Numerical Simulation and Inversion of Pressure Data Acquired With Permanent Sensors

Faruk O. Alpak, SPE, Carlos Torres-Verdín, SPE, and Kamy Sepehrnoori, SPE, The University of Texas at Austin

Abstract

We have developed a new axisymmetric solution for the numerical simulation of single-phase fluid flow in permeable media. In this new solution, the governing parabolic partial differential equation (PDE) is transformed into a linear operator problem that is subsequently solved with an Extended Krylov Subspace Method (EKSM). The algorithm yields solutions for pressure at a multitude of times with the same efficiency of that of a single-time solution. Extensive comparisons with analytical solutions show that our simulator yields pressure results with accuracies better than 1%, and favorably competes with commercial simulators in CPU execution times and memory efficiency. Examples of numerical simulation are considered for water injection and variable flow-rate problems. In both cases, we have focused our attention to the description of pressure data acquired with permanent sensors deployed in direct hydraulic contact with the producing formation.

The inverse problem considered in this paper consists of mapping time and space variations of pressure and flow rate into a local distribution of permeabilities. We solve this inverse problem by minimizing a cost function written as the sum of the square differences between the measured pressure data and the corresponding pressure data yielded by our Krylov subspace simulator. A nonlinear Gauss-Newton fixed-point iteration search is used to minimize the quadratic cost function. We also test a dual coarse- and fine-grid approach to accelerate the solution of the inverse problem. Examples of inversion are shown with noise-free and noisy data aimed at understanding the role played by the flow-rate function and the location, spacing, and number of permanent sensors into the accuracy and stability of the inverted permeability values.

Introduction

The availability of permanently installed downhole pressure and temperature sensors has opened a new window of opportunities to probe hydrocarbon reservoirs. A continuous stream of time-domain data is now available to perform real-time monitoring of the variation of fluid-flow parameters resulting from production. In turn, time-domain monitoring provides the basic hardware for a much-needed active feedback loop to control hydrocarbon production in real time. Very recently, prototypes of permanent sensors have been constructed to be deployed behind casing and hence to be in direct contact with the producing formations. When commercially available, these in-situ sensors will allow the possibility of delivering real-time images of the spatial distribution of fluid-flow parameters in the vicinity of a well and also between existing wells. Interpretation work is therefore in order to quantify how a variation in the measurements acquired by in-situ permanent sensors will translate into a variation in the spatial distribution of fluid-flow properties. It is also imperative to optimally design the spacing and number of permanent sensors in light of existing reservoir conditions and potentially deleterious noisy measurements. The work described in this paper is an attempt to quantify the spatial resolution properties of in-situ permanent pressure measurements. To do so, we consider a hypothetical water injection experiment and make use of the extensive body of estimation techniques available from the field of geophysical inverse theory. Our goal is to construct an efficient algorithm to quantify the sensitivity of permanent pressure data to lateral and vertical variations in the distribution of permeability around the injection well.

In the past, several nonlinear optimization procedures have been applied to solve inverse problems in petroleum engineering and ground water hydrology studies. Jacquard and Jain\(^1\) presented the first numerical algorithm for the estimation of two-dimensional (2D) permeability variations from pressure measurements. Later, Carter \textit{et al.}\(^2\) developed an
efficient method to calculate sensitivity coefficients for 2D single-phase flow. He et al. extended Carter’s method to 3D geometries. Likewise, Chen et al. and Chavent et al. made use of a least-squares formulation based on adjoint operators to compute the gradient of the objective function with respect to the model parameters. In the ground water literature, Carrera and Neuman used an optimal control method to estimate fluid-flow parameters of aquifers. Quite recently, Wu developed an efficient Newton-Raphson nonlinear inversion technique for mapping multiphase production data into distributions of permeability and fluid saturations.

The inversion algorithm employed in this paper to evaluate permanent pressure data is based on an efficient least squares minimization technique adapted from the work of Torres-Verdín et al. We also implement a novel dual finite-difference gridding approach to accelerate the inversion associated with a large number of unknown parameters. A Spectral Lanczos Decomposition Method (SLDM) is used to simulate numerically the single-phase pressure data acquired by the permanent sensors. An important feature of the SLDM algorithm is that pressures can be simulated for a multitude of times with almost the same computer efficiency as that of one single time simulation (Knizhnerman and Druskin and Knizhnerman). Our paper concludes with several examples of parametric inversion drawn from the interpretation of pressure, pressure derivative, and flow-rate formations. We solve Eq. (5) to simulate numerically the time-domain measurements acquired with the in-situ pressure gauges for specific flow-rate (time) schedules of water injection. A pressure solution is approached using a finite-difference formulation in cylindrical coordinates via the SLDM. Details of this method of solution are described in the Appendix. Extensive comparisons with simulations performed using analytical models, as well as ECLIPSE 100™ consistently showed that the SLDM formulation yielded pressure results with accuracies better than 1%. Also, in our comparison tests the SLDM algorithm favorably competed with several commercial simulators in CPU execution times and memory efficiency (Alpak).

Forward Modeling: Numerical Simulation of the Single-Phase Diffusion Problem

Let us consider a Newtonian fluid in a rigid porous medium occupying a bounded domain $\Omega \subset R^3$ with a smooth boundary $\partial \Omega$. The conservation of mass over a representative control volume leads to the continuity equation:

$$- \nabla \cdot [ \rho(\mathbf{r},t) \mathbf{v}(\mathbf{r},t)] = \phi(\mathbf{r}) \frac{\partial p(\mathbf{r},t)}{\partial t},$$

(1)

where $\rho$ is the mass density, $\mathbf{v}$ is the velocity, $\phi$ is the time-invariant porosity distribution, $\mathbf{r}$ is a point in $R^3$ and $t$ is time. According to Darcy’s law, the fluid velocity can be written as

$$\mathbf{v}(\mathbf{r},t) = \frac{\mathbf{K}}{\mu} \cdot \nabla p(\mathbf{r},t),$$

(2)

where $p$ is pressure, $\mu$ is fluid viscosity, and $\mathbf{K}$ is the second-order permeability tensor, given by

$$\mathbf{K} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}.$$

(3)

We further assume the existence of a principal coordinate system in which the permeability tensor takes on the simple diagonal form

$$\mathbf{\tilde{K}} = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}.$$

(4)

Finally, for a slightly compressible fluid with constant compressibility ($C$) and viscosity ($\mu$), the pressure diffusion equation (1) can be written as (Muskat)

$$\nabla (\mathbf{\tilde{T}}(\mathbf{r}) \cdot \nabla p(\mathbf{r},t)) = \phi(\mathbf{r}) C \frac{\partial p(\mathbf{r},t)}{\partial t},$$

(5)

where $\mathbf{\tilde{T}} = \mathbf{\tilde{K}} / \mu$ is the mobility tensor.

The specific geometrical model considered in this paper is illustrated in Fig. 1. We focus our attention to a hypothetical test case in which water is injected from a vertical well into the surrounding oil-saturated rock formations. Moreover, for simplicity but without sacrifice of generality, we assume that the permeability field exhibits azimuthal symmetry around the well. Permanent in-situ pressure gauges are then positioned along the well’s water injection interval, to remain in direct hydraulic contact with the surrounding rock formations. We solve Eq. (5) to simulate numerically the time-domain measurements acquired with the in-situ pressure gauges for specific flow-rate (time) schedules of water injection. A pressure solution is approached using a finite-difference formulation in cylindrical coordinates via the SLDM. Details of this method of solution are described in the Appendix. Extensive comparisons with simulations performed using analytical models, as well as ECLIPSE 100™ consistently showed that the SLDM formulation yielded pressure results with accuracies better than 1%. Also, in our comparison tests the SLDM algorithm favorably competed with several commercial simulators in CPU execution times and memory efficiency (Alpak).

Inverse Modeling: Nonlinear Inversion as Minimization

In the context of data acquired with in-situ permanent sensors, the inverse problem consists of estimating an axisymmetric distribution of permeability from a finite collection of discrete time-domain measurements of pressure. We further assume that the unknown permeability distribution (or permeability model) can be parameterized with a finite number of values. Let $\mathbf{m}$ be the size-$N$ vector of unknown parameters that fully describe the axisymmetric permeability distribution, and $\mathbf{m}_0$ a reference vector of the same size as $\mathbf{m}$ that has been determined from some a-priori information. We undertake the estimation (inversion) of $\mathbf{m}$ from the measured in-situ permanent pressure data by minimizing a quadratic cost function, $C(\mathbf{m})$, defined as (Torres-Verdín and Habashy)
\[ 2C(m) = \left\| W_d \left[ d(m) - d^{\text{obs}} \right] \right\|^2 + \lambda \left\| W_m \cdot (m - m_R) \right\|^2, \]

(6)

where \( d^{\text{obs}} \) is a size-\( N \) vector that contains the measured data in an organized fashion, \( W_d \) is the inverse of the covariance matrix, \( \chi^2 \) is the prescribed value of data misfit, \( d(m) \) is the data vector numerically simulated for specific values of \( m \), \( W_m \) is the inverse of the model covariance matrix, and \( \lambda \) is a Lagrange multiplier or regularization parameter.

To determine a stationary point, \( m \), where the cost function attains a minimum, we make use of a Gauss-Newton fixed-point iteration search. This method considers only first-order variations of the cost function in the vicinity of a local iteration point. The corresponding iterated formula can be written as

\[ m^{k+1} = \left\{ T \left( m^k \right) \cdot W_d^T \cdot W_d \cdot J(m^k) + \lambda \left\| W_m \cdot (m - m_R) \right\|^2 \right\} ^{-1} \cdot \left\{ T \left( m^k \right) \cdot W_d^T \cdot W_d \cdot \left[ (d(m^k) - d^{\text{obs}}) + J(m^k) \cdot m^k \right] \right\} , \]

(7)

subject to

\[ l_i \leq m^{k+1}_i \leq u_i, \]

(8)

In the above expression, the superscript \( k \) is used as an iteration count, the superscript \( T \) denotes transpose, and \( J(m) \) is the Jacobian matrix of \( C(m) \), given by

\[ J(m) = \begin{bmatrix}
\partial d_1 / \partial m_1 & \ldots & \partial d_1 / \partial m_i & \ldots & \partial d_1 / \partial m_N \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\partial d_M / \partial m_1 & \ldots & \partial d_M / \partial m_i & \ldots & \partial d_M / \partial m_N
\end{bmatrix}_{M \times N}. \]

(9)

The upper and lower bounds enforced on \( m^{k+1} \) are intended to have the iterated solution yield only physically consistent results.

When the linear system of equations embodied by Eq. (7) is solved for subsequent values of \( m \) in the search of a minimum of the quadratic cost function, the evaluation of the Jacobian matrix is the most computationally demanding operation. The fixed-point iteration search for a minimum of \( C(m) \) is concluded when the measured data have been fit within the prescribed tolerance, \( \chi^2 \).

**Single-Phase Flow Test Examples**

We have applied the foregoing nonlinear inversion procedure to the estimation of single-phase axisymmetric permeabilities. Several test cases are described below.

**Three-layer, six-block test case.** In this first test case, we constructed a relatively simple reservoir model consisting of three layers and six blocks. The input permeability field is described in Fig. 2. That figure shows a vertical cross-section (radial distance vs. vertical location) of the axisymmetric permeability field around an injection well. Geometrical and fluid-flow parameters associated with this test case are listed in Table 1. Input data, assumed to be the permanent-sensor measurements, were computed using our single-phase flow simulator (SPRS). We considered two modalities of data input to the inversion algorithm: (a) pressure with respect to time \( (\Delta p vs. t) \), and (b) pressure derivative with respect to time \( (d\Delta p/dt vs t) \). In addition, our inversion study assumed two types of flow-rate time schedules (injection/production), namely, (a) a step-function pulse (conventional constant injection rate fall-off test) and (b) a sinusoidal pulse. The associated equations for injection/production flow-rate time schedules are as follows:

- **Step-function pulse:** \( q(t) = U(t - t_{\text{shut}}) \cdot q_{\text{inj}}, \)
- **Sinusoidal pulse:** \( q(t) = q_{\text{max}} \cdot \sin \left( \frac{\pi t}{\tau} \right) \)

where \( q_{\text{max}} \) is the maximum attainable injection/production flow rate, and \( \tau \) is the period of the sinusoidal pulse.

A finite-difference grid of size \( 105 \times 281 \) (radial and vertical nodes, respectively) was constructed to perform the numerical simulations and inversions of pressure following an extensive sensitivity study. This grid, shown in Fig. 3, consists of logarithmic steps in the radial direction and linear steps in the vertical direction. Figures 4 and 5 are plots of the simulated pressure data, \( \Delta p \) vs. \( t \), together with the associated flow-rate schedule, \( q \) vs. \( t \), for step-function and sinusoidal flow-rate pulses, respectively. Notice that for the case of a step-function pulse, pressure measurements are acquired during the shut-in time interval, whereas for the case of the sinusoidal pulse, pressure measurements are acquired at the same time the flow-rate pulse is taking place. Figure 4 shows a slight time delay between the pressure response and the injection/production flow-rate pulse.

**Validation of the numerical simulation algorithm.** Prior to performing the inversions, an extensive validation exercise was carried out to quantify the accuracy and efficiency of our single-phase fluid-flow simulator (SPRS). Benchmark comparisons were carried out against the widely commercially used ECLIPSE 100™ simulator. Figures 6 and 7 show a comparison of numerical simulations of pressure computed for the step-function flow rate pulse (45 hours of water injection and 120 hours of observation while in shut-in mode) with both ECLIPSE 100™ and the SPRS. The input permeability model...
corresponded to the same three-layer, six-block reservoir shown in Fig. 2. Both SPRS and ECLIPSE 100™ numerical simulations agree within 1% of each other. **Noise-free three-layer, six-block test case.** We simulated pressure data in the form of pressure changes, \( \Delta p \), with respect to time, \( t \), using a 135 hour-long step-function pulse, and amounting to a total of 500BBL/D of water injected along the borehole wall. A sensor deployment was assumed consisting of 11 equally spaced pressure sensors along the 10m zone of interest. Both lateral boundaries as well as permeability values for each of the six blocks comprised the set of unknown parameters \((a total of 9 unknown parameters). Input data were also simulated for a hypothetical array of 5 pressure sensors. The inversion algorithm was initialized with a uniform permeability field of 5md and with radial block boundaries located at a uniform distance of 10m away from the borehole wall. A plot of the actual permeability field is shown in Fig. 2. This figure describes the permeability values assigned to each of the blocks as well as the radial location of the corresponding block boundaries (within brackets). **Figures 8 through 11** show the results of the inversion assuming noise-free input pressure data. Two sets of results are shown in these figures. One of these sets is associated with a deployment of 11 sensors and the remaining set is associated with a deployment of 5 sensors. Comparisons are also shown between the input pressure data and the pressure data simulated with the set of inverted permeability and boundary location values. In both cases, sensor locations are shown along the vertical axis of the model plots. In the two examples of inversions, the estimated parameters (permeabilities and block boundary locations) are within 0.7% of the actual input parameters. On the other hand, the \( \Delta p \) vs. \( t \) data produced by the inversion remains within 0.55% and 0.7% of the input data for the case of 5 and 11 sensors, respectively.

We repeated the inversion exercise described above assuming a sinusoidal flow-rate pulse such as the one described by Eq. (12), and constructed with a period, \( \tau \), of 60hr, and a maximum injection rate of 500 BBL/D \((q_{\text{max}})\). The time interval for the pressure measurements was chosen to be the same as in the previous case, i.e. 120 hours. **Figures 12 through 17** summarize the results obtained in conjunction with this second inversion exercise. Results are shown in those figures of the inverted unknown model parameters together with the corresponding fit to the input pressure data. We also considered a slight variation to the same problem in which the input data consisted of time derivatives of pressure, \( dp/dt \) vs. \( t \), instead of pressure changes, \( \Delta p \) vs. \( t \). It is observed that the inverted model parameters are all within 0.75% of the original values for all the inversions associated with the sinusoidal flow-rate pulse.

**Noisy three-layer, six-block test case.** This inversion exercise was aimed at assessing the influence of noisy measurements in the inverted model parameters. We contaminated the simulated pressure data with various amounts of additive random noise. Noise was synthesized numerically with a zero-mean Gaussian random number generator of standard deviation equal to a given percentage of the pressure change amplitude. For reference, we considered the same 3-layer 6-block model described in the previous example. A sinusoidal pulse was chosen for the flow-rate function and we examined deployments with 11 and 23 sensors evenly spaced along the well’s injection interval. **Figures 18 and 19** describe the first set of inversion results obtained with an array of 11 pressure sensors and data contaminated with 1% Gaussian random additive noise. Pressure measurements were acquired during a time interval of 120hrs. We notice that inverted parameters associated with model features located far away from the borehole are the most severely affected by the presence of noise. On the other hand, model parameters associated with near-borehole features remain relatively unscathed by the presence of noise.

A second test was intended to assess the effect of sensor spacing and number of sensors in the inverted model parameters assuming data contaminated with several levels of noise. Pressure data were simulated for a total of 23 in-situ pressure sensors. Two of these sensors were positioned 0.5m above and below the zone of interest. **Figures 20 and 21** describe the inversion results for the case of \( \Delta p \) vs. \( t \) input data contaminated with 1% Gaussian random additive noise. **Figure 22**, on the other hand, shows the model parameters inverted from pressure data contaminated with 2% Gaussian noise. As expected, the quality of the inversions degrades with increasing levels of noise in the input data. The largest relative errors in the estimated model parameters correspond to 1.2% and 2.5% for the cases of 1 and 2% Gaussian random additive noise, respectively.

**Analysis of inversion results for the three-layer, six-block test case.** In general, our study shows that in a reservoir model consisting of large lateral and vertical variations in permeability, near-borehole parameters can be robustly estimated even in the presence of noisy measurements. Conversely, parameters corresponding to model features located away from the borehole remain adversely affected by even relatively small amounts of noise. This behavior is consistent with the diffusive properties of pressure phenomena in porous media, and there is practically little one can do about it. Redundancy in the measurement scheme only partially helps to mitigate the deleterious effect of noise.

In the case of noisy measurements, the possibility also exists that the inversion be rendered unstable. A way to prevent such instability is to include a regularization term in the minimization of the cost function. This is accomplished by setting the matrix \( W_m \) equal to a unity diagonal matrix in Eq. (6)\(^2\). The Lagrange multiplier, \( \lambda \), in Eq. (6) then takes the role of a regularization constant. For the inversions presented in this paper, we have used a non-zero value of \( \lambda \) only in the cases of noisy pressure measurements, and have set it to be a small percentage of the ratio between the largest and smallest eigenvalues of the matrix \( J'(m^8) \cdot W^T_{m} \cdot W_p \cdot J(m^8) \) in Eq.(7). The value of this percentage was chosen in proportion to the estimated noise level.
Multi-block test case. Relatively more complex test cases were analyzed as part of our assessment of permanent pressure measurements. One of these test cases, shown in Fig. 23, and described in Table 1, consists of 20 radial permeability blocks within the same vertical layer. Permanent pressure data were simulated for this model assuming an injection flow rate in the form of a 135-hour step-function. A constant permeability value of 5 md was assigned to all of the blocks to initialize the inversions. The sensor deployment consisted of only one pressure gauge located at the mid-point of the vertical zone of interest. In this case, the inversion was formulated to render estimations of the 20 unknown permeability values. Figures 24 through 26 describe the inversion results for noise-free pressure data. The inverted permeabilities are all within 0.65% of the original values.

In the second test case, we constructed a 3-layer, 20-block reservoir model exhibiting the same rock and fluid parameters described in the previous case. Noise-free pressure measurements were simulated with a 135 hr step-function pulse in the injection flow rate. Figure 27 shows the actual permeability model. In this case, we considered a deployment of 11 pressure sensors in order to capture the vertical variability in pressure change due to cross-flow between layers. We initialized the inversion by assigning a constant, 5 md permeability value, to each of the existing model blocks. As indicated in Fig. 27, this inversion exercise involved 20 parameters per layer, thus adding up to a grand total of 60 unknown permeability values. Inversion results are described in Figs. 28 through 30. All of the inverted permeabilities are within 0.1% of the original values.

Application of a dual-grid inversion technique to the three-layer, six-block test case. We implemented a novel dual-grid strategy to reduce the computer execution times associated with the inversion of a large number of unknown parameters. Details of the dual-grid nonlinear inversion technique are discussed extensively by Torres-Verdin et al. We set a subset (54 × 281) of the fine finite-difference grid (105 × 281) described in Fig. 3 to perform the dual-grid inversions. The minimization strategy performs computations of the Jacobian matrix on the coarse finite-difference grid, while the fine grid is used only to carry out periodic checks of the fit to the measured data. Figures 31 and 32 show the inversion results obtained with the dual-grid minimization technique. All of the inverted model parameters are within 0.55% of the original values. Table 2 shows a comparison of CPU times required by the inversions performed with and without the dual-grid minimization approach. In this particular problem, the dual-grid inversion technique yielded a ~2.8 fold reduction in CPU execution time with respect to the standard procedure.

Figure 33 describes the route to convergence of the dual-grid inversion procedure used to obtain the permeability model shown in Fig. 31. In Fig. 33a, the relative data misfit is plotted as a function of the number of data misfit evaluations. We computed the relative data misfit using the formula reported by Torres-Verdin et al., namely,

\[ \frac{\| W_d \cdot (d(m) - d_{obs}) \|_2}{\| W_d \cdot d_{obs} \|_2}, \]

where \( W_d \) is a diagonal matrix with elements equal to the inverse of the measurement times the standard deviation of the noise (in the noise-free cases, \( W_d \) is set to a diagonal matrix with elements equal to the inverse of the measurement). In Fig. 33b, the nonlinear inversion was accomplished with only 5 calls to the forward-modeling code implemented on the fine-grid (105 × 281). Figure 33b is a plot of the data misfit with respect to the iteration number within one of the successive minimization within the coarse finite-difference grid (54 × 281). By contrast, Fig. 34 is a plot of the relative data misfit as a function of iteration number evaluated with the fine grid (54 × 281). A comparison of the routes to convergence in the two cases indicates a quicker reduction of residuals for the dual-grid implementation that is consistent with the CPU execution times described in Table 2.

Application of Nonlinear Inversion to Dual-Phase Flow Models

An attempt was made to adapt the inversion algorithm described above for the estimation of dual-phase relative permeabilities in homogeneous layers. Accordingly, a single-layer reservoir model was constructed with a homogeneous and isotropic slab of permeability equal to 100 md. We assumed that initially the reservoir was saturated with oil and irreducible water. Time variations of pressure were simulated assuming a fall-off flow-rate pulse enacted after 135 hrs. of water injection at a constant injection rate of 500 BBL/D. Measurements of pressure were acquired during a time interval of 45 hrs after the onset of fall-off. The geometrical and fluid-flow parameters of the assumed reservoir model are described in Table 3. As explained in Fig. 35, we assumed a Corey type oil-water relative permeability curve and a capillary pressure curve of the type introduced by Sanchez-Bujanos. Simulations of the dual-phase injection experiment were carried out with the commercial simulator ECLIPSE 100TM. A 200 × 200 fine grid was designed in order to fully capture the expected variability in the lateral and vertical distributions of pressure and fluid saturations. Measurements consisted of time-domain pressure data gathered by a single permanent pressure gauge located at the mid-point of the vertical zone of interest.

Even though the pressure sensor data were generated with a dual-phase nonlinear fluid-flow simulator operating over a uniform permeability slab, our objective was to use the single-phase inversion algorithm to estimate radial variations of fluid saturations. The idea was to detect radial variability in the water saturation profile from the corresponding profile of relative permeabilities. We assumed that the water-oil relative permeability curves were also part of the input data and that so was the layer’s absolute permeability. Thus, if the inversion algorithm were to detect radial variations of permeability we would transform these variations into radial variations of...
water saturation. We proceeded to discretize the otherwise uniform slab into 20 blocks of constant permeability using logarithmic steps in the radial direction. Figures 36 shows the inverted radial profile of relative permeabilities. This profile is consistent with the corresponding profile of water saturation. For instance, the inverted profile of relative permeabilities monotonically decreases as the water saturation decreases in the immediate vicinity of the well (water is to become the dominant fluid phase in this radial zone after 135hrs. of injection). Moreover, the inverted profile of relative permeability exhibits a minimum at a distance of approximately 0.3m away from the borehole wall. It is at this point where none of the two fluid phases becomes dominant. Lastly, the inverted relative permeability profile monotonically increases away from the borehole wall. This behavior is consistent with the fact that oil becomes the dominant fluid phase for large radial distances away from the borehole wall. At a distance of 3m away from the borehole wall the inverted relative permeability becomes constant and approximately equal to 80md. This value corresponds to the upper limit imposed on the inverted permeabilities by the inversion algorithm. Figure 37 shows the comparison between the input time-domain pressure data and the corresponding pressure data simulated from the permeability data yielded by the inversion. We notice an excellent agreement between the two data sets, thereby giving credence to the inversion results.

Conclusions

We carried out a relatively simple study to quantify the sensitivity of in-situ permanent pressure measurements to detecting spatial distributions of permeability. This study was based on a hypothetical numerical example of single-phase fluid flow, and on the availability of an array of permanent pressure gauges deployed along a vertical well. The well was subject to water injection and the pressure gauges were positioned in direct hydraulic contact with the surrounding rock formations. For simplicity, we also assumed that the rock formations exhibited azimuthal symmetry about the axis of the injection well. Techniques borrowed from the field of geophysical inverse theory were used to perform the sensitivity study. As part of this work, we also developed a novel numerical algorithm to simulate single-phase fluid-flow in axisymmetric porous media. We also introduced an efficient dual-grid inversion approach that can substantially reduce computation times in cases of a large number of unknown model parameters.

Our test cases indicate that in-situ permanent pressure sensors have the potential of accurately detecting complex spatial distributions of permeability. Unlike standard pressure measurements acquired within the borehole, in-situ pressure measurements are highly sensitive to individual rock formations, and to hydraulic communication among formations. A great deal of flexibility in the acquisition system is provided by (a) the location, spacing, and number of permanent sensors, (b) the time sampling schedule of the measurements, and (c) the way in which the injection flow rate is pulsed to produce a perturbation in the pressure field. The sensitivity studies also showed that noisy pressure measurements could considerably bias the detection of spatial variations of permeability located far away from the sensor array. This behavior is due to the diffusive nature of the flow of fluids in porous media, and there is hardly anything one can do about it. However, it was found that both sensor redundancy and an appropriate selection of the flow-rate schedule (e.g. a low frequency sinusoidal pulse) could improve the sensitivity of the sensor array to permeability variations located far away from the borehole wall.

An attempt was also made to invert a radial profile of relative permeabilities from pressure measurements simulated with a dual-phase nonlinear fluid-flow simulator. It was shown that, in principle and subject to several operating assumptions, a single-phase inversion algorithm could be used to provide a quantitative indication of the corresponding water saturation profile.

In the future, much more exciting work is anticipated to quantify the detection properties of in-situ pressure arrays deployed in horizontal and highly deviated wells. We also envision the possibility of carrying out several time-dependent inversions during the course of fluid production. Alternatively, a global, or predictor-corrector inversion could be formulated to take into account data gathered at different times during the production history of the reservoir. In so doing, flow-rate pulses could be performed periodically in time to monitor dynamic reservoir conditions such as fluid movements. Data redundancy in time could also help improve the accuracy of the inverted permeability distributions in the presence of noise.

Nomenclature

\( A \) = functional operator and symmetric matrix obtained via \( A[u] \)

\( C(m) \) = quadratic cost function

\( d(m) \) = numerically simulated data vector

\( d^{\text{obs}} \) = observed data vector

\( C_i \) = total compressibility

\( G \) = impulse response function

\( G_w \) = impulse response function at the open section of the wellbore

\( G_t \) = impulse response function at time \( t = 0 \)

\( H \) = Heaviside's step function

\( h \) = formation thickness

\( I \) = identity matrix

\( J(m) \) = Jacobian matrix of \( C(m) \)

\( \bar{K} \) = second order permeability tensor

\( k \) = single-phase permeability

\( k_r \) = single-phase permeability in radial direction

\( k_z \) = single-phase permeability in vertical direction

\( l \) = lower bound

\( l_1, l_2 \) = vertical bounds of the open interval
m = size-N vector of unknown variables

m_e = reference vector of the same size with m

p = pressure

Δp = pressure drop

p_i = pressure at the open section of the wellbore

p_i = initial formation pressure

q = volumetric flow rate

R^2 = two dimensional space

r = radial coordinate axis

r_e = reservoir external radius

r_w = wellbore radius

s = Laplace domain variable

S_o = residual oil saturation

S_o = water saturation

S_o = irreducible water saturation

T = mobility Tensor

T = mobility

t = time

U = Laplace transform of Green’s function solution

u = variable that results from substitution in (A-19), upper bound

v = velocity vector,

W_d = inverse of the covariance matrix

W_m = inverse of the model covariance matrix

z = vertical coordinate axis

Greek Symbols

δ(t) = Dirac delta function

Ω_cyl = cylindrical surface (section of wellbore)

Ω_i = open section of the wellbore

Ω_2 = no-flow section of the wellbore

Δp_{skin} = pressure drop due to the presence of skin factor

λ = regularization parameter (Lagrange multiplier)

λ_r = radial mobility

μ = fluid viscosity

Ω = bounded domain

ρ = mass density

σ = initial condition for variable u

σ_i = vector of initial conditions at every spatial location for u

ϕ = porosity

τ = integration variable, period of sinusoidal function

ω = integration variable

χ^2 = prescribed value of data misfit

Subscripts

e = external

i = ith layer, ith upper (or lower) constraint

irr. = irreducible

max. = maximum

o = initial or original condition, oil phase

r = entity in radial direction, residual

sf = sandface

t = total

w = well, wellbore, water phase

x, y, z = Cartesian coordinates

z = entity in vertical direction

Superscripts

- = fictitious outer domain

k = iteration number

T = transpose of an arbitrary matrix

Acknowledgements

We would like to express our gratitude to Baker-Atlas, Halliburton, and Schlumberger for funding of this work through UT Austin’s Center of Excellence in Formation Evaluation.

References


3. He, N., Reynolds, A.C., and Oliver, D.S.: “Three-Dimensional Description From Multiwell Pressure Data and Prior Information,” SPEJ. 2(3) (September 1997), 312-327.


Next, on the open section of the wellbore $(\partial \Omega_1)$, where the total volumetric flow rate $(q_{sf})$ is the prescribed source condition, we write (Muskat\textsuperscript{17})

$$
- \int_{\partial \Omega_1} \lambda_r(\tilde{r}) \frac{\partial p(\tilde{r}, t)}{\partial r} d\omega = q_{sf}(t) \text{ on } \partial \Omega_1, \quad t > 0,
$$

(A-3)

whereas for the no-flow section of the wellbore we have

$$
\frac{\partial p(\tilde{r}, t)}{\partial r} = 0 \text{ on } \partial \Omega_2, \quad t > 0,
$$

(A-4)

\[ \text{Figure A-1: Description of the spatial domain considered in the numerical solution of axisymmetric pressure.} \]

where $U_{l_z l_r}^2 \partial \Omega_1 = \partial \Omega$, $\tilde{r} : r = r_w$, $l_1 < z < l_2$, $l_1$ and $l_2$ are the vertical bounds of the open interval, and $r_w$ is the radius of the internal cylindrical boundary of the wellbore. In Eq. (A-3), the mobility $\lambda_r \in \mathbb{T}$ in the $r$ direction is defined as

$$
\lambda_r(\tilde{r}) = \frac{k_r(\tilde{r})}{\mu}.
$$

(A-5)

A geometrical description of the spatial domain and boundary conditions for this problem are shown in Figure A-1.

We also require that the pressure be uniform on the open surface of the cylinder ($\partial \Omega_2$) (i.e. that it be independent of $r$ and $z$) and exclusively a function of time, i.e.,

$$
p(r_w, t) = p_n(r_w, t) \text{ on } \partial \Omega_1, \quad t > 0.
$$

(A-6)

The no-flow boundary condition described in Eq. (A-4) equally applies to the exterior boundary of the reservoir. A Green’s function representation of the problem described above can be used to derive a canonical time-domain solution. Accordingly, the differential equation satisfied by the Green’s function and its associated initial and boundary conditions are:
\[ \nabla \cdot \left[ \mathbf{T}(\mathbf{r}) \cdot \nabla G(\mathbf{r}, t) \right] = \phi(\mathbf{r}) C_i \frac{\partial G(\mathbf{r}, t)}{\partial t}, \quad (A-7) \]

\[ G(\mathbf{r}, t) = 0 \quad \text{at} \quad t = 0, \quad (A-8) \]

\[ - \int_{\partial \Omega_1} \lambda_r(\mathbf{r}) \frac{\partial G(\mathbf{r}, t)}{\partial r} \, d\omega = \delta(t) \quad \text{on} \quad \partial \Omega_1, \quad \text{where} \quad t > 0, \]

\[ \frac{\partial G(\mathbf{r}, t)}{\partial r} = 0 \quad \text{on} \quad \partial \Omega_2 \quad \text{for} \quad t > 0, \]

\[ G(r_w, t) = G_w(r_w, t) \quad \text{on} \quad \partial \Omega_1 \quad \text{for} \quad t > 0, \]

\[ G(\mathbf{r}, 0) = G_1(\mathbf{r}) \quad \text{at} \quad t = 0. \quad (A-12) \]

An asymptotic representation of Eq. (A-7) can be written as

\[ \phi(\mathbf{r}) C_i \frac{\partial G(\mathbf{r}, t)}{\partial t} \]

where \( H \) is Heaviside’s step function. Therefore, the boundary and initial conditions associated with canonical Green’s function can be equivalently written as

\[ - \int_{\Omega_2} \lambda_r(\mathbf{r}) \frac{\partial G(\mathbf{r}, t)}{\partial r} \, d\omega = 0 \quad \text{for} \quad t > 0 \quad \text{on} \quad \partial \Omega_1, \quad (A-13) \]

where

\[ \frac{\partial G(\mathbf{r}, t)}{\partial r} = 0 \quad \text{on} \quad \partial \Omega_2 \quad \text{for} \quad t > 0, \]

\[ G(r_w, t) = G_w(r_w, t) \quad \text{on} \quad \partial \Omega_1 \quad \text{for} \quad t > 0, \]

\[ G(\mathbf{r}, 0) = G_1(\mathbf{r}) \quad \text{at} \quad t = 0. \]

In cylindrical coordinates \((r - z)\), Eq. (A-7) is given by

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial G(r, z, t)}{\partial r} \right] + \frac{\partial}{\partial z} \left[ T_z \frac{\partial G(r, z, t)}{\partial z} \right] = \phi(r, z) C_i \frac{\partial G(r, z, t)}{\partial t}. \quad (A-17) \]

By introducing

\[ u(r, z, t) = (r \phi(r, z) C_i)^{1/2} G(r, z, t), \quad (A-18) \]

we obtain

\[ - A[u] = \frac{\partial u}{\partial t} \quad (A-19) \]

where \( A \) is a functional operator defined as

\[ A[u] = -(r \phi(r, z) C_i)^{-1/2} \left[ \frac{\partial}{\partial r} (r T_r) \frac{\partial}{\partial r} + \frac{\partial}{\partial z} (T_z) \frac{\partial}{\partial z} \right] \left[ r \phi(r, z) C_i \right]^{1/2} u. \quad (A-20) \]

It can be easily shown that the spatial operator \( A \) given by Eq. (A-20) is self-adjoint and non-negative. Moreover, using the change of variables introduce by Eq. (A-18) gives rise to the modified initial condition

\[ \sigma = u(r, z, 0) = (r \phi(r, z) C_i)^{1/2} G(r, z, 0), \quad (A-21) \]

which has the asymptotic representation

\[ \sigma = (r \phi(r, z) C_i)^{1/2} \frac{\delta(r - r_c) H(z - l_1) H(z - l_2)}{2 \pi r_c (l_1 - l_2) \phi(r, z) C_i} \]

as \( t \to 0. \quad (A-22) \]

The explicit solution to Eq. (A-19) is then given by

\[ u(r, z, t) = \exp(-tA) \cdot \sigma(r, z). \quad (A-21) \]

To solve numerically for \( u(r, z, t) \), we approximate the spatial operator \( A \) above by finite differences using a 5-point second-order stencil on a 2D grid spanning the semi-plane \((r > 0, z)\). In turn, a solution for the matrix functional \( \exp(-tA) \cdot \sigma(r, z) \) is obtained using the Extended Krylov Subspace Method (EKSM). This method of solution provides solutions for a multitude of values of \( t \) with practically the same efficiency as that of a single time solution. Subsequent substitution from Eq. (A-18) yields the corresponding Green’s function \( G(r, z, t) \).

Finally, the solution for the pressure distribution associated with an arbitrary time variation of the flow-rate is obtained via the convolution operation

\[ p(\mathbf{r}, t) = p_0 - \int_{-\infty}^{t} q_f(\tau) \left[ G(\mathbf{r}, t - \tau) + \delta(t - \tau) \Delta p_{\text{skin}} \right] \, d\tau, \quad (A-23) \]

where \( \Delta p_{\text{skin}} \) represents the pressure drop due to the presence of a skin factor, and \( p_0 \) is the initial formation pressure.
TABLE 1 - GEOMETRICAL AND FLUID-FLOW PARAMETERS USED TO CONSTRUCT THE SINGLE-PHASE TEST CASES DESCRIBED IN THIS PAPER

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure, $p_0$</td>
<td>2500 psi</td>
</tr>
<tr>
<td>Water viscosity, $\mu_w$</td>
<td>1 cp</td>
</tr>
<tr>
<td>Total compressibility, $C_t$</td>
<td>0.00002 psi$^{-1}$</td>
</tr>
<tr>
<td>Effective porosity, $\phi$</td>
<td>0.2 fraction</td>
</tr>
<tr>
<td>Wellbore radius, $r_w$</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Reservoir thickness, $h$</td>
<td>10 m</td>
</tr>
<tr>
<td>Injection rate, $q$ (Step-function pulse)</td>
<td>500 BBL/D</td>
</tr>
<tr>
<td>Max. attainable injection rate, $q_{max}$ (Sinusoidal pulse)</td>
<td>500 BBL/D</td>
</tr>
</tbody>
</table>

TABLE 2 - COMPARISON OF CPU EXECUTION TIMES FOR INVERSIONS PERFORMED WITH AND WITHOUT A DUAL-GRID. THREE-LAYER, 6-BLOCK TEST CASE

<table>
<thead>
<tr>
<th>Grid I Size (outer loop)</th>
<th>Grid II Size (inner loop)</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>105 x 281</td>
<td>105 x 281</td>
<td>7804.052</td>
</tr>
<tr>
<td>105 x 281</td>
<td>54 x 281</td>
<td>2807.188</td>
</tr>
</tbody>
</table>

TABLE 3 - GEOMETRICAL AND FLUID-FLOW PARAMETERS USED TO CONSTRUCT THE MULTI-PHASE TEST CASES DESCRIBED IN THIS PAPER

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure, $p_0$</td>
<td>2500 psi</td>
</tr>
<tr>
<td>Water viscosity, $\mu_w$</td>
<td>1 cp</td>
</tr>
<tr>
<td>Oil viscosity, $\mu_o$</td>
<td>1 cp</td>
</tr>
<tr>
<td>Water density, $\rho_w$</td>
<td>62.3127 lb/ft$^3$</td>
</tr>
<tr>
<td>Oil density, $\rho_o$</td>
<td>48.6234 lb/ft$^3$</td>
</tr>
<tr>
<td>Total compressibility, $C_t$</td>
<td>0.00002 psi$^{-1}$</td>
</tr>
<tr>
<td>Effective porosity, $\phi$</td>
<td>0.2 fraction</td>
</tr>
<tr>
<td>Initial water saturation, $S_{wi}$</td>
<td>0.3 fraction</td>
</tr>
<tr>
<td>Irreducible water saturation, $S_{wirr}$</td>
<td>0.3 fraction</td>
</tr>
<tr>
<td>Residual oil saturation, $S_{or}$</td>
<td>0.2 fraction</td>
</tr>
<tr>
<td>Wellbore radius, $r_w$</td>
<td>0.1 m</td>
</tr>
<tr>
<td>Reservoir thickness, $h$</td>
<td>10 m</td>
</tr>
<tr>
<td>Injection rate, $q$ (Step-function pulse)</td>
<td>500 BBL/D</td>
</tr>
</tbody>
</table>

Fig. 1- Graphical description of a generic permanent sensor installation. Pressure gauges are deployed in direct hydraulic contact with the formation. In this example, water is injected through an open interval thereby displacing in-situ oil. Water invasion fronts in the form of cylinders are used to indicate variability in the vertical distribution of permeability.
**Fig. 2**-Input permeability model for a 3-layer 6-block reservoir with communicating layers.

**Fig. 3**-Finite-difference grid used for the numerical simulation and inversion examples presented in this paper.

**Fig. 4**-Example $\Delta p$ vs. $t$ and $q$ vs. $t$ plots (superimposed) for a step-function flow-rate pulse.

**Fig. 5**-Example $\Delta p$ vs. $t$ and $q$ vs. $t$ plots (superimposed) for a step-function flow-rate pulse.

**Fig. 6**-Comparison of sandface pressure fall-off responses for a 3-layer 6-block reservoir with communicating layers.

**Fig. 7**-Comparison of reservoir pressure fall-off responses 1m away from the wellbore for a 3-layer, 6-block reservoir with communicating layers.
Fig. 8-Permeability model inverted using a step-function pulse with $t_{inj}=135\text{hr}$, and a deployment of 11 sensors ($\Delta p$ vs. $t$ input data).

Fig. 9-Post-inversion fit to the input data (step-function pulse, deployment of 11 sensors, $t_{inj}=135\text{hr}$, $\Delta p$ vs. $t$ input data). Continuous curves identify the data yielded by the inversion.

Fig. 10-Permeability model inverted using a step-function pulse with $t_{inj}=135\text{hr}$, and a deployment of 5 sensors ($\Delta p$ vs. $t$ input data).

Fig. 11-Post-inversion fit to the input data (step-function pulse, deployment of 5 sensors, $t_{inj}=135\text{hr}$, $\Delta p$ vs. $t$ input data). Continuous curves identify the data yielded by the inversion.

Fig. 12-Permeability model inverted using a sinusoidal pulse with $\tau=60\text{hr}$, and a deployment of 11 sensors ($\Delta p$ vs. $t$ input data).

Fig. 13-Post-inversion fit to the input data (sinusoidal pulse, deployment of 11 sensors, $\tau=60\text{hr}$, $\Delta p$ vs. $t$ input data). Continuous curves identify the data yielded by the inversion.
Post-Inversion Permeability Field (md)

Logarithm of radial position from wellbore (m)

Fig. 14-Permeability field inverted using a sinusoidal pulse with τ=60hr, and a deployment of 11 sensors (dp/dt vs. t input data).

Post-Inversion Permeability Field (md)

Vertical position (m)

Fig. 15-Post-inversion fit to the input data (sinusoidal pulse, deployment of 11 sensors, τ=60hr, dp/dt vs. t input data). Continuous curves identify the data yielded by the inversion.

Post-Inversion Permeability Field (md)

Vertical position (m)

Fig. 16-Permeability model inverted using a sinusoidal pulse with τ=60hr, and a deployment of 5 sensors (dp/dt vs. t input data).

Post-Inversion Permeability Field (md)

Logarithm of radial position from wellbore (m)

Fig. 17-Post-inversion fit to the input data (sinusoidal pulse, deployment of 5 sensors, τ=60hr, Δp vs. t input data). Continuous curves identify the data yielded by the inversion.

Post-Inversion Permeability Field (md)

Vertical position (m)

Fig. 18-Permeability model inverted using a sinusoidal pulse with τ=60hr, and a deployment of 5 sensors. Gaussian, 1% random noise was added to the Δp vs. t input data. Continuous curves identify the data yielded by the inversion.

Post-Inversion Permeability Field (md)

Logarithm of radial position from wellbore (m)

Fig. 19-Post-inversion fit to the input data (sinusoidal pulse, deployment of 5 sensors, τ=60hr). Gaussian, 1% random noise was added to the Δp vs. t input data. Continuous curves identify the data yielded by the inversion.
Fig. 20-Permeability model inverted using a sinusoidal pulse with \( \tau = 60 \) hr, and a deployment of 23 sensors. Gaussian, 1% random noise was added to the \( \Delta p \) vs. \( t \) input data.

Fig. 21-Post-inversion fit to the input data (sinusoidal pulse, deployment of 23 sensors, \( \tau = 60 \) hr, \( \Delta p \) vs. \( t \) input data). Gaussian, 1% random noise was added to the input data. Continuous curves identify the data yielded by the inversion.

Fig. 22-Permeability model inverted using a sinusoidal pulse with \( \tau = 60 \) hr, and a deployment of 23 sensors. Gaussian, 2% random noise was added to the \( \Delta p \) vs. \( t \) input data.

Fig. 23-Input permeability model for a single-layer, 20-block reservoir.

Fig. 24-Permeability model inverted using a step-function pulse with \( t_{inj} = 135 \) hr, and a deployment of a single sensor (\( \Delta p \) vs. \( t \) input data) for a single-layer 20-block reservoir.

Fig. 25-Percent absolute error in the inverted permeability model (step-func. pulse with \( t_{inj} = 135 \) hr, and a deployment of a single sensor, using \( \Delta p \) vs. \( t \) data) for a single-layer 20-block reservoir.
Fig. 26-Post-inversion fit to the input data for a single-layer 20-block reservoir model (step-func. pulse, deployment of a single sensor, \( t_{\text{w}} = 135 \text{hr} \), \( \Delta p \) vs. \( t \) input data). Continuous curves identify the data yielded by the inversion.

Fig. 27-Input permeability model for a 3-layer, 60-block reservoir.

Fig. 28-Permeability model inverted using a step-function pulse with \( t_{\text{w}} = 135 \text{hr} \), and a deployment of 11 sensors (\( \Delta p \) vs. \( t \) input data) for a 3-layer, 60-block reservoir.

Fig. 29-Percent absolute error in the inverted permeability model (step-func. pulse with \( t_{\text{w}} = 135 \text{hr} \), and deployment of 11 sensors, using \( \Delta p \) vs. \( t \) input data) for a 3-layer, 60-block reservoir.

Fig. 30-Post-inversion fit to the input data fit for a 3-layer, 60-block reservoir (step-function pulse, deployment of 11 sensors, \( t_{\text{w}} = 135 \text{hr} \), \( \Delta p \) vs. \( t \) input data). Continuous curves identify the data yielded by the inversion.

Fig. 31-Permeability model inverted using a step-function pulse with \( t_{\text{w}} = 135 \text{hr} \), and a deployment of 11 sensors (\( \Delta p \) vs. \( t \) input data). The inversion was performed with a dual-grid algorithm.
Fig. 32-Post-inversion fit to the input data. The inversion was performed with a dual-grid nonlinear inversion technique.

Fig. 33-Plots of data misfit versus iteration number in the search of a minimum of the cost function. (a) is the normalized data misfit evaluated with the fine grid (105 × 281). (b) shows the normalized data misfit as a function of the iteration number within one of the auxiliary cost functions constructed with the coarse grid (54 × 281).

Fig. 34-Plot of data misfit versus iteration number in the search of a minimum of the cost function. The normalized misfit was evaluated using a fine grid (105 × 281 nodes).

Fig. 35-Oil-water relative permeability and capillary pressure curves used for the simulation of a dual-phase fall-off test case.

Fig. 36-Relative permeability model inverted for a dual-phase flow test case. The inversion was carried out assuming a homogeneous and isotropic single-phase permeability model.

Fig. 37-Post-inversion fit to the input data for multiphase flow in a homogeneous and isotropic single-phase permeability model (step-function pulse, deployment of 1 sensor, \( t_{inj}=135\) hr, \( \Delta p \) vs. \( t \) input data).