspace{1cm} (21)

where $n$ is the normal vector. The domain of interest is given by $\Omega = \{(x, y, z) : x_{\text{min}} \leq x \leq x_{\text{max}}, y_{\text{min}} \leq y \leq y_{\text{max}}, z_{\text{min}} \leq z \leq z_{\text{max}}\}$.

The solution of Eq. (20) is implemented by approximating Eq. (17) with a finite-difference stencil in a 3D Cartesian coordinate frame, $\mathbb{R}^3$, and with a staggered grid (Yee, 1966). A homogenization technique (Moskow et al., 1999) is also used within the SLDMINV algorithm that does not require the finite-difference grid to conform to the interfaces of the inhomogeneities. This homogenization technique provides a finite-difference approximation for complex geometries with the same degree of numerical accuracy as that of a finite element method, but at a much lower computer cost. In dipping formations, bed boundaries are modeled using a staircase deformation. At low frequencies, the solution of Eq. (20) can be rewritten as

$$E = -i \omega^{-1} (A^{-1} - i \omega^{-1} I) A^{-1} \varphi.$$  \hspace{1cm} (22)

This expression is not only highly efficient to simulate low frequency electromagnetic phenomena, but it also considerably reduces the influence of spurious (non-rotational) numerical modes. SLDMINV provides additional advantages over standard finite-difference procedures such as the ability to produce results for multiple frequencies in a single simulation run, and the usage of matrix operations in real arithmetic. The efficiency of the SLDMINV algorithm relies on fast evaluation of the inverse powers of the stiffness operator, $A$. In turn, the latter operator is computed from a decomposition of its curl-free and divergence-free projections. Solutions for these projections are computed by making use of discrete Fourier transforms (DFT) and preconditioned conjugate gradient (PCG) iterations. Druskin et al. (1999) show that the convergence rate of the SLDMINV method increases as frequency decreases, which makes the SLDMINV particularly attractive for low-frequency applications. Extensive details of the SLDMINV are presented in Druskin et al. (1999).

**THE LINK BETWEEN SATURATION AND ELECTRICAL RESISTIVITY**

For simplicity, Archie’s law (Archie, 1942) is used to establish a quantitative relationship between fluid saturation and electrical resistivity. For the hydrocarbon-bearing virgin formation, Archie’s relationship is given by

$$R_i(r) = \frac{R_w(r)}{\phi^m S_{w,i}^n(r)}.$$  \hspace{1cm} (23)

On the other hand, for the invaded zone one has

$$R_{x,i}(r) = \frac{R_{mf}(r)}{\phi^m S_{x,i}^n(r)} \quad \text{where} \quad i = 1, \ldots, n_{S_{w}}.$$  \hspace{1cm} (24)

In the above equations, $R_i$, $R_w$, $R_{mf}$, $R_{x,i}$, $S_w$ and $S_{x,i}$ denote resistivities of virgin zone, formation water (connate water), mud-filtrate, and invaded zone; and saturations of virgin and invaded zones, respectively. The porosity and saturation exponents, $m$ and $n$, respectively, are empirical constants. The latter form of Archie’s law, given in Eq. (24), is used to describe the transformation from water saturation to electrical resistivity for multiple invasion zones.

**SENSITIVITY ANALYSIS**

We assess the sensitivity of AIT* measurements to the presence of water-base mud-filtrate invasion in a horizontal wellbore for various combinations of petrophysical parameters and fluid properties. A schematic description of the modeling geometry adopted in our study is shown in Fig. 2.

*Operating Assumptions for the Simulation of Mud-filtrate Invasion*

We assume that the radial Darcy speed at which the mud-filtrate flows into the formation, $u_f$, is entirely controlled by the mud properties. Hence, the volume of filtrate per unit time entering the formation can be calculated from

$$q_{mf} = 2\pi u_f r_w h,$$  \hspace{1cm} (25)

where $r_w$ is the wellbore radius, and $h$ is the formation thickness in which mud-filtrate invasion takes place. Equation (25) is a direct consequence of the large hydrodynamic resistance encountered by filtrate as it flows through the mud cake, in comparison to the formation. The above assumption also follows from the presence of a...
mud cake, implying that the horizontal permeability, \( k_h \), of the bed is above some minimum value. A minimum value of \( k_h \) required for the development of the mud cake depends primarily on the difference between wellbore and formation pressures as well as on the speed at which the mud is pumped through the wellbore (more accurately, the shear stress at the wellbore wall). Under nominal conditions, this requires \( k_h \) to be greater than 10 to 20md (Dussan V. et al., 1994). Once the horizontal permeability exceeds this value (a mud cake is present), then the specific value of \( u_f \) is independent of \( k_h \). In this paper, we consider the case in which the rate of filtrate invasion is controlled by mudcake properties rather than by formation permeability. Mud-filtrate invasion rates are calculated assuming a time-invariant invasion speed. A spatially uniform local invasion speed condition is imposed along the borehole wall. We assume that the value of the speed of radial filtrate invasion is \( 5 \times 10^{-5} \text{cm/s} \), which remains consistent with the invasion speeds reported by Dussan V. et al. (1994).

**Synthetic Permeable Medium**

Our synthetic reservoir is constructed using a 60ft-long horizontal section drilled through the center of a 98ft thick and 98ft wide hydrocarbon-bearing zone saturated at irreducible water saturation. For multiphase flow simulations, a constant rate inner boundary condition and a no-flow outer boundary condition are imposed on the permeable medium. Equation (25) is used to calculate an invasion rate of 2.29rbbl/d assuming an average wellbore radius of 0.3ft. We make use of an average equivalent wellbore radius given that the horizontal wellbore penetrating through the permeable medium is constructed with a Cartesian grid.

A 171×172×2 \((x, y, \text{ and } z, \text{ respectively})\) Cartesian grid is implemented to simulate detailed images of the near-wellbore saturation field. This grid consists of small spatial steps around the wellbore and coarser steps away from the borehole. Figure 3 shows a two-dimensional (2D) cross-section of the finite-difference grid enforced for fluid flow simulations. The grid is superimposed on an example of simulated saturation field. Average initial formation pressure and temperature are assumed to be 3000psi and 220F°, respectively. Furthermore, the assumption is made that for the simulated pressure range, the rock compressibility is small with respect to the fluid compressibility. Hence, total compressibility is assumed to be solely a function of the compressibilities of the saturating fluids. A permeability-porosity relationship is also enforced in our fluid flow simulations through an empirical correlation commonly reported in the well-logging literature, namely,

\[
\phi = 0.1 \log_{10} (k_h) + 0.05, \quad (26)
\]

where horizontal permeability is given in md. The latter empirical relationship is typical of shaly sandstones (Dussan V. et al., 1994). Parameters for Archie’s law are assumed as follows: connate water resistivity, \( R_w = 0.56 \text{ohm-m} \), mud-filtrate resistivity, \( R_{mf} = 0.2 \text{ohm-m} \), \( m = 2 \), and \( n = 2 \). We further assume a negligible salinity gradient with respect to the well location. A 2D schematic of the horizontal cross-section of the zone of interest is shown in Fig. 4. The petrophysical and fluid flow properties displayed in this figure summarize the ranges of variability considered in our sensitivity studies.

**Sensitivity Studies**

We analyze the sensitivity of mud-filtrate invasion to a permeability variation in a homogeneous and isotropic porous formation (Study 1). Porosity is coupled to horizontal permeability by Eq. (26). Properties of both mud-filtrate and 30°API-oil considered in this study are shown in Tables 1 and 2, respectively. We assume a water-wet rock formation and characterize the fluid displacement process as imbibition governed by a Corey-type model of relative permeabilities consistent with the rock wettability (see Eqs. (13) and (14)). In addition, we neglect hysteresis in relative permeability. Capillary effects are also assumed to be negligible. The top-most panels of Fig. 5 are cross-sections of the simulated near wellbore saturation fields for rock formation permeabilities of 100md, 500md, and 1000md, respectively. A color gradient from dark blue to red is used to describe the water saturation range (\( S_w \)) from irreducible water saturation (\( S_{wirr} \)) to water saturation at residual oil saturation (\( 1-S_{oir} \)). The wellbore region is depicted with a circle in the cross-sections. We utilize multiple degrees of smoothing on the post-simulation saturation field. Consequently, multiple levels of geometrical complexity are considered in the resistivity domain subsequent to the application of Archie’s law on
the spatially smoothed saturation fields. The mid and bottom panels of Fig. 5 display the simplest and the most detailed smoothed saturation images, respectively. Figures 12(a) through 12(c) show the simulated electromagnetic responses corresponding to rock formation permeabilities of 100md, 500md, and 1000md, respectively. Electromagnetic responses for each of the five AIT* channels are presented for the simplest and most detailed invaded region geometry considered as a result of the spatial smoothing procedure. For comparison, the resistivity of the uninvaded zone, $R_s$, is also shown in the AIT* sensitivity plots. All fluid-flow sensitivity simulations described in this paper are presented as time snapshots taken on the 3rd day of mud-filtrate invasion and subject to the prescribed invasion rate.

A second sensitivity study is intended to assess the effect of a variation in oil type saturating the zone of interest. To this end, we simulated mud-filtrate invasion into heavy, intermediate, and light hydrocarbon-bearing formations. The liquid hydrocarbon phase is characterized using typical °API gravities for heavy, intermediate, and light oils. Variation of °API inherently embodies variations in density, formation volume factor, compressibility, and viscosity of the hydrocarbon phase. From the displacement viewpoint, this study incorporates two governing parameters, namely, density contrast and viscosity ratio, which in turn control the effect of gravity and phase mobilities. Tables 1 and 2 are organized summaries of the fluid parameters considered in our sensitivity study. Two values of homogeneous and isotropic permeability, $k_r$, for the rock are also considered in this study: a high $k_r$ of 500md (Study 2), and a moderate $k_r$ of 100md (Study 3). Remaining geometrical and petrophysical properties are assumed to be equal to those described in Study 1 (base case). Figures 6 and 7 show the simulated saturation fields and their spatially smoothed counterparts for Studies 2 and 3, respectively. AIT* sensitivities are shown in Figs. 13(a) through 13(c) for Study 2 and 14(a) through 14(c) for Study 3, respectively.

The impact of permeability anisotropy on mud-filtrate invasion, and consequently, on the AIT* response is considered in Study 4. Permeability anisotropy ratio, $R_{anis}$, is defined as the ratio of horizontal to vertical permeability, $k_h/k_v$. In Study 4, we analyzed cases where $R_{anis}$ is equal to 1, 10, and 100, respectively. We further assumed a typical horizontal permeability, $k_h$, of 100md, and assigned the vertical permeability, $k_v$, values of 100md, 10md, and 1md, respectively. The remaining fluid and petrophysical parameters are equal to those of the base case. Figure 8 displays the ensuing fluid saturation plots, and Figs. 15(a) through 15(c) describe the sensitivity of the AIT* response to permeability anisotropy.

Sensitivity of the AIT* response to a variation in rock wettability is examined by modifying the model-based Corey type relative permeability, $k_r$, curves used for fluid flow simulations (see Eqs. (13) and (14)). Typical end-point relative permeabilities, $k_{ro}$ and $k_{ow}$, and residual saturations, $S_{wirr}$ and $S_{orr}$, along with curvature parameters, $e_w$ and $e_o$, are used for a water-, intermediate- and oil-wet rock. Additionally, a typical imbition capillary pressure, $P_c$, relationship is employed to assess the effect of capillary pressure on filtrate invasion for the case of a water-wet rock. We assume two homogeneous and isotropic permeabilities, $k$: a high $k$ of 500md (Study 5), and a moderate $k$ of 100md (Study 6). Figures 9 and 10 show the results of fluid flow simulations in the form of 2D cross-section of near-wellbore fluid saturation at the 3rd day of invasion for Studies 5 and 6, respectively. The range of fluid saturation is unique to each case of rock wettability owing to the variability in endpoint saturations. Hence, the color gradient used in the saturation plots should be interpreted independently for each case. Figure 11 shows the values of $k_r$ as well as the $P_c$ relationship used in the fluid flow simulations. The corresponding AIT* responses are shown in Figs. 16(a) through 16(d) for Study 5 and Figs. 17(a) through 17(d) for Study 6, respectively.

**Interpretation of the Simulation Results**

Several significant observations can be made from the interpretation of the simulation of both mud-filtrate invasion (saturation distribution and geometry) and openhole electromagnetic logging (AIT*) response. The simulated near-wellbore saturation fields clearly indicate that increasing values of permeability or density contrast accelerate fluid movement and increase the depth of sagging in the vertical direction at the bottom of the horizontal well. However, the extent of invasion becomes narrower at the sides and, especially, at the top of the horizontal wellbores.
Higher values of mud-filtrate saturation, characterized by less spatial spread, are observed almost symmetrically around the wellbore. This leads to cylindrical invasion shapes for the cases in which sagging at the bottom of the well is less significant. Invasion geometries evolve from simple cylindrical to off-centered cylindrical shapes. Thereafter, those shapes evolve into elliptical and skewed elliptical forms and even spatially more complex prisms as the value of permeability or density contrast increases.

Time evolution of invasion fronts differs substantially from the homogeneous and isotropic case as the anisotropy ratio becomes more severe. Invasion fronts evolve from cylindrical to wellbore-centered elliptical prisms that extend toward the sides of the wellbore for increasing values of anisotropy ratio. Spatial distributions of mud-filtrate associated with anisotropic rock formations are characterized by symmetrical, wide-extent fronts on the sides, and spatially concentrated fronts at the top and bottom of the wellbore. Typically, the higher the anisotropy ratio, the more skewed the shape of the elliptical invasion prism. This spatial distribution of mud-filtrate is analogous to that resulting from the flow of filtrate along a horizontal fracture intersecting the horizontal wellbore. Increasing the anisotropy ratio suppresses the effect of density contrast or high permeability by reducing the fluid movement in the vertical direction at the bottom of the borehole.

Variability in the rock’s wettability directly affects displacement efficiency thereby causing significant variations in average saturations and invasion geometries around the wellbore. The distributions of fluid saturation shown in Studies 5 and 6 indicate larger invasion zones with increasingly higher oil saturations as the rock wettability is varied from water- to oil-wet. This result is consistent with the physics of immiscible displacement dictated by the relationships of relative permeability. The background effect of the variability in absolute permeability is resolved by comparing the fluid saturation plots for Studies 5 and 6. We observe that the overall invasion geometry is determined by absolute permeability as it controls the sagging at the bottom of the well. However, the spatial distribution of fluid saturation within the invaded zone remains a strong function of wettability.

We examine a limited number of cases intended to assess capillary pressure effects. It is observed that capillary pressure suppresses the effects of both density contrast and high permeability. It also disperses the saturation front, and reduces the displacement efficiency, thereby causing larger wellbore-centered cylindrical invasions with higher oil saturations within the invaded zone.

Signatures of both fluid and petrophysical parameters can be observed on the AIT* tool response. A characteristic increase in the shallower resistivity channel is consistently observed as a function of increasing filtrate sagging at the bottom of the borehole. The response from the shallower resistivity channels becomes closer to the resistivity of the virgin zone, \( R_v \), as filtrate sagging increases at the bottom of the well. Quite obviously, the AIT* response is sensitive to the average medium properties around the borehole. Uninvaded zones at the top, and slightly invaded zones at the sides of the wellbore constructively influence the tool response. This causes a reduction in the deviation of the resistivity response due to severe invasion at the bottom of the well, whereupon the AIT* response yields resistivity values closer to those of the virgin zone. Consequently, petrophysical and fluid flow conditions in the presence of high permeability and/or high gravity contrasts between invading and in-situ fluids (for a relatively low permeability anisotropy ratio) have a distinct imprint on the shallower AIT* resistivities.

Increasing the anisotropy ratio suppresses the invasion at the top and at the bottom of the well and results in deeper horizontal invasion. As a function of increasing anisotropy ratio, the shallower AIT* channels monotonically yield resistivity values closer to those of the virgin zone. An increase in the anisotropy ratio causes a thinning in the lense-shaped distribution of horizontal mud-filtrate invasion. Uninvaded zone saturations at the top and at the bottom of the well strongly influence the AIT* tool response as a function of increasing anisotropy.

Average hydrocarbon saturation around the wellbore increases and the overall invasion zone grows larger as the rock’s wettability is shifted from water- to intermediate- and finally to oil-wet. The corresponding AIT* response consistently yields higher resistivity values from shallower channels as a function of the above mentioned
wettability shift. An enlargement of the shape of the invasion zone is observed as more deep-sensing channels are influenced by the invasion process. The front-dispersing effect produced by capillary pressure yields high average oil saturations as well as wellbore-centered deep invasions. Again, shallow AIT* channels yield resistivity values closer to $R_t$, but the effect of deep invasion remains observable in the deep-sensing channels. Despite the adverse influence of capillary pressure, for all of the investigated cases, the deepest penetrating AIT* channel continues to exhibit a strong influence of the virgin zone resistivity, $R_v$.

Prior knowledge of one or more of the controlling petrophysical and fluid parameters together with the AIT* log response can significantly enhance the interpretation of both the extent and the geometry of the invasion zone. In addition, for a significant number of investigated cases, the AIT* response remains sensitive to the gross geometry of invasion rather than to the detailed geometrical features of the invaded zone. Despite this insensitivity to the detailed geometry of invasion, the AIT* still exhibits sufficient sensitivity to enable a correction for invasion effects on the deeper channels.

CONCLUSIONS

The following conclusions can be drawn from the sensitivity study presented in this paper:

- Petrophysical and fluid parameters play a central role in determining (a) the time evolution of the shape and lateral extent of the mud-filtrate invasion and (b) the distribution of saturations within the invaded zone of a horizontal well. Consequently, understanding the influence of petrophysical and fluid parameters in array induction measurements acquired in a horizontal well requires of a rigorous quantitative interpretation approach that addresses multiphase flow phenomena. It has been found that the role played by petrophysical and fluid parameters in the time evolution of mud-filtrate invasion can be more significant in the case of horizontal wells than in the case of vertical wells.
- Array induction tools exhibit limited sensitivity to several of the complex geometrical features of the saturation distribution around a horizontal borehole. Therefore, it has been found that often simple parametric representations of both invasion geometry and invaded zone saturations can be utilized to quantitatively interpret array induction measurements into fluid saturations in the virgin zone.
- Test cases designed to quantify the effect of capillary pressure on mud-filtrate invasion show that deep invasion can occur in the presence of relatively low mud-filtrate saturations. Clearly, quantifying the capillary pressure effects on the AIT* response requires of a more extensive study than the one presented in this paper.
- For all of the test cases considered in our sensitivity study, including those aimed at quantifying the effect of capillary pressure, the deepest penetrating AIT* channel remains strongly influenced by the virgin zone’s resistivity, $R_v$. Hence, we conclude that the AIT* 5 channels embody sufficient information to enable accurate and robust estimation of $R_v$ through nonlinear inversion.

ACKNOWLEDGEMENTS

We would like to express our gratitude to Schlumberger for the funding of this work through a summer internship position provided to Faruk. O. Alpak. The authors also wish to thank Drs. Vladimir Druskin, Barbara Anderson, and Ping Lee, for valuable discussions and support throughout the completion of this project.

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Table 1. Summary of mud-filtrate (aqueous phase) properties at a pressure of 3000psi and a temperature of 220°F.

| Formation Volume Factor, \( B_w \), (rbbl/stb) | 1.0410 |
| Density, \( \rho_w \), (lbm/cuft) | 60.5446 |
| Compressibility, \( C_w \), (1E-06 x psi\(^{-1}\)) | 3.1801 |
| Viscosity, \( \mu_w \), (cp) | 0.3245 |

Table 2. Summary of hydrocarbon (oleic phase) properties at a pressure of 3000psi and a temperature of 220°F.

<table>
<thead>
<tr>
<th>oAPI</th>
<th>( \rho_o ) (lbm/cuft)</th>
<th>( B_o ) (rbbl/stb)</th>
<th>( c_o = c_{fl} \times 10^6 ) (psi(^{-1}))</th>
<th>( \mu_o ) (cp)</th>
<th>( \Delta \rho = \rho_w - \rho_o ) (lbm/cuft)</th>
<th>( \Delta \rho ) (g/cm(^3))</th>
<th>( \frac{\mu_o}{\mu_w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>60.2375</td>
<td>1.1913</td>
<td>9.6185</td>
<td>5.7945</td>
<td>0.3071</td>
<td>0.004920</td>
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<td>1.4223</td>
<td>91.4322</td>
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<td>0.094540</td>
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<td>55</td>
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<td>2.0657</td>
<td>156.3497</td>
<td>0.1558</td>
<td>13.2267</td>
<td>0.211872</td>
<td>0.4799</td>
</tr>
</tbody>
</table>

Figure 1. Organization flow chart used to quantitatively assess the sensitivity of array induction tools to mud-filtrate invasion in horizontal wells.

Figure 2. Geometrical model considered for the simulation of two-phase flow (simultaneous flow of oleic and aqueous phases) and electromagnetic induction (adapted from Anderson et al., 1999).
Figure 3. Two-dimensional cross-section perpendicular to the borehole axis of the finite-difference grid constructed for the simulation of two-phase flow. The simulation grid is shown superimposed to an example water saturation field.

Figure 4. Two-dimensional cross-section of the zone of interest traversed by a horizontal well. The petrophysical and fluid flow properties displayed in the figure summarize the ranges of variability considered in our sensitivity studies.

Figure 5. Study 1: Sensitivity of the near-wellbore fluid saturation field to a variation in absolute permeability. Porosity is empirically coupled to absolute permeability. The top panels are detailed images of near-wellbore water saturation simulated for various values of absolute permeability. Mid and bottom panels display the simplest and the most detailed smoothed images, respectively, used to transform near-wellbore saturations into electrical resistivities.

Figure 6. Study 2: Sensitivity of the near-wellbore fluid saturation field to a variation in oil type within a high-permeability formation. The top panels are detailed images of near-wellbore water saturation simulated for various °API oils. Oil types are characterized by the values of fluid density, viscosity, and compressibility shown in Table 2. Refer to Fig. 5 for further details on the plotting conventions.

Figure 7. Study 3: Sensitivity of the near-wellbore fluid saturation field to a variation in oil type within a moderate-permeability formation. Compare with Fig. 6 (results of Study 2). The top panels are detailed images of near-wellbore water saturation simulated for various °API oils. Refer to Fig. 5 for further details on the plotting conventions.

Figure 8. Study 4: Sensitivity of the near-wellbore fluid saturation field to a variation in permeability anisotropy. The top panels are detailed images of near-wellbore water saturation simulated for various permeability anisotropy ratios. Here, anisotropy ratio is defined as the ratio of horizontal to vertical absolute permeabilities. Refer to Fig. 5 for further details on the plotting conventions.
Figure 9. Study 5: Sensitivity of the near-wellbore fluid saturation field to a variation in the wettability characteristics of a high-permeability formation. The top panels are detailed images of near-wellbore water saturation simulated for typical wetting characteristics of the reservoir rock. Refer to Fig. 5 for further details on the plotting conventions.

Figure 10. Study 6: Sensitivity of the near-wellbore fluid saturation field to a variation in the wettability characteristics of a moderate-permeability formation. Compare with Fig. 9 (results of Study 5). The top panels are detailed images of near-wellbore water saturation simulated for typical wetting characteristics of the reservoir rock. Refer to Fig. 5 for further details on the plotting conventions.

Figure 11. Wettability characteristics of the permeable medium. The figures show how the wettability properties are changed by modifying the relative permeability relationships used for fluid flow simulations. Typical end-point relative permeabilities and residual saturations along with curvature parameters are used to describe water-, intermediate- and oil-wet rocks. An additional plot describes the imbibition capillary pressure relationship employed for fluid flow simulation in the water-wet case.
Figure 12. Simulated electromagnetic responses for Study 1 Cases 1, 2, and 3. Five AIT* electrical resistivity channels are shown for each of the panels (a), (b), and (c).

Figure 13. Simulated electromagnetic responses for Study 2 Cases 1, 2, and 3. Five AIT* electrical resistivity channels are shown for each of the panels (a), (b), and (c).
Figure 14. Simulated electromagnetic responses for Study 3 - Cases 1, 2, and 3. Five AIT electrical resistivity channels are shown for each of the panels (a), (b), and (c).

Figure 15. Simulated electromagnetic responses for Study 4 - Cases 1, 2, and 3. Five AIT electrical resistivity channels are shown for each of the panels (a), (b), and (c).
Figure 16. Simulated electromagnetic responses for Study 5 Cases 1, 2, 3, and 4. Five AIT* electrical resistivity channels are shown for each of the panels (a), (b), (c), and (d).

Figure 17. Simulated electromagnetic responses for Study 6 Cases 1, 2, 3, and 4. Five AIT* electrical resistivity channels are shown for each of the panels (a), (b), (c), and (d).