A Quantitative Study to Assess the Value of Pressure Data Acquired with In-Situ Permanent Sensors in Complex 3D Reservoir Models Subject to Two-Phase Fluid Flow

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Abstract

Recent advances in smart well completion technologies have enabled the dynamic acquisition of in-situ pressure measurements with sensors in direct hydraulic contact with rock formations. In addition to the immediate impact of in-situ sensors as tools for real-time, reactive reservoir management and control, usage of in-situ pressure sensors has long-term benefits. Devising an optimal macro-management strategy for hydrocarbon reservoirs requires more than a tool for instantaneous monitoring and control. Precise knowledge of the spatial distribution of petrophysical properties is essential for accurate reservoir delineation, management, and production forecasting. From the formation evaluation viewpoint, large volumes of flow-related data constitute an attractive prospect for robust and accurate characterization of reservoirs. In addition to static information in the form of geostatistical, seismic, and geologic data, usage of dynamic measurements remains imperative to construct accurate reservoir models amenable to production forecast.

In this paper, we address the quantitative estimation of three-dimensional (3D) spatial distributions of permeability and porosity from pressure measurements acquired with in-situ permanent sensors. A pilot waterflood operation conducted with a conventional five-spot pattern is chosen as example for our numerical experiments. We assume in-situ permanent pressure sensors to be an integral part of the production-well completion and to remain completely isolated from the hydraulics of the wellbore. Therefore, these sensors perform uncorrupted measurements of in-situ formation fluid pressures. Quantitative estimation of spatial distributions of permeability and porosity is approached with a novel subspace approach and a modified Gauss-Newton inversion algorithm. This inversion strategy incorporates an adjoint formulation for the efficient computation of model sensitivities. In our inversions, the physics of two-phase fluid-flow in the 3D spatial domain is rigorously incorporated into the assessment of distributions of petrophysical properties from in-situ permanent-sensor pressure data. Comparisons are shown of the enhancement in spatial resolution and reduction of uncertainty when using in-situ permanent sensors with respect to estimations performed via standard history matching techniques.

Introduction

The availability of permanently installed downhole pressure, resistivity, and temperature sensors has opened a new window of opportunities to probe hydrocarbon reservoirs. Permanent sensors and monitoring systems provide continuous streams of measurements that facilitate real-time reservoir management and, therefore, help to increase hydrocarbon recovery. Several publications have quantified the added value of permanent downhole pressure gauges when used as a part of the well completion, i.e., Baker et al.1 and Athichanagorn et al.2. Data sets with enhanced resolution properties can be acquired with pressure gauges cemented behind casing and in direct hydraulic communication with the formation. In-situ sensors of this type are placed in the annulus between the formation and the casing, thereby remaining exposed to the hydraulics of formation fluids. Patents have been granted for cemented formation pressure sensors3 and cemented resistivity arrays4. Oilfield experiments have been conducted to test the practical feasibility and the added value of in-situ permanent sensors (see, for instance, van Kleef et al.5, Bryant et al.6). These proof-of-concept field tests of in-situ permanent sensors have created a renewed interest in the dynamic monitoring for reactive management of complex hydrocarbon reservoirs.

When linked with a feedback loop to completion hardware such as remotely operated valves, interpretations of continuous stream of data can be employed to implement an optimal management strategy for the real-time control of fluid production/injection rates. In contrast to sensor development and deployment issues, advances reported in the area of the interpretation of data acquired with in-situ permanent sensors have been scarce. Athichanagorn et al.2 describe a wavelet analysis technique for the interpretation of permanent downhole pressure measurements and discuss practical issues related to the processing of large amounts of data. Belani et al.8 describe the utilization of permanent sensor pressure data...
to monitor pressure transients with repeated fall-off tests. In the latter development, a method is described to jointly interpret cemented pressure and resistivity sensor data into estimates of front position and fluid mobility ratios. Raghuraman and Ramakrisnan also combined in-situ permanent resistivity array and cemented pressure-gauge data to further constrain the petrophysical assessment of the reservoir model under investigation. Charara et al. performed a numerical experiment to demonstrate the use of permanent resistivity and transient-pressure measurements for time-lapse saturation mapping. Wang and Horne integrated permanent resistivity sensor and production data to improve the estimation of spatial distributions of single-phase permeability.

Traditional production sampling strategies consist of time records of bottom-hole pressure (BHP) and surface flow rates expressed in terms of water-oil ratio (WOR) for two-phase flow behavior. Albeit intrinsically useful, these measurements at best can provide large-scale average properties of flow units commingled through a well rather than the response of individual flow units. This paper quantifies the use of in-situ pressure sensors to sample time records of individual layer pressures during the hydrocarbon production. As shown in Fig. 1, pressure sensors considered in this study are deployed in-situ behind casing, are hydraulically isolated from the wellbore, and remain in hydraulic communication with the formation during hydrocarbon production. It is shown that pressure data measured with an array of in-situ pressure sensors installed across multi-layer formations provide enhanced vertical resolution in reconstructing depth dependent petrophysical properties. Additionally, injection/production responses sampled with in-situ pressure arrays can provide enhanced inter-well spatial sensitivity to resolve main features of the lateral distribution of petrophysical properties.

The majority of the interpretation work for permanent sensor measurements reported in the literature have been based on the limiting assumption of single-phase fluid flow regime throughout the production/injection history of the reservoir. On the contrary, numerical examples described in this study are intended to assess the spatial resolution properties of in-situ permanent sensors in the presence of two-phase flow within the context of a waterflood secondary recovery scheme.

Estimation of petrophysical properties is approached with a novel inversion algorithm that efficiently combines multidirectional-resolution properties and an adjoint formulation for sensitivity computations. This inversion algorithm also incorporates the use of water-cut data to further reduce uncertainty in the estimation of a 3D petrophysical model. Inversions of measurement data are performed to quantify the lateral and vertical resolution of in-situ permanent pressure sensor measurements. Reconstructions of the permeability and porosity model using pressure data acquired with in-situ permanent pressure sensors are compared to the inversion results of BHP data acquired with conventional wellbore sensors. We also consider the integration of in-situ permanent sensor data with water-cut measurements acquired in the late-time history of the waterflood. Comparisons of inversion results indicate substantial vertical resolution enhancement in the reconstruction of depth dependent petrophysical properties when in-situ permanent pressure data are included as input to the inversion. In comparison to the inversions of conventional wellbore data, relatively more accurate reconstructions of interwell porosity and permeability are achieved with the use of in-situ permanent pressure data. Although to a less degree when compared to the vertical resolution enhancement, in-situ permanent-sensor data clearly extend the lateral resolution of inversions. Inversion results for the numerical test examples considered in this paper constitute a practical proof of the reduction of uncertainty when using in-situ permanent sensors with respect to estimations performed with history matching techniques.

Simulation of Fluid-Flow Measurements
Quantitative estimation of the spatial distribution of permeability and porosity is posed as an optimization problem in this paper. Accordingly, reservoir parameters such as permeability and porosity are adjusted to minimize the misfit between the observed and predicted time records of an arbitrary combination of in-situ permanent-sensor pressure, BHP, and water-cut measurements. The predicted measurements are generated from the numerical solution of coupled partial differential equations (PDE) that describe two-phase flow phenomena in porous media. In our formulation, we disregard the presence of chemical reactions, rock/fluid mass transfer, and diffusive/dispersive transport. The mass balance equation for the $i$th fluid phase in a 3D spatial domain can be stated as follows:

$$\frac{\partial}{\partial t} \left( \phi S_i \right) + \nabla \cdot \left( \frac{u_i}{B_i} \right) = R^{sc}, \quad (1)$$

where $B$, $u$, $\phi$, $R^{sc}$, and $S$ denote formation volume factor, fluid velocity vector, porosity, source/sink term for each phase at standard conditions, and phase saturation, respectively. No-flow boundary conditions are assumed at the external boundaries of the porous medium subject to simulation. For the modeled fluid-flow phenomenon, we assume mutual immiscibility between both fluid components (water and oil) meaning that the phases and components are the same. Darcy’s law is the governing transport equation, i.e.,

$$u_i = -k \frac{k_i}{\mu_i} (\nabla p_i - \gamma_i \nabla D_z), \quad (2)$$

where $k$ is the single-phase permeability tensor of the porous medium and $k_i$ denotes relative permeability. In addition, $\mu$, $\rho$, $\gamma$, and $D_z$ denote the phase viscosity, pressure, specific gravity, and the vertical location below some reference level, respectively. Finally, the constituent equation follows from the equation of state. The assumption is made that both fluid and rock compressibilities are functions of the pressure range of interest for the simulations and are given by

$$c_i = -\frac{1}{V_i} \frac{\partial V_i}{\partial p} \left|_T \right. = \frac{1}{\rho_i} \frac{\partial \rho_i}{\partial p} \left|_T \right. , \quad (3)$$

and
\[ c_r = \frac{1}{\phi} \frac{\partial \phi}{\partial \phi} \]  \hspace{1cm} (4)

respectively. Equations (3) and (4) are incorporated into Eq. (1) via constitutive relationships given by

\[ B_i = B_i^0 [1 - c_i (p - p^*)], \]  \hspace{1cm} (5)

and

\[ \phi(p) = \phi^o [1 + c_i (p - p^*)], \]  \hspace{1cm} (6)

where \( B_i^0, p^o, \) and \( \phi^o \) denote formation volume factor, pressure, and rock porosity at reference conditions. In our simulator, standard conditions are used as reference. On the other hand, capillary pressures and fluid saturations are governed by

\[ P_c (S_w) = p_o - p_w, \]  \hspace{1cm} (7)

and

\[ S_o + S_w = 1.0, \]  \hspace{1cm} (8)

respectively. The coupled set of PDEs described by Eq. (1) is solved numerically using a finite-difference stencil and an IMPES approach\(^1.5\).

**A Subspace Adjoint Inversion Algorithm**

By making use of a Bayesian statistical rule, inversion can be stated as the minimization of the quadratic objective function\(^1^2\) given by

\[ J = (m - m_o)^T C_D^{-1} (m - m_o) + (g - d_{obs})^T C_M^{-1} (g - d_{obs}), \]  \hspace{1cm} (9)

where \( m = [\log(k), \phi]^T \) is the size-\( M \) vector of reservoir parameters, \( g \) is the vector of simulated measurements, \( d_{obs} \) is the vector of measurement data, \( C_D \) and \( C_M \) denote the covariance matrices of measured data and the model matrix, respectively. Given that, in general, physical values of absolute permeability encompass a large range, a convenient logarithmic transform for permeability remains suitable. This transformation is also useful in enforcing positivity constraints.

When the number of unknown reservoir parameters is large, numerous combinations of model domain parameters can equally satisfy the time record of fluid-flow measurements. Therefore, the inclusion of the first term in Eq. (9) aims at reducing the non-uniqueness of the solution as well as to stabilizing the estimation in the presence of noisy data. Precisely speaking, the first term biases the inversion toward a specific set of solutions in parameter space when the degree of non-uniqueness in the inversion remains high. The covariance matrix of the model may not be available due to the lack of a priori knowledge about the model. In general, the Laplacian or an equivalent isotropic operator can be applied for the regularization of model parameters. In particular, Tarantola\(^1^4\) showed that the first-order difference approximation is analogous to the model covariance matrix. In the inversions performed in this paper, we adopt an exponential covariance function introduced by Oliver\(^1^5\) to compute the covariance matrix.

A modified Gauss-Newton technique\(^1^2,1^4\) is employed for the minimization of Eq. (9). The iterative form of the modified Gauss-Newton method is given by

\[ m^{l+1} = m_o - C_M G_l^T [C_D + G_l C_M G_l^T]^{-1} [g - d_{obs} + G_l (m^l - m_o)]. \]  \hspace{1cm} (10)

In Eq. (10), \( G_l \) is the matrix of sensitivity coefficients calculated at the iteration level \( l \). At each iteration level, we solve the linear equation given by

\[ [C_D + G_l C_M G_l^T] x = g - d_{obs} + G_l (m^l - m_o). \]  \hspace{1cm} (11)

for the model update vector, \( x \). In this formulation, the dimension of the equations is equal to the number of observed data. Such a modified Gauss-Newton method operates efficiently for the cases where the size of \( d_{obs} \) (or the number of fluid-flow measurements) is relatively small. We also employ an adjoint procedure\(^1^2\) for the computation of the sensitivity coefficients of fluid-flow measurements with respect to petrophysical model parameters. At each iteration, the sensitivity coefficients are computed at the cost of a single forward simulation run and the solution of an adjoint system with multiple right-hand (RHS) vectors. The number of RHS vectors equals the size of \( d_{obs} \).

For inversion cases involving a large number of observed data, such as in the case of time records of production data, the size of the sensitivity matrix is considerably large. The high computational cost and computer memory requirements for calculating the sensitivity matrix makes the inversion algorithm in its current form impractical. Kennett et al.\(^1^6\) and Abacioglu et al.\(^1^7\) proposed a subspace approach to invert seismic data and pressure data, respectively. Similarly, in this paper, we propose a new subspace approach that makes large-scale inversion tractable. Details of this inversion approach are described below.

Suppose that the modeling errors can be neglected and that only measurement errors are of interest. The relationship between the modeled response and the measured data are given by

\[ g_{i,j} = d_{i,j} + e_{i,j} \text{ for } i = 1, \ldots, N_W \text{ and } j = 1, \ldots, N_D, \]  \hspace{1cm} (12)

where \( g \) is the modeled response. In the above equation, \( e \) denotes the observed error for each data point, \( N_D \) is the number of producing wells, \( N_W \) is the number of measured data, such as in-situ permanent-pressure sensor data, water-oil data at each producer, etc.

It is now remarked that the second term in Eq. (9) can be modified to read as

\[ \sum_{i=1}^{N_W} \sum_{j=1}^{N_D} \frac{(g - d_{obs})^2}{\sigma_{i,j}^2}. \]  \hspace{1cm} (13)

A motivation for developing a new subspace approach is based on the central limit theorem. The latter theorem states that the distribution of the sum of a large number of identically distributed random variables will be approximately normal, regardless of the individual distributions. If the error between the observed data and the model predicted data fall within the
Here, the matrix minimization of the objective function in Eq. (15) is much smaller than the size of the sensitivity coefficient matrix associated with the adjoint systems to be solved at each iteration. Moreover, the automatically translates into a reduction in the number of Therefore, a reduction in the number of observed data RHS vectors depends on the number of observed data. A linear system with multiple RHS vectors. The number of computation of sensitivity coefficients requires the solution of a linear system with multiple RHS vectors. The number of RHS vectors depends on the number of observed data. Therefore, a reduction in the number of observed data automatically translates into a reduction in the number of adjoint systems to be solved at each iteration. Moreover, the size of the sensitivity coefficient matrix associated with the minimization of the objective function in Eq. (15) is much smaller than the size of the sensitivity coefficient matrix associated with the minimization of the original objective function given by Eq. (9). Because of this, large volumes of observed data can be efficiently inverted into large reservoir petrophysical models using the subspace adjoint inversion algorithm.

Numerical Examples
With the purpose of quantifying the resolution properties of in-situ permanent sensors in a two-phase fluid-flow environment, we designed a realistic set of numerical examples. The reservoir volume is saturated with oil. A waterflood operation to sweep the reservoir volume is assumed to be the physical background of the inversion problem. We consider the deployment of cemented in-situ permanent pressure sensors along the production wells. Pressure measurements acquired with these sensors are inverted to yield spatial distributions of permeability and porosity. In order to establish a rigorous basis of comparison for the information content of in-situ permanent pressure measurements, we additionally consider the acquisition of conventional BHP and WOR measurements and perform inversions of these data to estimate spatial distributions of permeability and porosity. Assessment of the value of cemented in-situ permanent pressure sensors is performed directly in model space.

Reservoir Model. The reservoir model consists of a spatially heterogeneous 3D porous medium and is displayed in Fig. 2. Spatial dimensions are 1680 ft × 1680 ft × 50 ft and the model is discretized with a 21×21×5 Cartesian grid in the x, y, and z directions, respectively. The gridsize is uniform in each direction. Reservoir thickness is assumed uniform and equal to 50 ft. Each gridblock exhibits specific values of permeability and porosity. The permeability field is homogeneous, yet vertically anisotropic, with the vertical permeability assumed to be one-tenth of the horizontal permeability. Oil production in the reservoir of interest is assumed to be driven by a waterflood operation. A conventional 5-spot well pattern is used to sweep the hydrocarbons saturating the reservoir volume. The central well is used for water injection while the remaining wells are dedicated to oil production. Well locations are shown in Fig. 2. The injector well is located in the gridblock [x, y] = [11, 11] and four producers, numbered 1 through 4, are located in the gridblocks [5, 17], [17, 5], [5, 5], and [17, 17]. The total flow rate at each producer is fixed at 400 STB/D and the flow rate at the injector is fixed at 2100 STB/D. Two-phase relative permeability and capillary pressure curves employed in the forward modeling of fluid-flow measurements are shown in Figs. 3 and 4, respectively.
Permeability and porosity distributions for the synthetic 3D reservoir are generated by means of geostatistical simulation. Here, the mean for log-permeability is assumed to be 4.5 with variance equal to 0.5. On the other hand, the mean for porosity is assumed to be 0.18 with variance equal to 0.0025. Vario gram ranges for both permeability and porosity are 800 ft. Each layer is assumed to exhibit a spherical variogram model. No correlation is assumed between the porosity and log-permeability. Spatial distributions of the permeability and porosity are generated using unconditional simulation computed from Cholesky decomposition of the covariance matrix and are shown in Figs. 5 and 6, respectively. Measurements used in the inversions are generated using the permeability and porosity distributions in a forward simulation. Measurement errors are assumed independent and uncorrelated among themselves. Details of the reservoir fluid and rock properties along with the characteristics of the model’s spatial architecture are given in Table 1.

Inversion of In-Situ Permanent Sensor Pressure Measurements. Three in-situ permanent pressure sensors are assumed cemented behind casing in each production well. Sensor locations correspond to the centers of the upper-most, mid, and bottom-most layers, respectively. In-situ permanent sensor pressure measurements are acquired in response to oil production in each of the production wells. A 10 hr-long time-record of pressure responses is input to the inversion algorithm. Synthetically generated pressure measurements are contaminated with additive random Gaussian noise. The standard deviation of noise is assumed equal to 2 psi. Inversions are initialized with uniform spatial distributions of permeability and porosity.

Figures 7 and 8 display results of the simultaneous inversion of in-situ pressure data for spatial distributions of permeability and porosity, respectively. Estimation of the spatial distribution of permeability remains successful in
capturing the main features of the original distribution in both lateral and vertical directions. The main characteristic of the inversion results is the blurring of permeabilities for model domain voxels located far away from measurement locations. The quality in the reconstruction of permeability increases in the vicinity of the sensors. Yet, the inversion tends to yield an effective medium response close to that of average permeability away from the sensors. In this paper, we explore not only the information inherent to in-situ pressure measurements but also attempt to integrate dynamic information provided by fluid-flow measurements into geostatistical models. In the absence of any a-priori information about the spatial statistics of the permeability model, a very good reconstruction of the permeability model is attained with in-situ permanent sensor data. Similar conclusions concerning spatial resolution apply to the spatial distributions of porosity. Overall, inversion results characteristically yield relatively enhanced vertical resolution in the vicinity of the in-situ sensors. However, reconstructions undergo a significant blurring in both lateral and vertical directions away from the sensors. On the other hand, approximate locations and parameter values of large model features are reconstructed in a robust manner. Post-inversion data-domain matches of measurements remain very good for all sensor locations. Figures 9 through 12 show the agreement between observed data and the predicted pressure data for various sensor locations. In-situ pressure sensors 1, 2, 3, and 4 are located in the production wells 1, 2, 3, and 4 from the top of the reservoir, respectively. For completeness, time records of pressure computed from spatial distributions of initial guess model parameters are also shown in each of these figures.

Quantitative assessment of the model domain reconstructions is carried out by computing the correlation coefficient of post-inversion and true model domain parameters. The correlation coefficient varies between -1 and 1. It is equal to 1 when there is a perfect match between two model sets. In other words, for the case when inverted and true model parameters, e.g. permeabilities, match identically then the correlation coefficients are equal to 1. As inverted and true permeability fields become increasingly different, the correlation coefficient decreases toward zero. Thus, correlation coefficients can be utilized to diagnose the closeness of two parameter fields. Figures 13 and 14 display maps of correlation coefficients for initial guess and inverted permeability fields, respectively. In both figures, correlation coefficients are computed with respect to the actual spatial distributions of permeability. Figure 13 indicates that the majority of the correlation coefficients are below 0.5 at the level of the initial guess. Yet, as displayed in Fig. 14, most of the correlation coefficients increase to approximately 0.70 after matching the in-situ permanent sensor measurements. The enhancement in the correlation coefficients can be effectively interpreted as the reduction of non-uniqueness achieved with the integration of dynamic in-situ measurements with respect to that of an initial guess model lacking a-priori information. On the other hand, Figs. 15 and 16 display maps of correlation coefficients for initial guess and inverted porosity fields, respectively. In both these figures, correlation coefficients are computed with respect to the actual spatial distributions of porosity. The comparison of Figs. 15 and 16 also indicates a trend of the correlation coefficients to increase for the post-inversion spatial distributions of porosity. However, the magnitude of the enhancement is relatively less significant in comparison to the case of spatial distributions of permeability. Consistent with the physics of pressure measurements, in-situ permanent sensor data appear to be more sensitive to the spatial distribution of permeabilities in comparison to the spatial distribution of porosities.

If the oil production rate is measured on a layer-by-layer basis along each of the production wells, this information can play a similar role as in-situ sensor pressure does to reconstruct spatial distributions of permeability and porosity. Of course, in-situ flow rate measurements require special completions and permanent downhole flowmeters. To demonstrate the information content of in-situ flow rate measurements, we plot post-inversion layer-by-layer oil production rates in Fig. 17 for the case of the inversion of in-situ pressure data. Although layer flow rates do not vary as a function of time until the time of water breakthrough (not shown), individual flow rate substantially differ from each other. However, at the initial guess level, when uniform permeability and porosity distributions were assigned to all gridblocks, in-situ flow rates remain approximately equal to each other. This information alone evidences the sensitivity of in-situ flow rates to near-wellbore distributions of petrophysical model parameters. A full inversion of in-situ flow rates is still necessary to further assess the spatial resolution properties of this type of measurements.

Comparison of the Inversion of In-Situ Permanent Sensor Pressure Measurements to the Inversion of BHP Measurements. In the following numerical example, attention is focused to the inversions of BHP data acquired at each production well with a conventional downhole pressure gauge. We compare the inversion results to those yielded by the measurements acquired with in-situ permanent sensors.

The main difference between in-situ permanent sensor pressure and BHP data lies in the measurement conditions. Cemented in-situ permanent sensors are designed to establish a direct hydraulic communication with the formation fluids and are isolated from the wellbore. As such, in-situ pressure gauges measure the time record of fluid pressures at a particular location within the reservoir. Under this measurement condition, pressure responses are acquired before the produced fluids from various layers undergo mixing and pressure equilibration within the wellbore. Hence, in-situ permanent sensor measurements exhibit a significant amount of sensitivity to the local distributions of flow properties of the formation. Subsequent to the flow of fluids from reservoir layers to the wellbore, fluid pressures from these layers equilibrate at wellbore conditions. This equilibrium pressure is also referred to as BHP. As such, BHP measurements can only be available on a well-to-well basis rather than on a sensor-by-sensor basis. Moreover, a significant amount of spatial resolution inherent to the in-situ pressures is sacrificed to the pressure equilibration within the wellbore. In contrast to in-situ pressure measurements, BHP measurements incorporate limited sensitivity to the local distributions of flow properties. Hence, effective average medium properties in the vicinity of
the wellbore characterize the level of spatial resolution that can be recovered from BHP measurements.

Confirmation of the physical insights discussed above is provided by the inversions of BHP data acquired within the production wells. Measurement data are corrupted with the same noise contamination mechanism described in the previous numerical example. Spatial distributions of permeability and porosity yielded by the inversion of BHP data are shown in Figs. 18 and 19, respectively. Post-inversion BHP data matches are shown in Figs. 20 through 21 along with BHP time records computed from spatial distributions of initial guess model parameters. As shown in Figs. 18 and 19, the spatial resolution of BHP measurements suffices only to asymptotically approximate an average permeability and porosity for the entire reservoir. Variations in the inverted permeability occur only in the vicinity of wells, where one can observe slightly improved spatial resolutions inherent to the BHP measurements. However, the above conclusion does not hold true for porosity distributions. Such an observation is consistent with the first-order sensitivity of pressure measurements to permeability. Information about the location and magnitude of the main reservoir features is nonexistent in the inversions of BHP measurements. Poor reconstructions of the spatial distributions of permeability and porosity coupled to good matches between BHP measurements and post-inversion BHP simulations (shown in Figs. 20 through 23) clearly indicate the high level of non-uniqueness in the inversions of BHP data.

Comparisons of Figs. 7 and 17 for permeability, and Figs. 8 and 19 for porosity domains indicate major differences in the resolution properties of BHP and in-situ permanent sensor data. Spatial distributions of permeability and porosity yielded by the inversions of in-situ pressure measurements exhibit significantly enhanced lateral as well as vertical resolution in comparison to the reconstruction of distributions inverted from BHP measurements.

Comparison of the Inversion of In-Situ Permanent Sensor Pressure Measurements to the Inversion of WOR Measurements. We also investigate the use of WOR measurements for the quantitative estimation of spatial distributions of permeability and porosity. In general, WOR data constitute a relatively late-time measurement in the economic life of a reservoir. Until the water breakthrough occurs in one or more production wells, WOR information is literally nonexistent. After water breakthrough, the ratio of the volumetric production rates of water and hydrocarbon components is measured at surface conditions. Hence, in this paper, similar to BHP measurements, we focus on conventional WOR measurements acquired on a well-by-well basis. WOR measurements are contaminated with additive random noise using a procedure similar to that explained for the case of in-situ permanent pressure measurements. Figures 24 and 25 show inversion results from WOR data in the form of spatial distributions of permeability and porosity, respectively. Post-inversion data domain fits are displayed in Figs. 26 through 29 for each production well along with the WOR data simulated using a homogeneous distribution of parameters as initial guess.

Model domain reconstructions indicate that, in comparison to the inversion of BHP data, a relatively better reconstruction is obtained for the main features of the reservoir. However, when we compare the inversions obtained from in-situ pressure measurements to the inversions from WOR measurements, in-situ pressure measurements clearly yield higher resolution reconstructions of the spatial distributions of both permeability and porosity.

Similar to the BHP, WOR measurements contain average vertical information of reservoir parameters. Therefore, the vertical resolution of in-situ pressure measurements is far superior to that of WOR data especially in the vicinity of production wells. Moreover, as discussed before by Wu et al., the inverted distributions remain highly influenced by the actual values of porosity and permeability in the vicinity of wells since the fluid flow rates are extremely sensitive to these gridblock parameters near production wells. On the other hand, in contrast to BHP measurements, WOR measurements incorporate a relatively extended lateral length of penetration evidenced by the relatively improved sensitivity to the general reservoir features. As far as the lateral resolution of BHP and WOR measurements is concerned, in this paper, we refer to “improved lateral sensitivity” rather than “improved lateral reconstruction” when considering the use of WOR data. As shown in Fig. 24 for the permeability domain, and in Fig. 25 for the porosity domain, WOR measurements clearly yield the lateral resolution necessary to identify the location of the main reservoir features. Yet, the comparison of Figs. 18 and 24 for permeability domain and Figs. 19 and 25 for porosity domain indicate that, in the lateral direction, WOR measurements encompass more information in comparison to BHP measurements. However, when we compare the inversion results of in-situ permanent sensor measurements to that of WOR and BHP measurements, the lateral resolution of in-situ permanent sensor measurements is significantly superior to that of both WOR and BHP data. From a formation evaluation viewpoint, acquisition of pressure data with arrays of in-situ permanent sensors deployed in various lateral locations in the reservoir replicate multiple local pressure interference tests that are sensitive to local formation properties. In a much coarser sense, these measurements can be regarded as sensitive to the interwell reservoir volume.

Figures 26 through 29 show good post-inversion data domain fits yielded by the inversion of WOR data. Along with the comparison of inverted and true spatial distributions for model domain parameters, the above observation provides a proof of concept for the high-level of non-uniqueness when estimating permeabilities and porosities from WOR data.

Another disadvantage of WOR measurements lies in the fact that, in general, they are not available in the early stages of the life of the reservoir. Once water breakthrough occurs, a significant reduction in the production rate takes place very quickly unless a remediation measure is taken. Late arrival of the information is a significant factor to reduce the value of WOR measurements.

Regularization. It must be pointed out that, in order to robustly invert parameter fields, one can incorporate the prior information under the Bayesian framework to bias the inversion toward a specific region in model space. In the
absence of \textit{a-priori} information, regularization can be used to impose a desired degree of freedom of the parameter fields. Thus, regularization appears to improve the resolution of permeability and porosity around producers and injector wells. In all cases, we make use of the regularization term discussed within of the specific formulation implemented to solve the inverse problem. In the absence of a regularization term, very rough parameter fields caused convergence problems. We also observed that the regularization did contribute to improve the vertical resolution of the spatial distribution of permeability and porosity.

\textbf{Figure 30} shows the convergence rates associated with the subspace algorithm for inversions that incorporate different types of measurements. Minimization of the subspace objective function is performed for the inversions of in-situ permanent sensor pressure, BHP, and WOR measurements. The convergence behavior for each of these cases is shown in Fig. 30 as a function of the required number of iterations. For the purpose of comparison, convergence histories for all of the cases are plotted on the same graph. The objective function is efficiently reduced for all of the cases. The final value of the objective function is the lowest for the case of the inversion of WOR data followed by BHP data. Comparisons of the inverted and true spatial distributions of permeability and porosity for the inversions of all types of measurements indicate a significantly lower degree of non-uniqueness for the inversion of in-situ permanent sensor data.

\textbf{Summary and Conclusions}
A robust and efficient subspace inversion algorithm was developed for the inversion of fluid-flow measurements acquired in two-phase flow environments, i.e., waterflood operations. The subspace inversion algorithm was applied to the quantitative appraisal of in-situ permanent sensor pressure measurements in the estimation of spatial distributions of permeability and porosity. Reconstruction of model parameters yielded by permanent pressure sensor pressure data was rigorously compared to the spatial distributions of model parameters inverted from BHP and WOR data, respectively. When compared to the true permeability and porosity fields, reconstructions of spatial distributions of model parameters yielded by noisy in-situ permanent pressure measurements are more accurate than reconstructions performed with noisy BHP and WOR data. In both lateral and vertical directions the relative spatial resolution of in-situ permanent sensor data is significantly higher than the resolution provided by BHP and WOR measurements. Our test cases also indicate that the degree of sensitivity of in-situ permanent sensor measurements is higher for the spatial distributions of permeability than for the spatial distributions of porosity.

Inversion results clearly show the added value of in-situ permanent sensor measurements in providing enhanced spatial resolution of permeability and to a relatively less degree for porosity. Having established a practical proof of concept for the value of dynamic in-situ pressure measurements in the absence of \textit{a-priori} information, in-situ pressure measurements could be further integrated into the construction of static geostatistical reservoir models. Such a strategy could provide more accurate spatial descriptions of porosity and permeability and hence a significant reduction of uncertainty in the forecast of hydrocarbon production.

\textbf{Acknowledgements}
We would like to express our gratitude to Baker Atlas, Halliburton, Schlumberger, Anadarko Petroleum Corporation, and Shell International Exploration & Production Inc. for partial funding of this work through UT Austin’s Center of Excellence in Formation Evaluation. Partial funding of this work was also provided by the U.S. Department of Energy under contract No. DE-FC26-00BC15305.

\textbf{Nomenclature}

\begin{tabular}{ll}
$B_o$ & formation volume factor of oil \\
$B_w$ & formation volume factor of water \\
$C$ & covariance matrix \\
$c$ & water compressibility, oil compressibility \\
$d_{obs}$ & vector of observed data \\
$D$ & depth of reservoir \\
e & observed error \\
$F$ & vector of discretized flow equation \\
$G$ & sensitivity matrix of data \\
g & vector of simulated flow \\
$J$ & objective function \\
k & permeability \\
K & number of penetrated layers \\
kro & oil relative permeability \\
kw & water relative permeability \\
m & vector of model parameters \\
$M$ & number of simulator gridblocks \\
$N_{id}$ & number of observed data \\
$N_w$ & number of production well \\
$q_o$ & oil flow rate \\
$q_w$ & water flow rate \\
$q_t$ & total flow rate \\
p & pressure \\
$P_c$ & capillary pressure \\
r & well radius \\
$S$ & skin factor \\
$S_w$ & water saturation \\
$S_o$ & oil saturation \\
t & time \\
u & vector of fluid velocity \\
$WI$ & well index \\
X & vector of pressure and saturation
\end{tabular}

\textbf{Greek Symbols}

\begin{tabular}{ll}
$\rho$ & fluid density \\
$\mu$ & fluid viscosity \\
$\lambda$ & adjoint variable, mobility of fluid \\
$\phi$ & porosity \\
$\sigma$ & standard deviation \\
$\gamma$ & specific gravity
\end{tabular}

\textbf{Subscripts}

\begin{tabular}{ll}
f & formation \\
i & \textit{ith} phase, \textit{ith} well, \textit{ith} layer \\
j & \textit{jth} observed data \\
l & iteration number \\
o & oil phase \\
w & water phase \\
r & rock
\end{tabular}
Superscript

- \( T \) = matrix transpose
- \( n \) = time steps
- \( o \) = end-point

References


Appendix – Computation of the Subspace Gradient via an Adjoint Operator

Wu et al.\(^{12}\) describe an efficient adjoint method for the computation of sensitivities of production data with respect to model parameters for two-phase flow. Here, we extend the adjoint method to calculate the gradient of the sub-objective function with respect to permeability and porosity. The discretized form of the two-phase flow equation can be stated as follows:

\[ F_m(X^{n+1}, X^n, t) = 0, \quad (A-1) \]

where \( X = (x_1, x_2, \ldots, x_M)^T \) is the vector of pressure and saturation in all \( M \) gridblocks and the superscript \( n \) stands for the \( n^{th} \) time-step.

For all \( n+1 \) time steps, the two \( M \)-dimensional vectors of adjoint variables are defined by \( \lambda_m^n = (\lambda_{m,1}^n, \lambda_{m,2}^n, \ldots, \lambda_{m,M}^n)^T \), where \( m \) stands for either water or oil phase. The flow equation (A-1) can be written in terms of an adjoint operator, whereupon a new functional \( J \) can be constructed using the expression,

\[ J = J_i + \sum_{m=0, w}^N \sum_{n=0}^{N-1} \lambda_{m}^{n+1} F_{m}^{n+1}, \quad (A-2) \]

where the objective function \( J_i \) is used to quantify the data misfit function for a given production well, for instance, and the subscript \( i \) designates a given well. Taking the variation of the objective functional into account and using the necessary condition for a functional extremum, one can obtain the adjoint equations\(^{12}\)

\[ \frac{\partial F_m^n}{\partial X^n} \lambda_m^n = - \frac{\partial F_m^{n+1}}{\partial X^n} \lambda_m^{n+1} - \frac{\partial J_i}{\partial X^n} \lambda_m^n \quad \text{for } n = 1, 2, \ldots, N-1, \quad (A-3) \]

where \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_M)^T \) denotes the vector of adjoint variables. Once the adjoint variable \( \lambda \) is computed, the gradients with respect to permeability and porosity can be computed with the expressions

\[ \frac{\partial J_i}{\partial k} = \sum_{m=0, w}^N \sum_{n=0}^{N-1} \left[ \frac{\partial F_m^n}{\partial k} \right]^T \lambda_m^n, \quad (A-4) \]

and

\[ \frac{\partial J_i}{\partial \phi} = \sum_{m=0, w}^N \sum_{n=0}^{N-1} \left[ \frac{\partial F_m^n}{\partial \phi} \right] - \left[ \frac{\partial F_m^{n+1}}{\partial \phi} \right]^T \lambda_m^n, \quad (A-5) \]
respectively. For the objective function \( J_i \), one needs to compute its derivatives with respect to both pressure and saturation. These derivatives can be calculated via the equation

\[
\frac{\partial J_i}{\partial X} = \frac{\partial}{\partial X} \sum_{n=0}^{N_D} (\text{WOR}_n^i - \text{WOR}_{\text{obs},n}^i)^2, \tag{A-6}
\]

where \( N_D \) is the number of the observed data, and

\[
\text{WOR}_n^i = \sum_{k=1}^{K} \left( q_{w,k}^i \right) \sum_{k=1}^{K} \left( q_{o,k}^i \right).	ag{A-7}
\]

In Eq. (A-7) \( K \) denotes the number of the penetrated layers. If the total flow rate is specified, the total flow rate at the \( k \)th layer can be written as

\[
q_{i,k}^r = q_i^r \frac{W T_k^i \left( \lambda_{o,k}^i / B_{o,k}^i + \lambda_{w,k}^i / B_{w,k}^i \right)}{\sum_{k=1}^{K} W T_k^i \left( \lambda_{o,k}^i / B_{o,k}^i + \lambda_{w,k}^i / B_{w,k}^i \right)}, \tag{A-8}
\]

where \( W T_k^i \) denotes the well index for the \( k \)th layer at well \( i \), given by

\[
W T_k^i = \frac{2\pi 1.127 \times 10^{-3} k_i^r \sqrt{k_i x_k^r k_i y_k^r}}{\ln(r_{o,k}^i / r_w^i) + S_i}. \tag{A-9}
\]

In Eq. (A-9) \( r_{o,k}^i \) is computed from

\[
r_{o,k}^i = \frac{0.2873 \Delta x^i \sqrt{1 + k_i x_k^i (\Delta y_k^i)^2 / (k_i y_k^i (\Delta x_k^i)^2)}}{1 + \sqrt{k_i x_k^i / k_i y_k^i}}, \tag{A-10}
\]

where \( r_w^i \) is the radius of well \( i \), \( S_i \) is the skin factor for well \( i \), and \( \Delta x_k^i \) and \( \Delta y_k^i \) denote the lengths of the gridblock containing well \( i \) (at \( k \)th layer) in the \( x \) and \( y \) directions, respectively. Phase flow rates at each layer are computed via

\[
q_{w,k}^i = q_{i,k}^w \frac{\lambda_{w,k}^i / B_{w,k}^i}{\lambda_{o,k}^i / B_{o,k}^i + \lambda_{w,k}^i / B_{w,k}^i} \tag{A-11}
\]

and

\[
q_{o,k}^i = q_{i,k}^o \frac{\lambda_{o,k}^i / B_{o,k}^i}{\lambda_{o,k}^i / B_{o,k}^i + \lambda_{w,k}^i / B_{w,k}^i}. \tag{A-12}
\]

Here, water and oil mobilities (\( \lambda_{w,k}^i \) and \( \lambda_{o,k}^i \)) are given by

\[
\lambda_{w,k}^i = \frac{k_{r_m,k}^i}{\mu_w}, \tag{A-13}
\]

and

\[
\lambda_{o,k}^i = \frac{k_{r_m,k}^i}{\mu_o}. \tag{A-14}
\]

By substituting equations (A-8) through (A-14) into (A-7), one obtains derivatives of WOR with respect to pressure and saturation in a straightforward manner.
Table 1. Summary of geometrical, fluid, and reservoir properties of the synthetic reservoir model considered in this paper.

<table>
<thead>
<tr>
<th>Property</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fluid</strong></td>
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</tr>
<tr>
<td>Water density</td>
<td>62.40 lb/ft³</td>
</tr>
<tr>
<td>Oil density</td>
<td>52.88 lb/ft³</td>
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<tr>
<td>Water viscosity</td>
<td>1.00 cp</td>
</tr>
<tr>
<td>Oil viscosity</td>
<td>0.92 cp</td>
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<tr>
<td>Water compressibility</td>
<td>$3.20 \times 10^{-6}$ psi⁻¹</td>
</tr>
<tr>
<td>Oil compressibility</td>
<td>$1.00 \times 10^{-5}$ psi⁻¹</td>
</tr>
<tr>
<td>Water formation volume factor</td>
<td>1.00 RB/STB</td>
</tr>
<tr>
<td>Oil formation volume factor</td>
<td>1.16 RB/STB</td>
</tr>
<tr>
<td><strong>Reservoir</strong></td>
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</tr>
<tr>
<td>Initial water saturation</td>
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<tr>
<td>Residual oil saturation</td>
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<tr>
<td>Average porosity</td>
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<tr>
<td>Average permeability</td>
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</tr>
<tr>
<td>$k_{rw}^o$ endpoint of water relative permeability</td>
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</tr>
<tr>
<td>$k_{ro}^o$ endpoint of oil relative permeability</td>
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</tr>
<tr>
<td>$k_x / k_y$ ratio of the two principal permeability directions</td>
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</tr>
<tr>
<td>$k_z / k_z$ ratio of horizontal permeability to vertical permeability</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Simulation</strong></td>
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<tr>
<td>Number of gridblocks</td>
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<tr>
<td>Gridblock size</td>
<td>$80 \times 80 \times 10$ ft</td>
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<tr>
<td>Injection rate</td>
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<tr>
<td>Production rate</td>
<td>2100 STB/D</td>
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<tr>
<td>Perforation information</td>
<td>All layers</td>
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<tr>
<td>Number of production wells</td>
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</tr>
<tr>
<td>Number of injection wells</td>
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</tbody>
</table>

**Fig. 1**– Graphical description of the components of an in-situ pressure gauge. The gauge is cemented behind casing and operates in direct hydraulic contact with the formation.

**Fig. 2**– Graphical description of the 3D reservoir model. The diagram shows the 21x21 Cartesian finite-difference grid used to simulate fluid-flow measurements. Well locations for the conventional five-spot pattern are indicated with vertical lines penetrating through the reservoir. The injection well is located in the middle of the reservoir. Four production wells are distributed symmetrically, and remain closer to the edges of the reservoir.
Fig. 3– Water-oil relative permeability curves used in the numerical simulation of water injection.

Fig. 4– Capillary pressure curve used in the numerical simulation of water injection.

Fig. 5– Graphical description of the true spatial distribution of log-permeability.

Fig. 6– Graphical description of the true spatial distribution of porosity.

Fig. 7– Graphical description of the spatial distribution of log-permeability estimated from time-lapse in-situ permanent pressure measurements.

Fig. 8– Graphical description of the spatial distribution of porosity estimated from time-lapse in-situ permanent pressure measurements.
Fig. 9– Post-inversion match between simulated and measured in-situ permanent sensor pressure data for Sensor No. 1.

Fig. 10– Post-inversion match between simulated and measured in-situ permanent sensor pressure data for Sensor No. 2.

Fig. 11– Post-inversion match between simulated and measured in-situ permanent sensor pressure data for Sensor No. 3.

Fig. 12– Post-inversion match between simulated and measured in-situ permanent sensor pressure data for Sensor No. 4.

Fig. 13– A 2D map of correlation coefficients for log-permeability computed to quantify the correlation between the initial guess and true distributions of log-permeability.

Fig. 14– A 2D map of correlation coefficients for log-permeability computed to quantify the correlation between the inverted and true distributions of log-permeability.
Fig. 15– A 2D map of correlation coefficients for porosity computed to quantify the correlation between the initial guess and actual spatial distributions of porosity.

Fig. 16– A 2D map of correlation coefficients for porosity computed to quantify the correlation between the inverted and actual spatial distributions of porosity.

Fig. 17– Layer-by-layer oil production rates computed using post-inversion spatial distributions of permeability and porosity. Model parameters were estimated from time-lapse in-situ permanent sensor pressure data.

Fig. 18– Graphical description of the spatial distribution of log-permeability estimated from time-lapse bottom-hole pressure (BHP) measurements.

Fig. 19– Graphical description of the spatial distribution of porosity estimated from time-lapse bottom-hole pressure (BHP) measurements.

Fig. 20– Post-inversion match between simulated and measured bottom-hole pressure (BHP) data for the Production Well No. 1.
Fig. 21– Post-inversion match between simulated and measured bottom-hole pressure (BHP) data for the Production Well No. 2.

Fig. 22– Post-inversion match between simulated and measured bottom-hole pressure (BHP) data for the Production Well No. 3.

Fig. 23– Post-inversion match between simulated and measured bottom-hole pressure (BHP) data for the Production Well No. 4.

Fig. 24– Graphical description of the spatial distribution of log-permeability estimated from time-lapse water-oil ratio (WOR) measurements.

Fig. 25– Graphical description of the spatial distribution of porosity estimated from time-lapse water-oil ratio (WOR) measurements.

Fig. 26– Post-inversion match between simulated and measured water-oil ratio (WOR) data for the Production Well No. 1.
Fig. 27– Post-inversion match between simulated and measured water-oil ratio (WOR) data for the Production Well No. 2.

Fig. 28– Post-inversion match between simulated and measured water-oil ratio (WOR) data for the Production Well No. 3.

Fig. 29– Post-inversion match between simulated and measured water-oil ratio (WOR) data for the Production Well No. 4.

Fig. 30– Plot of the normalized objective function as a function of iteration number observed with the use of the subspace inversion algorithm. Curves are shown for inversions performed with three different types of time-lapse input data, namely, in-situ permanent sensor pressure, bottom-hole pressure (BHP), and water-oil ratio (WOR) measurements.