Assessing the Value of 3D Seismic Data in Reducing Uncertainty in Reservoir Production Forecasts
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Abstract
Using three-dimensional (3D) seismic data has become a common way to identify the size and shape of putative flow barriers in hydrocarbon reservoirs. It is less clear to what extent determining the spatial distribution of engineering properties (e.g., porosity, permeability, pressures, and fluid saturations) can improve predictions (i.e., improve accuracy and reduce uncertainty) of hydrocarbon recovery, given the multiple non-linear and often noisy transformations required to make a prediction. Determining the worth of seismic data in predicting dynamic fluid production is one of the goals of the study presented in this paper.

We have approached the problem of assessing uncertainty in production forecasts by constructing a synthetic reservoir model that exhibits much of the geometrical and petrophysical complexity encountered in clastic hydrocarbon reservoirs. This benchmark model was constructed using space-dependent, statistical relationships between petrophysical variables and seismic parameters. We numerically simulated a waterflood in the model to reproduce time-varying reservoir conditions. Subsequently, a rock physics/fluid substitution model that accounts for compaction and pressure was used to calculate elastic parameters. Pre-stack and post-stack 3D seismic data (i.e., time-domain amplitude variation of elastic responses) were simulated using local one-dimensional approximations. The seismic data were also contaminated with noise to replicate actual data acquisition and processing errors. We then attempted to estimate the original distribution of petrophysical properties and to forecast oil production based on limited and inaccurate spatial knowledge of the reservoir acquired from wells and 3D seismic data.

We compared the multiple realizations of the various predictions against predictions with a reference model. Adding seismic data to the static description affected performance variables in different ways. For example, the seismic data did not uniformly improve the variability of the predictions of water breakthrough time; other quantities, such as cumulative recovery at a later time, did exhibit an uncertainty reduction as did a global measure of recovery. We evaluate how different degrees of spatial correlation strength between seismic and petrophysical parameters may ultimately affect the associated uncertainty in production forecasts.

Most of the predictions exhibited a bias in that there is a significant deviation between the medians of the realizations and that the value from the reference case. This bias is evidently caused by noise in the various transforms (some of which we introduced deliberately) coupled with nonlinearity. The key nonlinearities seem to be in the numerical simulation itself, specifically in the transform from porosity to permeability, in the relative permeability relationships and in conservation equations themselves.

Introduction
Flow simulations are routinely used as the main input to the economical evaluation of hydrocarbon recovery. Predictions from these simulations have proven to be sensitive to the reservoir description, which is normally known through geology and petrophysics. Because the latter are based primarily on often sparsely-spaced wells, there is usually considerable uncertainty in the description and, hence, uncertainty in the prediction.

Relatively few reservoir characterization studies have made use of quantitative information contained in amplitude variations of 3D seismic data.1 Three dimensional seismic data sample the entire reservoir and thereby offer the possibility of filling the spatial gap between usually sparse well locations. We are encouraged by this possibility because in the past 3D seismic data have been successfully used to generate geometrical and structural maps, to assess the spatial distribution and size of flow units, and to volumetrically infer some petrophysical properties such as porosity and fluid saturations.2,3

However, there are limits to the use of 3D seismic data for quantitative reservoir description. For instance, (a) the lateral...
(horizontal) resolution, being largely determined by the distance between adjacent traces, is often no better than 20-50 m, (b) the vertical resolution remains controlled by the frequency content of the underlying seismic wavelet, and is often no better than 5-10 m, hence normally greater than what is needed to model the spatial detail of fluid-flow phenomena, and (c) the transformations between what the seismic data measures and the input to a fluid-flow model are complicated, noisy and non-linear. It is not automatic obvious, therefore, that the inclusion of seismic data will improve simulation predictions, even though they are spatially exhaustive. Determining the benefits and trade-offs of the quantitative use of 3D seismic data in model construction is the goal of this paper.

We consider different reservoir characterization techniques to determine the impact of the static reservoir description (i.e., porosity model) on a dynamic production forecast. Inference and forecast are accomplished using several alternative procedures, namely, (a) a homogeneous reservoir model, (b) a layered reservoir model, (c) 3D geostatistical techniques, and (d) a 3D geostatistical inversion technique that jointly honors 3D seismic data and well logs. The construction procedure implicitly considered (a) the uncertainty associated with statistical relations between petrophysical and elastic parameters, and (b) the effect of relative differences in geometrical support between the well logs and the seismic data. Comparisons of results are performed in model space (e.g., porosity) and data space (e.g., volume of oil production and seismic data). The conceptual geological representation of the model as well as the recovery process are the same for all cases so that the differences obtained in dynamic behavior can be traced back to the information available in constructing each of the models.

**Model Definition**

**Reservoir Model.** The synthetic earth model consists of a reservoir sand embedded in a background shale. Figure 1 shows the geometry and dimensions of the synthetic reservoir sand. The figure also shows the spacing and location of the wells and the distribution of water saturation within the reservoir sand after 4 years of production. Approximately 30 million cells were used to construct the grid used to simulate the synthetic seismic data associated with the earth model. However, only the reservoir sand is discretized for fluid-flow simulation with about half a million cells. The size of the blocks used to simulate seismic data and those used to simulate fluid-flow behavior are the same, hence mathematical upscaling was not necessary.

The initial model of porosity was constructed stochastically (Gaussian simulation) using probability density functions (PDF's) and semivariograms for each of the two lithologies (sand and shale). We assumed the porosity field to be second-order stationary, normally distributed, and having a spatial structure described by a prescribed semivariogram. This model is hereafter used as the truth reference case (referred to as case T). Appendix A presents a detailed summary of the conditions and relations used to simulate the

Waterflood. Relationships between porosity, permeability, and water saturation were enforced using well-documented paradigms. These were subsequently used to determine the initial conditions of the reservoir. Relative permeability curves representative of a water-wet medium were scaled using power-law functions that depend on residual saturation and endpoints. Figure 2 shows the set of capillary pressure and normalized relative permeability curves used in the fluid-flow simulations. These petrophysical relations are spatially invariant. A five-spot waterflood process (one injection well and four production wells) with an unfavorable mobility ratio (endpoint mobility ratio of 1.67) was simulated using a finite-difference algorithm. Seismic data are not strongly sensitive to the density contrast between oil and water; hence, a waterflood becomes a stringent test for the sensitivity analysis pursued in this paper. A second reason for picking a waterflood recovery process is so that our results can provide some insights into potential waterfloods in deepwater reservoirs where seismic is a main data source. The production wells were set to a constant bottomhole pressure and the injector well by a constant injection pressure. Fluid and rock properties and fluid-flow simulation conditions associated with case T are described in Table 1.

Permeability is not directly available from seismic information. We used the transformation \( \log k = 10 \phi - 0.5 \) to infer permeability (in md) from porosity (as a fraction). The nonlinear form of this equation is consistent with empirical observations that generally show a linear relationship between permeability plotted on a logarithmic scale and porosity. As our results will show, the nonlinearity of this relation contributes significantly to the accuracy in predictions. Permeability-porosity relations, however, are notoriously noisy, a factor that we are neglecting here. The interplay between the nonlinearity and the noise is known to lead to additional bias in predictions (reference 7, p.212). Addressing this complication is left to future work.

**Simulation of Seismic Data.** Elastic parameters, such as compressional velocity, shear velocity, and density, were calculated using a rock physics/fluid substitution model (Appendix B) that includes the effect of compaction. Rock physics/fluid substitution models relate the elastic properties to fluid and rock properties (e.g., density, porosity, and fluid saturation). Models that include compaction provide a realistic depth trend for the elastic parameters; hence making the synthetic seismic data consistent with actual burial conditions. We assumed locally one-dimensional distributions of acoustic impedance (AI), the product of seismic velocity and bulk density, to simulate post-stack seismic data across the reservoir model. This was accomplished by a convolution operator implemented with a zero-phase Ricker wavelet centered at 35 Hz. Figure 3 shows the Ricker wavelet used in this study and a cross-section of post-stack seismic data along well 1. In addition, pre-stack seismic data were simulated for three angle intervals: near (0-15°), mid (15-30°), and far (30-45°), respectively. The seismic
wavelets associated with these three angle stacks are a simple modification of the Ricker wavelet shown in Figure 3. Each angle interval is equivalent to what is normally referred to as an angle pseudo-stack in reflection seismology. The three angle pseudo-stacks were generated with a distinct synthetic wavelet for each angle-stack by making use of the Knott-Zoeppritz equations. These equations describe the amplitude of transmitted and reflected plane waves as a function of their angle of incidence at a boundary separating regions with unequal elastic properties. Subsequently, random noise (i.e., 10% additive zero-mean, uncorrelated Gaussian noise, where the noise percentage is in proportion to the global energy of the seismic data set) was added to the simulated seismic data in an effort to replicate actual noise in seismic measurements.

Numerical Experiments

In the model described above all the variables are completely known. However, in the numerical experiments, the reservoir properties are partially and imperfectly known. Figure 4 is a flow diagram that describes the method adopted for modeling and validating these reservoir characterization procedures.

The amount of data available for quantitative analysis increases as production proceeds. Most of these data are dynamic, in the form of production rates and pressures. Before production begins, the available data are mostly static (i.e., they do not stem from fluid-flow in the reservoir) and it is the value of this type of data that is the subject of this study. The kind of information we use is geologic interpretation, noisy seismic data, seismic interpretation (i.e., horizons), well logs, and the degrees of correlation between petrophysical and elastic properties. Well information (e.g., logs and core data) is the most important and direct way to obtain insight about the reservoir properties. This information can be biased because the well locations are not commonly representative of the entire population and because of their relatively short spatial support. Core data, especially, is subject to biased sampling. Aside from bias considerations, all of the well data substantially undersample the reservoir. It is said that the knowledge of the reservoir is better at the end of its life; but even then the knowledge is restricted to the inferences made from tests and production history, and to the spatial distribution of the hard data (i.e., wells).

Normally, major uncertainties in the geologic model are not fully considered in the modeling prior to the time production begins because there is a substantial amount of work involved in developing alternative models. The static models evaluated here include different degrees of information in their construction. They comprise simple models (e.g., homogeneous and layered), seismic inversion models, and stochastic models (e.g., geostatistical and geostatistical seismic inversion models). Table 2 summarizes the nomenclature of the estimation models considered in this paper. Since we are interested in evaluating the static models and their impact on a production forecast, all variables remain the same in the waterflood except for porosity and other petrophysical properties (e.g., permeability), which are assumed to be porosity-dependent. This allows one to perform a direct comparison between model construction, influence of seismic data, and production forecast.

Simple Models. We use two simple models, homogeneous and uniformly layered models that are mainly used when relatively few data are available, at which time it is necessary to use the average understanding of the field. In the homogeneous case (H), the porosity is spatially constant and equal to the mean value. Case H contains the mean statistical information but can not capture vertical and lateral spatial variability of the petrophysical properties. The uniformly layered model (L) makes use of the well-log data (i.e., porosity) to calculate average properties of each of the 51 simulation layers. Case L contains the vertical spatial variability but can not capture lateral spatial variability of the petrophysical properties.

Seismic Inversion Models. Seismic inversion is an estimation procedure whereby acoustic impedance (AI) is derived from post-stack seismic data. Related to the acoustic properties of the rocks, AI is often correlated with reservoir parameters. If there is a relationship between AI and petrophysical parameters then a direct transformation can be used to generate the reservoir parameters (see Figure 5). This is case DAI. Here, the AI recovered from the post-stack seismic inversion is transformed into porosity using the relationship shown in Figure 5 (top panel), which was calculated using well-log data. The more correlated the variables are, the more accurate the transformation of AI into the corresponding petrophysical property becomes. Although the Figure 5 shows correlation between AI and porosity, and AI and bulk density, there is some scatter around the main trend. However, there is not always a relationship between AI and petrophysical parameters. This is case AIW. Here, the AI recovered from the post-stack seismic inversion is transformed into porosity using the relationship shown in Figure 6 with a small correlation coefficient ($r^2 = 0.1$).

Stochastic Models. Stochastic modeling allows the generation of equally probable statistical realizations of the spatial distribution of reservoir properties. If these realizations are subject to fluid-flow simulations then the dynamic behavior of the reservoir can also be interpreted in terms of statistical properties. Normally, the range of possible solutions is an important part of the reservoir evaluation since in practical cases an analytical solution to the fluid-flow equations is not available. The stochastic approach is one of the techniques that allow one to integrate different kinds of information into the static description of the reservoir. In this section, the cases studied include: geostatistical models, and geostatistical seismic inversion of the post-stack and far-offset volumes for porosity and bulk density.

Bias and accuracy are important issues when evaluating results of stochastic realizations since the value of the inferences can be jeopardized by a potential bias in the results. Bias is a statistical sampling or testing error caused by systematically favoring some outcomes over others. Then, it becomes imperative to identify the source of bias in the
estimation procedures to properly evaluate the results. In our study, the main sources of bias originate from nonlinear equations, noisy relationships between variables, the nature of the production scheme, and the correctness of the physical model, to name but a few.

**Geostatistical Models.** Geostatistical modeling (case G) makes use of the information acquired along the five existing wells to build PDF’s of reservoir properties (i.e., porosity). Then, through the use of semivariograms, it is possible to build many realizations on the desired variable (i.e., porosity). Each realization has the same probability of occurrence and honors the well data that have been imposed in the process of Gaussian stochastic simulation of porosity.

The calculation of horizontal semivariograms (x- and y-direction) for each lithology is difficult because there are only a few number of points available (i.e., wells), which tends to produce pure nugget semivariograms. Figure 7 illustrates the semivariograms used in this study. We used zero-nugget spherical semivariograms to construct the porosity distribution in the truth reference case (case T). These semivariograms have two parameters as input: a range, which indicates the extent or size of the spatial autocorrelation, and a variance. The range is different for each of the three coordinate directions in case T. But because of the difficulty of estimating the range, in the statistical models we used horizontal ranges equal to one-half ($\ell / \ell_T = 0.5$) and twice ($\ell / \ell_T = 2$) those used in the reference case ($\ell_T$). For the vertical semivariograms, wells provide sufficient spatial sampling to calculate the corresponding parameters. The horizontal ranges used in the reference case were approximately equal to the well spacing. Variances for porosity and density were set to the values calculated from the sampled well-log data.

**Geostatistical Inversion Models.** Geostatistical inversion provides a framework to quantitatively integrate seismic data, well logs, and geological information in one step. In geostatistical inversion, a prior AI model is built and then modified until the global misfit between the measured seismic data and the simulated seismic data is reduced to a prescribed value (usually the global misfit is less than 5% depending on the amount of noise present in the seismic data). Because AI can often be related to petrophysical parameters, it is possible to directly obtain stochastic models of reservoir parameters that jointly honor the seismic and the well-log data.

In this study, a geostatistical inversion of the noisy post-stack seismic data from case T was performed for porosity (case IP) and bulk density (case ID). The PDF’s of those two variables for each lithology are shown in Figure 8. Semivariograms used in the inversions were the same as those described earlier (Figure 7). The relationships used in the geostatistical inversion between AI and porosity, and AI and density for each lithology were calculated from well-log data. These are shown in Figure 5. Given that we also want to make use of the partial offsets of the previously generated pre-stack seismic data, a geostatistical inversion was also performed of the far offset seismic data for porosity (case IPEI) and bulk density (case IDEI). Far offsets of seismic data are important because the AI of the encasing shale is larger than the AI of the reservoir sand. The properties obtained from this inversion (porosity and density) are subsequently used in the static description of the reservoir.

**Evaluation of Results and Discussion**

The two main assumptions underlying the reservoir construction methods described above are the second-order stationarity of the data and the existing relationship between AI and petrophysical parameters. Another important issue is the degree of representativeness of the data. It is known that, statistically speaking, well information is rarely representative of the spatially variability and volume under study. Often such a fact is overlooked but the information is nevertheless used because they are a primary and direct source of rock and fluid properties.

**Consistency in Data Space for Seismic Data.** To ascertain the consistency of the inferred hydrocarbon reservoir models, we performed an assessment of the error in predicting the 3D seismic data. This was accomplished by simulating the seismic data at the onset of production for each of the construction methods described above. Subsequently, we calculated a correlation coefficient between the seismic data of each case and the seismic data associated with the reference model (case T).

Figure 9 is a map of the correlation coefficient in data space (i.e., seismic data) for an arbitrary statistical realization of case G1. The average correlation coefficient ($r^2$) is 0.21. Table 3 summarizes the results obtained for the remaining cases considered in this paper. Cases H, L, and G exhibit the smallest correlation coefficients. By construction, cases that make use of seismic data in the definition of the reservoir properties must exhibit large correlation coefficients. For instance, Cases DAI and AIW exhibit the largest correlation since the AI is calculated through seismic inversion. Cases IP, ID, IPEI, and IDEI exhibit a large correlation coefficient. Obtaining a correlation map like the one shown in Figure 9 helps one to validate the predicted results against other sources of data.

**Consistency in Model Space for Porosity.** We performed an error assessment in model space (i.e., porosity). We compared the porosity model of the reference case to the porosity models of all cases. Figure 10 shows a map of correlation coefficients between the actual and predicted porosity for the hydrocarbon reservoir model inferred from an arbitrary realization of case G-1. Table 3 summarizes the results obtained for other cases considered in this paper and shows that cases H, L, and G exhibit the smallest correlation coefficient, whereas cases IP, ID, IPEI, and IDEI exhibit a larger correlation coefficient.

The average correlation coefficients between the seismic data are necessarily larger than that between the seismic-inferred porosity data. This is because the latter makes use of additional petrophysical relationships that tend to degrade the correlation. The correlation coefficients between the predicted and true seismic data are primarily measures of the errors introduced in the forward and inversion steps.
When determining global dynamic behavior (e.g., cumulative oil production) the agreement in model space is secondary. For instance, we can get a close prediction of oil recovery with a simple model. However, this agreement becomes important when detailed studies are necessary such as in the determination of an infill drilling location. Here, the cases with high correlation in model space consistently yield the closest fluid distribution to that of the reference model.

Many of the following results are shown in the form of Box plots. A Box plot enables one to examine a number of variables and to extract the more salient characteristics of their distributions. It also gives one insight to the global behavior of the corresponding variable. In a Box plot the y-axis displays the variation of the data and the x-axis displays the names of each case. Each vertical box encloses 50% of the data with the median value of the variable displayed as a horizontal line in the box. Bottom and top boundaries of the box define the 25 and 75 percentiles of the variable population. Lines extending from the top and bottom of each box define the minimum and maximum values that fall within a population range. Any value outside of this range, called an outlier, is displayed as an individual point.

In a Box plot, the reproducibility of a prediction is given by the size of the vertical boxes. Bias shows itself as the median value being significantly different from the truth value, or when the vertical box does not cover the truth case. In a sense, then, increasing the precision of a prediction can contribute the bias if the median value is not brought closer to the truth value. We note also that in nearly every practical case, the truth value is unknown.

Semivariograms and Property Relationships. Increasing the range in the property semivariograms amplifies the variability of the dynamic behavior response for cases that involve the use of semivariograms (cases G, IP, ID, IPEI, and IDEI). The increased variability is consistently observed in different dynamic variables. A larger range semivariogram produces slightly smaller correlation coefficients when comparing the results in data and model space (see Table 3).

If the construction of the static model is based on AI but there is no correlation between AI and petrophysical parameters (see Figure 6) then the initial static description of the reservoir is poor. Case AIW was designed to show that the lack of correlation between acoustic and petrophysical properties causes the seismic data not to contribute positively in the construction of a model of reservoir properties. Nevertheless, seismic data could still be useful for boundary identification. Noisy (scattered) relationship between AI and porosity deteriorates the correlation in model space (see Table 3) and leads to dynamic results that are biased, hence not representative of the reference case T. Figure 11 describes the original oil in place and cumulative oil recovery after 7 years of production for case AIW. Case AIW is evidently incorrect and therefore excluded from further analysis. Since the static model is not accurate, this case underpredicts the oil in place by 82.6% and oil recovery after 7 years of production by 84.6% compared to case T.

Oil in Place. Estimation of the original oil in place (OOIP) is an important appraisal tool in the early stages of the life of the reservoir. In our study, OOIP is not critical since all the models exhibit the same geometry (i.e., the same geometrical boundaries). The assumption of a known geometry is based on the fact that normally available seismic data can be used to construct a geometrical model of reservoir compartments. However, it is easily seen that each constructed model produces a different set of static distributions of properties (porosity and porosity-dependent variables) and therefore the OOIP is different in each case. For comparison, the OOIP of each case was normalized against that of case T.

The Box plots of Figures 12 and 13 show that the range of variation of normalized OOIP is small (within ±8% of case T) because it generally satisfies the same global statistics. Variations of OOIP entailed by the realizations for a particular case are also small because the realizations, while varying locally, exhibit identical average properties. OOIP, being itself a global quantity, is more sensitive to averages than to variability. More accurate predictions are obtained for those cases that involve the use of seismic data. Results within a case present more variability for the larger range semivariogram. The geostatistical inversion for density overpredicts the OOIP whereas the one for porosity underpredicts the OOIP. This can be related to the correlation strength between porosity and AI, and density and AI (see Figure 5).

Figures 12 and 13 embody a conceptual insight that will be a major conclusion of this work. For none of the geostatistical or seismic inversion cases (G, IP, IPEI, ID, and IDEI) do the 25-75 percentile vertical boxes overlap the prediction yielded by the reference case. It is difficult to make firm conclusions about this because of the paucity of realizations (10) on which the results were based. The bias has been exacerbated by the reduction in uncertainty caused by adding more data, which is most evident in Figure 12. In neither case, Figure 12 or 13, is the bias large; however, it will prove to be significant in the global dynamic responses described below. The source of the bias is the noise and the non-linearity of the various transforms needed to make the description.

The OOIP for the realizations in all the following cases was set to that of the reference case (case T) so that the dynamic reservoir predictions were performed assuming a reservoir with the same initial volumetrics.

Oil Recovery. Oil recovery represents a global dynamic response at a specific time in the life of the reservoir as shown in Figure 14 for an arbitrary realization of cases with $\varepsilon/\varepsilon_{T} = 0.5$. It depends mainly on the recovery mechanism, production strategy, and time. Figures 15 and 16 show the results of evaluating the normalized oil recovery after 2010 days of production. For none of the geostatistical or seismic inversion cases do the 25-75 percentile vertical boxes overlap the prediction yielded by the reference case. The recovery for cases H, L, and DAI is less than that of case T by 39%, 36%, and 25%, respectively. Median oil recovery for cases with $\varepsilon/\varepsilon_{T} = 2$ (bottom panel) is within ±19% of case T. For those
cases with $\varepsilon/\varepsilon_T = 0.5$ (top panel) the results are within ± 15% of case T. Even though the outcome of this global variable remains biased, the decrease in relative error in oil recovery comes as a direct consequence of adding new information in the construction of the property models. Table 4 shows the difference between the maximum and minimum values of the normalized oil recovery of the cases shown in Figures 15 and 16. Cases involving seismic data (IP, ID, IPEI, and IDEI) provide better precision than the realizations obtained only through the geostatistical case (case G). The latter statement is clear for cases with $\varepsilon/\varepsilon_T = 0.5$. For cases with $\varepsilon/\varepsilon_T = 2$ the differences are small as shown in Table 4. If oil recovery is evaluated at a given pore volume of water injected, there are small differences and results are not biased. However, the water injected is a result of the selected injection strategy, constant injection pressure in our case, and the initial model description.

**Time of Water Breakthrough.** Figure 17 shows the normalized time of water breakthrough for all cases considered in our study. A wider variability is observed than with the variables analyzed before (e.g., recovery). The range of variation is between 0.5 and 2 times the water breakthrough time for case T. For some of the cases the 25-75 percentile vertical boxes overlaps the prediction yielded by the reference case. Results shown in Figure 17 are less biased than those of Figures 15 and 16 because they do not show an average dynamic behavior response as in the case of oil recovery. Time of water breakthrough represents a dynamic response of the spatial distribution of the reservoir properties, especially the permeability distribution.

**Value of Information.** Figure 18 shows the oil recovery at the time of water breakthrough normalized with respect to case T for $\varepsilon/\varepsilon_T = 0.5$. Oil recovery represents a global dynamic behavior and, as discussed earlier, time of water breakthrough is closely related to the spatial distribution of properties. For all cases that involves seismic data the 25-75 percentile vertical boxes overlaps the value from the reference case. In Figure 18, one can quantitatively assess the benefits of including more information (i.e., seismic data) into the process of model construction.

Since the measure of accuracy of a prediction depends on the time at which the prediction is taken, we compared results with the L2 norm of the cumulative oil recoveries. The L2 norm is a global measure of recovery that does not depend on a specific time in the life of the waterflood. Figure 19 illustrates the results of performing such a calculation. Values were normalized against the homogeneous case (T). The horizontal axis identifies the particular case and can also be interpreted as a measure of the information content (scant information content to the left and higher information content to the right). It is clearly seen that the cumulative time uncertainty decreases as more information is included in the construction of the initial model.

**Linear Relations Experiment.** As emphasized earlier, we hypothesize that the main source of bias in our study originate from nonlinear flow equations, noisy relationships between elastic and petrophysical variables, the production scheme, and the correctness of the physical model. We decided to investigate the importance of some of these biases in the results of oil recovery. To accomplish this objective, a special case was designed in which all the relationships used in the fluid-flow simulator were made linear and precise (relative permeability, porosity-permeability) and the fluids exhibited the same viscosity. This case is not realistic but provides more insight about the source of the bias in the oil recovery.

Simulations were redone for the reference case (T) and case G2 (referred as G2L). Results were compared with those presented in Figure 16 and are shown in Figure 20. The oil recovery of this experiment is less biased and more accurate than previous results. Results suggest that the source of the bias in oil recovery is caused by the nonlinearity implicit in the underlying multi-phase fluid-flow equations.

**Summary and Conclusions**

The work presented in this paper was an attempt to assess the value of 3D seismic data in the construction of a hydrocarbon reservoir model. Several strategies were considered to appraise the influence of the usage of seismic data in the construction of a reservoir model. We concentrated on the relatively difficult case of a waterflood production system in which water was injected to displace oil as a way to enhance production efficiency. Seismic data are relatively insensitive to detecting spatial variations in oil and water saturations, especially in the presence of low-porosity rock formations (porosities below 15%). Thus, a waterflood experiment constitutes a worst-case study for the usage of seismic data in reservoir characterization studies (as opposed to, for instance, the optimal seismic detection problem of water and gas saturations in thick, high porosity formations). The main appraisal tool used in this paper to assess the value of seismic data was the comparison of the time record of fluid production measurements with respect to that of a benchmark (truth) model. As expected, it was impossible to isolate the influence of the usage of seismic data in reservoir construction from technical issues concerning non-uniqueness and the definition of ancillary fluid and petrophysical variables unrelated to seismic measurements. Such ancillary variables included the choice of a porosity-permeability relationship, the choice of global relative permeability and capillary pressure curves, and the choice of degree of spatial smoothness of reservoir variables interpolated from well-log measurements. Despite this difficulty, we attempted to compare on equal footing a set of models with different degrees of spatial complexity by standardizing the role played by both initial fluid volumetrics and the choice of a production scheme on fluid production forecasts. Subsequently, we integrated the quantitative use of various types of seismic data into the construction of static reservoir models with increasing degrees of spatial complexity. Even with the usage of seismic data, the construction of reservoir models is non-unique (an
An uncountable set of models exist that honor the complete set of available measurements. Multi-phase fluid-flow simulations associated with each set of models (10 individual models per set) were performed in order to quantify the predictive power of each set of measurements and these time-domain simulations were compared against those of the benchmark model. Finally, an effort was made to take into account that time variability of the record of production measurements as it directly impacted the measure of appraisal. Global as well as time dependent measures of appraisal were explored to quantify the added value of seismic data. The following conclusions stem from our work:

1. Significant biases in predictions of fluid recovery can be associated with pure fluid-flow phenomena to which seismic measurement remain insensitive. Even with the usage of seismic data, sources of prediction bias can be more dominant than an incremental reduction in prediction bias due to the usage of seismic data. Sources of prediction bias associated with fluid phenomena include the nonlinear nature of the underlying multi-phase fluid-flow equations, nonlinear and inaccurate constitutive relationships (e.g., porosity vs. permeability), noisy measurements, variations in the spatial support of input measurements, and the choice of fluid production scheme, among others.

2. Reservoir models are often constructed with geostatistical methods that make use of spatial semivariograms. It was found that a considerable degree of variability in static and dynamic predictions of reservoir behavior could be caused by the usage of larger than necessary semivariogram ranges. Regardless of the usage of seismic data, accurate estimation of semivariogram functions and parameters thereof is crucial to performing reliable forecasts of fluid production. For instance, the accuracy of predicted oil recovery is adversely affected by an improper choice of semivariogram range.

3. Lack of correlation between elastic and petrophysical parameters causes the seismic data not to contribute positively to reduce uncertainty in production forecasts. Fluid production forecasts associated with poor input petrophysical-elastic correlation functions are rendered biased and inaccurate.

4. Static and dynamic predictions performed from reservoir models constructed with the use of seismic data normally exhibit an incremental decrease in their bias with respect to a nominal prediction bias due to pure fluid-flow phenomena. Global measures of prediction bias show a consistent improvement with respect to predictions derived from models that do not make use of seismic data. This conclusion is valid as long as a high degree of correlation exists between petrophysical and elastic parameters, and follows from comparison of production variables such as recovery efficiency, and time of water breakthrough, for instance.

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Nomenclature

- $Al$ = Acoustic impedance, mL/L^3/t
- $c$ = Fluid compressibility, L^2/mL/t^2
- $E$ = Young’s modulus, mL/t^2/L^2
- $k$ = Permeability, L^2
- $k_r$ = Relative permeability
- $L$ = Length, L
- $m$ = mass, m
- $p$ = Pressure, mL/t^2/L^2
- $p_{BHP}$ = Bottom hole pressure, mL/t^2/L^2
- $PDF$ = Probability density function
- $S$ = Saturation, fraction
- $t$ = Time, t
- $v$ = Velocity, L/t
- $V$ = Volume, L^3

Greek Symbols

- $\Delta$ = Difference
- $\partial$ = Derivative operator
- $\nabla$ = Gradient operator, L
- $\int$ = Integral operator
- $\kappa$ = Bulk modulus, mL/t^2/L^2
- $\mu$ = Viscosity, mL/Lt or shear modulus, mL/t^2/L^2
- $\rho$ = Density, m/L^3
- $\phi$ = Porosity, fraction
- $\varepsilon$ = Semivariogram range, L
- $\sigma$ = Standard deviation
- $i$ = Poisson’s ratio, dimensionless

Subscripts

- $b$ = Bulk
- $e$ = Effective
- $f$ = Formation or fluid
- $i$ = Initial or component
- $o$ = Oil
- $p$ = Compressional
- $s$ = Unsaturated or shear
- $sh$ = Shale
- $ss$ = Sand
- $r$ = Residual
- $T$ = Reference case or Temperature
- $t$ = Total
- $x, y, z$ = Coordinate directions
- $w$ = Water

Superscripts

- $m, n$ = Saturation exponents
- $o$ = Endpoint

References
Appendix A. Fluid-Flow Model

Modeling fluid-flow in a permeable medium requires mass conservation equations, constitutive equations, and fluid and rock property relations. The mass conservation equation for component $i$ is given by

$$\frac{\partial (\rho_i S_i)}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = -q_i,$$  \hspace{1cm} (A-1)

where $i$ is the component (water or oil), $\rho_i$ is the fluid density, $\mathbf{v}_i$ is the superficial velocity of phase $i$, $\phi$ is porosity, and $q_i$ is a source or sink term. For the fluid-flow modeled here, there is mutual immiscibility between both of the fluid components (water and oil) meaning that phases and components are the same.

The constitutive equation is Darcy’s law for phase $i$ (oil and water), given by

$$\mathbf{v}_i = -k_i \frac{\nabla p_i - \gamma_i \mathbf{v}_i}{\mu_i},$$  \hspace{1cm} (A-2)

where $k_i$ is the absolute permeability tensor of the permeable medium and is assumed to be diagonal, $k_i$ is the relative permeability function, $\mu_i$ is viscosity, and $\gamma$ is the specific weight of the fluid.

A fluid property relationship is given by the compressibility equation. We assume that fluid ($c_i$) and pore ($c_f$) compressibilities, given by

$$c_i = \frac{1}{\rho_i} \frac{\partial \rho_i}{\partial p},$$

and

$$c_f = \frac{1}{\phi} \frac{\partial \phi}{\partial p},$$  \hspace{1cm} (A-3)

respectively, are constant over the pressure range of interest.

Capillary pressure ($p_c$) and the fluid saturation constraint are governed by

$$p_c(S_w) = p_o - p_w,$$  \hspace{1cm} (A-4)

and

$$S_o + S_w = 1,$$  \hspace{1cm} (A-5)

respectively. Relative permeabilities are necessary to evaluate the fluid-flow performance of multi-phase systems. We adopted a deterministic power law to govern the dependency of relative permeability on water saturation. This power-law relationship is constructed in the following manner. First define a reduced water saturation as

$$S_w^* = \frac{S_w - S_{wi}}{1 - S_{or} - S_{wi}}.$$  \hspace{1cm} (A-6)

The relative permeability functions are then given by

$$k_{rw}^*(S_w^*) = k_{rw}^o S_w^n,$$  \hspace{1cm} (A-7)

and
\[
k_{ro} \left( S_w^+ \right)^m = k_{ro}^o \left( 1 - S_w^+ \right)^m, \tag{A-8}
\]
where, \( k_{ro}^o \) and \( k_{ro}^o \) are the end-point values of the water-oil relative permeabilities, and \( n \) and \( m \) are the water and oil saturation exponents, respectively. Values of fluid and rock parameters and simulation conditions considered in this paper are shown in Table 1.

**Appendix B. Rock Physics/Fluid Substitution Model and Elastic Relations**

There has been a great deal of work published concerning the relationships that link elastic properties of porous rocks to pore fluid properties, pressure, and composition. Most of the relationships are based on empirical correlations that only apply to a particular basin of the world. Others are based on wave theory, but are subject to specific and often restrictive operating assumptions.

In the present study, we adopted Duffy and Mindlin’s rock physics/fluid substitution model to generate the main elastic parameters, namely, the compressional \( (v_p) \) and shear \( (v_s) \) velocities. This model reproduces a wide variety of velocities measured on rock samples. The main results of the Duffy-Mindlin model are given by

\[
v_p^2 = \frac{C_{11} + \frac{\left( 1 - C_{11} + 2C_{12} \right)^2}{3\kappa_f}}{\rho_b}, \tag{B-1}
\]

and

\[
v_s^2 = \frac{C_{11} + C_{12}}{2\rho_b}, \tag{B-2}
\]

where the subscripted \( C \) variables are given by

\[
C_{11} = \frac{4 - 3\nu}{2 - \nu} \left\{ \frac{3E^2p_e}{8(1-\nu)^2} \right\}^{\frac{1}{3}}, \tag{B-3}
\]

and

\[
C_{12} = \frac{\nu}{2(2-\nu)} \left\{ \frac{3E^2p_e}{8(1-\nu)^2} \right\}^{\frac{1}{3}}. \tag{B-4}
\]

Equations B-5 to B-9 below summarize the basic definitions of the mechanical parameters used in the Duffy-Mindlin model. Poisson’s ratio, \( \nu \), can be written as

\[
\nu = \frac{3\kappa_f - 2\mu_f}{2(3\kappa_f + \mu_f)}, \tag{B-5}
\]

where, \( \kappa_f \) is the bulk modulus, and \( \mu_f \) is the shear (rigidity) modulus. The Young’s modulus, \( E \), is given by

\[
E = \frac{9\kappa_f \mu_f}{3\kappa_f + \mu_f}, \tag{B-6}
\]

and

\[
P_e = P_{overburden} - P_{pore}. \tag{B-7}
\]

where \( p \) is pressure, and the subscript ‘e’ stands for effective.

The bulk density \( (\rho_b) \) is a simple average weighted by the volume fraction of each component, i.e.,

\[
\rho_b = \left( \frac{1 - \phi}{\rho_w} + \frac{\phi}{\rho_f} \right) \rho_b + \left( \frac{1 - \phi}{\rho_w} + \frac{\phi}{\rho_f} \right) \rho_b, \tag{B-8}
\]

If \( \Delta p_{ore} \) is the change in pore pressure, then the change in water volume is given by

\[
-VS_w \Delta p_{pore}/\kappa_w, \tag{B-9}
\]

where \( \kappa_w \) is water bulk modulus (inverse of water compressibility), and the change in oil volume is given by

\[
-VS_o \Delta p_{pore}/\kappa_o. \tag{B-10}
\]

The total change in volume is the sum of the partial volume changes and is equal to \(-V\Delta p_{pore}/\kappa_f\). Consequently, the fluid bulk modulus \( (\kappa_f) \) is the harmonic average of each of the elemental component values weighted by their respective volume fraction, i.e.,

\[
\frac{1}{\kappa_f} = \frac{S_w}{\kappa_w} + \frac{S_o}{\kappa_o}. \tag{B-11}
\]

Determining elastic parameters of rocks from their petrophysical properties requires knowledge of the rock’s dry bulk modulus \( (\kappa_f) \). This is provided by the empirical equation proposed by Geertsman and Smith that relates the bulk modulus \( (\kappa_b) \), the rock’s dry modulus \( (\kappa_f) \), and the rock’s porosity \( (\phi) \), i.e.,

\[
\frac{\kappa_b}{\kappa_f} = \frac{1}{(1+50\phi)}. \tag{B-12}
\]

The main assumption made when estimating elastic parameters of rocks from their petrophysical properties is that the motion of interstitial fluid is independent from the motion of the matrix grains (low frequency approximation). This assumption causes the shear modulus of the fluid-saturated rock \( (\mu_b) \) to be the same as that of the unsaturated rock \( (\mu_f) \), i.e.,

\[
\mu_b = \mu_f. \tag{B-13}
\]

By making use of the flow diagram described in Figure 4, and by using equations (B-1) through (B-11), the elastic parameters \( (v_p, v_s, \rho_b) \) can be calculated for specific values of the rock’s petrophysical properties. These parameters constitute the input to the algorithm used to generate synthetic seismic data.
Table 1. Summary of fluid and petrophysical properties.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values and units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_w)</td>
<td>1000 kg/m(^3)</td>
</tr>
<tr>
<td>(\rho_o)</td>
<td>850 kg/m(^3)</td>
</tr>
<tr>
<td>(\mu_w)</td>
<td>1.0 cp</td>
</tr>
<tr>
<td>(\mu_o)</td>
<td>5.0 cp</td>
</tr>
<tr>
<td>(c_w)</td>
<td>3.1x10(^6) psi(^{-1})</td>
</tr>
<tr>
<td>(c_o)</td>
<td>2.0x10(^6) psi(^{-1})</td>
</tr>
<tr>
<td>Fluid</td>
<td></td>
</tr>
<tr>
<td>Average (S_w)</td>
<td>0.28</td>
</tr>
<tr>
<td>Average (S_o)</td>
<td>0.25</td>
</tr>
<tr>
<td>(\phi(\phi, \sigma))</td>
<td>N(0.21, 0.07)</td>
</tr>
<tr>
<td>(c_i)</td>
<td>1.7x10(^6) psi(^{-1})</td>
</tr>
<tr>
<td>(K^{w})</td>
<td>0.3</td>
</tr>
<tr>
<td>(K^{r})</td>
<td>0.9</td>
</tr>
<tr>
<td>(K/K_{r\phi})</td>
<td>0.1</td>
</tr>
<tr>
<td>(k_x/k_y)</td>
<td>0.7</td>
</tr>
<tr>
<td>Reservoir</td>
<td></td>
</tr>
<tr>
<td>Depth to top of sand</td>
<td>1219.2 m</td>
</tr>
<tr>
<td>Simulation</td>
<td></td>
</tr>
<tr>
<td>(p_{injection})</td>
<td>2500 psi</td>
</tr>
<tr>
<td>(p_{BHP})</td>
<td>300 psi</td>
</tr>
<tr>
<td>Number of cells</td>
<td>81x81x51</td>
</tr>
<tr>
<td>Cell size</td>
<td>~22x22x6 m</td>
</tr>
<tr>
<td>Perforations</td>
<td>All interval</td>
</tr>
</tbody>
</table>

Table 2. Summary of nomenclature for the numerical experiments.

<table>
<thead>
<tr>
<th>Case</th>
<th>Key</th>
<th>(\delta/\delta T = 0.5)</th>
<th>(\delta/\delta T = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference (True) model</td>
<td>T</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Homogeneous</td>
<td>H</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Layered</td>
<td>L</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Direct from Al</td>
<td>DAI</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Direct from Al (transform with poor correlation)</td>
<td>AIW</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Geostatistics*</td>
<td>- G-1</td>
<td>- G-2</td>
<td></td>
</tr>
<tr>
<td>GSI for porosity (post-stack)*</td>
<td>-</td>
<td>IP-1</td>
<td>IP-2</td>
</tr>
<tr>
<td>GSI for porosity (far offset)*</td>
<td>-</td>
<td>IPEI-1</td>
<td>IPEI-2</td>
</tr>
<tr>
<td>GSI for density (post-stack)*</td>
<td>-</td>
<td>ID-1</td>
<td>ID-2</td>
</tr>
<tr>
<td>GSI for density (far offset)*</td>
<td>-</td>
<td>IDEI-1</td>
<td>IDEI-2</td>
</tr>
</tbody>
</table>

GSI = Geostatistical Seismic Inversion
*10 realizations for each semivariogram

Table 3. Average correlation coefficients \(r^2\) in model space (porosity) and data space (seismic data) between an arbitrary-selected model realization and the reference model.

<table>
<thead>
<tr>
<th>Case</th>
<th>(r^2) model space (porosity)</th>
<th>(r^2) data space (seismic data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.16</td>
<td>0.18</td>
</tr>
<tr>
<td>L</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>DAI</td>
<td>0.44</td>
<td>0.98</td>
</tr>
<tr>
<td>AIW</td>
<td>0.09</td>
<td>0.98</td>
</tr>
<tr>
<td>G-1</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td>IP-1</td>
<td>0.54</td>
<td>0.87</td>
</tr>
<tr>
<td>IPEI-1</td>
<td>0.52</td>
<td>0.93</td>
</tr>
<tr>
<td>ID-1</td>
<td>0.53</td>
<td>0.87</td>
</tr>
<tr>
<td>IDEI-1</td>
<td>0.51</td>
<td>0.93</td>
</tr>
<tr>
<td>G-2</td>
<td>0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>IP-2</td>
<td>0.53</td>
<td>0.87</td>
</tr>
<tr>
<td>IPEI-2</td>
<td>0.54</td>
<td>0.91</td>
</tr>
<tr>
<td>ID-2</td>
<td>0.51</td>
<td>0.88</td>
</tr>
<tr>
<td>IDEI-2</td>
<td>0.52</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Table 4. Range of variation of normalized oil recovery at 2010 days of production.

<table>
<thead>
<tr>
<th>Case</th>
<th>Difference* in Normalized Oil Recovery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\delta/\delta T = 0.5)</td>
</tr>
<tr>
<td>G</td>
<td>0.072</td>
</tr>
<tr>
<td>IP</td>
<td>0.042</td>
</tr>
<tr>
<td>IPEI</td>
<td>0.048</td>
</tr>
<tr>
<td>ID</td>
<td>0.024</td>
</tr>
<tr>
<td>IDEI</td>
<td>0.036</td>
</tr>
</tbody>
</table>

\*Difference = (R_{maximum} – R_{minimum})

Figure 1. Three-dimensional view of the distribution of water saturation in the reservoir sand after 4 years of waterflood. Sand dimensions, well spacing, and well locations are as indicated on the figure.
Figure 2. Normalized set of relative permeability and capillary pressure curves used to model the waterflood.

Figure 3. Ricker wavelet used in the simulation of post-stack 3D seismic data (left panel) and cross-section of post-stack seismic data along well 1 (right panel).

Figure 4. Integrated flow diagram describing the method used in this study for validating static descriptions and dynamic predictions.

Figure 5. Relationship between acoustic impedance and porosity (top panel), and acoustic impedance and bulk density (bottom panel) constructed from well-log data sampled from the reference case T.

Figure 6. Relationship between acoustic impedance and porosity for case AIW. The correlation coefficient ($r^2$) is 0.1.
Figure 7. Semivariograms within the reservoir sand in the x, y, and z directions used for the stochastic simulations of porosity and density. The variable $\theta$ is the range of the spherical semivariogram used in the construction of the reference model, case T.

Figure 8. Histograms of porosity (top panel), bulk density (mid panel), and acoustic impedance (bottom panel) sampled from well-log data within the reservoir sand and the embedding shale for case T.

Figure 9. Map of correlation coefficient ($r^2$) between vertical columns of seismic data from the geostatistical case G-1 and the reference case T. A coefficient $r^2 = 1$ (dark shading) at a particular pixel indicates perfect correlation. The average $r^2$ for all pixels is 0.21. Table 3 gives average correlation coefficients for additional cases.

Figure 10. Map of correlation coefficient ($r^2$) between vertical columns of porosity from the geostatistical case G-1 and the reference case T. A coefficient $r^2 = 1$ (dark shading) at a particular pixel indicates perfect correlation. The average $r^2$ for all pixels is 0.19. Table 3 gives average correlation coefficients for additional cases.

Figure 11. Plot of the predicted original oil in place and oil recovery after 7 years of production when there is poor correlation between acoustic impedance and porosity (case AIW). See Table 2 for a definition of the cases.

Figure 12. Box plot of normalized original oil in place for cases with $\theta/\theta_T = 0.5$. See Table 2 for a definition of the cases.
Figure 13. Box plot of normalized original oil in place for cases with $\delta/\delta_T = 2$. See Table 2 for a definition of the cases.

Figure 14. Cumulative oil recovery as a function of time for an arbitrary-selected realization of cases with $\delta/\delta_T = 2$. See Table 2 for a definition of the cases.

Figure 15. Box plot of normalized oil recovery after 2010 days of production for cases with $\delta/\delta_T = 0.5$. All models were initialized with the same original oil in place. See Table 2 for a definition of the cases.

Figure 16. Box plot of normalized oil recovery after 2010 days of production for cases with $\delta/\delta_T = 2$. All models were initialized with the same original oil in place. See Table 2 for a definition of the cases.

Figure 17. Box plot of normalized time of water breakthrough. Top panel: $\delta/\delta_T = 0.5$. Bottom panel: $\delta/\delta_T = 2$. All models were initialized with the same original oil in place. See Table 2 for a definition of the cases.
Oil Recovery at BT (R@BT = 1 for case T)

Figure 18. Box plot of normalized oil recovery at time of water breakthrough for cases with $\varepsilon / \varepsilon_T = 0.5$. See Table 2 for a definition of the cases.

Oil Recovery (R = 1 for case T)

Figure 20. Box plot of normalized oil recovery after 2010 days of production for cases with $\varepsilon / \varepsilon_T = 2$. All relationships involved in the fluid-flow simulation for case G-2L are linear. See Table 2 for a definition of the cases.

Global Uncertainty for Oil Recovery

Figure 19. Box plot of global least-squares misfit ($U$) for cases with $\varepsilon / \varepsilon_T = 0.5$. $U(t) = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} (d(t)_\text{caseX} - d(t)_\text{caseT})^2 dt$, where $d(t)$ is cumulative oil recovery and $t_i$ is total time of simulation. See Table 2 for a definition of the cases.