A new test stand for measuring wall shear stress.

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The design, construction and preliminary measurements of a new test stand for accurately assessing the shear stress acting at the fluid surface interface of wall bounded flows is discussed. This stand is based on control volume analysis whereby a fully developed turbulent velocity profile produces shear forces which equate to the pressure drop measured between fixed points in a constant area pipe. The calibration stand is designed to facilitate both subsonic and supersonic flow. Subsonic flow conditions are achieved by placing different diameter nozzles at the exhaust of the test section thereby permitting different free stream velocities and mass flow rates for a given ratio of the total pressure to static pressure in the pipe. The advantages of this facility is in its ability to produce a broad range of Reynolds numbers (based on centerline velocity and pipe diameter) and elevated pressures that are required to gauge the sensitivity of modern shear stress sensors.

I. Introduction

The development of a reliable high-speed strike vehicle like the HIFiRE or the X-51 is plagued by a great many factors that require monumental achievements in not only material science and structural mechanics, but also aerodynamics, flight control and combustion. Quite simply, the vehicle must be capable of surviving prolonged exposures to extreme flight conditions encompassing elevated temperatures, and various unsteady phenomena, which are difficult to predict. Most efforts to predict the flight environment rely on ground based wind tunnel testing, or simulations using numerical modeling techniques. However, wind tunnel measurements are often restricted by the accuracy of the diagnostics, while numerical models (DNS, LES, uRANS) are confined to the meshing requirements needed for accurate solution to the governing flow equations. Thus, a synergy between wind-tunnel measurements, requiring robust and high fidelity sensing instruments, as well as the development of stable predictive models, is paramount to the future of hypersonic flight programs.

The most accurate simulations are constructed by directly solving the Navier-Stokes Equations. (DNS). However, simulations of this kind are impractical, given the broad range of scales that must be resolved at the kinds of Reynolds numbers that these hypersonic vehicles are exposed to. While both RANS and LES models are capable of accurately modeling complex flows, they are done so at the expense of employing too coarse a grid in vicinity of the fluid structure interface. Boundary layer physics, where heating and friction effects are greatest, are then modeled using empirical formulations that assume the shape of the velocity profile. The problem is further exacerbated by three-dimensional effects, surface texture variations, favorable/adverse pressure gradients, shock-shock or shock-turbulence interactions, freestream disturbances and multi-species combustion. The consequence of this, from a numerical standpoint, is the demand for more accurate empirical models for the shear stress at the wall in hypersonic and chemically reacting boundary layer flows. The development of a sensor capable of resolving the full spectrum of scales at the wall (both shear and normal forces) and at elevated temperatures would alleviate these restrictions by providing a direct measure of the variables important to characterizing the inner regions of the boundary layer needed for CFD validation.

Turbulent boundary layer flows (TBL) comprise the majority of the internal and external fluid-surface interfaces that a hypersonic vehicle is exposed to. For these kinds of flows, the friction Reynolds number is
the most relevant dimensionless number for the wall turbulence and is the ratio of the inner and outer length scales. In the classical view, the inner region is taken to be $0 < y^+ < 0.12 \text{Re}_\tau$ and is independent of the outer region.\textsuperscript{5} This viewpoint has been challenged in recent years based on evidence that the large scales undergo an amplitude modulation effect which link the large-scale superstructures to the events that reside in the near-wall region.\textsuperscript{6} This is tied to the meandering of low-momentum fluid in the buffer zone that is both intermittent and three-dimensional. Nevertheless, a simple scaling exercise following the work of Schewe,\textsuperscript{7} Klewicki et al.,\textsuperscript{8} Gravante et al.,\textsuperscript{9} and Dolder et al.,\textsuperscript{10} using a heated (1200 K) Mach 5, zero-pressure-gradient turbulent boundary, reveals the kinds of scales (temporal and spatial) that are required to accurately resolve the wall turbulence where the fluid-surface interface resides (and where the aerodynamic loads acting on a hypersonic vehicle reside). Given a wall unit of 2.312\textmu m, this equates to a time scale of 0.2\textmu s, and a sensor size no greater than 40\textmu m based on $d^+ = 18$. Likewise, given the three-dimensionality of TBL flows, such a sensor must be capable of selectively isolating in-plane shear and normal forces from cross-plane effects, all the while displaying excellent sensitivity.

Given the challenges associated with the development of a shear stress sensor capable of operating reliably in these kinds of harsh environments, there are very few scientific groups willing to take on this great challenge. Aside from the development of these sensors is the added challenge of calibrating them, and under the range of conditions that they are expected to perform at. Thus, at the forefront of this paper is the design and validation of a new test stand at the Applied Research Laboratories at The University of Texas at Austin for calibrating shear stress sensors for a range of conditions corresponding to high speed, (supersonic and hypersonic) flight.

### II. Model Equations for Predicting Shear Stress

The design and initial testing of a new test stand for characterizing the performance of modern shear stress sensors under high-speed and high-enthalpy flow conditions is here discussed. Its design utilizes indirect methods for estimating the theoretical shear stress acting on the wall of a constant area pipe of length $L$. An illustration of the pipe with control volume is provided in Fig. 1 while reviews of similar efforts can be found in Tropea et al.\textsuperscript{11}

![Figure 1: Control volume through a constant area pipe.](https://example.com/figure1.png)

Expressions for modeling these forces are obtained by applying the integral forms of the mass and momentum conservation equations to a control volume surrounding the pipe and flow. The generic forms of
these equations are as follows,

\[ \frac{\partial}{\partial \alpha} \int_{c_v} \rho \, d\mathbf{V} + \int_{c_s} \rho (\mathbf{V} \cdot \hat{n}) \, dA = 0, \tag{1} \]

\[ \sum \mathcal{F} = \mathcal{F}_a + \mathcal{F}_b = \frac{\partial}{\partial \alpha} \int_{c_v} \rho \, d\mathbf{V} + \int_{c_s} \rho (\mathbf{V} \cdot \hat{n}) \, dA, \tag{2} \]

where \( \mathbf{V} = U_j + V_j + \mathbf{W}_k \) and \( \mathbf{V} \) defines the control volume with control surfaces through which fluid flows uniformly. Neglecting body forces (hydrostatic and electrostatic), the \( x \) component force can be resolved independently \( (y \text{ and } z \text{ forces are neglected as they are assumed to be normal to the flow direction).} \) It is also assumed that the control volume is placed at the end of the entrance length to the pipe, where boundary layers are turbulent and merge to form a velocity profile that no longer changes with distance. Therefore, since the flow advects unchanged from the inlet (station 1) to the exit (station 2) of the control volume, then the right-hand-side of Eq. (2) reduces to the well known theoretical drag equation, which is a simple academic exercise for those studying fluid dynamics.

\[ \mathcal{F}_\tau = (P_2 - P_1) A_j \tag{3} \]

Here \( P_2 \) and \( P_1 \) are the static pressures measured at the inlet and exit of the control volume, respectively. The drag force is thus determined by integrating the shear force over the wetted surface area of the pipe so that, \( \mathcal{F}_\tau = \tau_w A_{12} \) and the wetted surface area is taken between stations 1 and 2 so that \( A_{12} = \pi D_p \ast L_{12} \).

The resultant expression for relating pressure drop to shear stress is then: \( \tau_w = (P_2 - P_1) D_p / (4L_{12}) \). The approach is only accurate so long as both the velocity profile (either laminar or turbulent) and the cross-sectional area of the pipe/duct remain unchanged between points 1 and 2, which poses certain design challenges for both subsonic and supersonic flows at elevated pressures. In what follows, a simple model for predicting the theoretical performance of this new test stand is described. While our initial focus is on unheated gases, the model is extended to account for high enthalpy flows.

A. General Expressions for a Compressible Gas

Many of the expressions found in the open literature for estimating pipe friction are based on the following Reynolds number definition,

\[ Re_D = \frac{\bar{u} D_p}{\nu} \tag{4} \]

where \( \bar{u} \) is the average velocity \( (\bar{u} \sim U_j) \) and \( \nu = \mu / \rho \) is the kinematic viscosity of the fluid. All fluids in this case are assumed to be calorically imperfect gases, \( \gamma = \gamma_j(T_j) \) and are assumed to obey the perfect gas law: \( P = \rho RT \). For compressed gases, and at elevated enthalpy states, solutions for the three terms, \( \bar{u}, D_p \) and \( \nu \) requires a roundabout calculation of the equations describing physical gas dynamics and at different points in the flow (in the plenum where the control volume is located; at the nozzle throat; at the exhaust plane downstream of the nozzle). Typical expressions for characterizing physical gas dynamics are as follows,

\[ \frac{T_o}{T_j} = \left[ 1 + \frac{\gamma_j - 1}{2} M_j^2 \right] = \left( \frac{P_o}{P_j} \right)^{\frac{\gamma_j - 1}{\gamma_j}} = \left( \frac{\rho_o}{\rho_j} \right)^{\gamma_j - 1} \tag{5} \]

\[ M_j = \left( \frac{2}{\gamma_j - 1} \left( \frac{P_o}{P_j} \right)^{\frac{\gamma_j - 1}{\gamma_j - 1}} - 1 \right)^{\frac{1}{2}} \tag{6} \]

where \( M_j = \bar{u}/a \) and \( a = \sqrt{\gamma_j RT_j} \). The specific heat ratio is thus \( \gamma = c_p/c_v \) where \( c_p = c_v + \Lambda \). Subscripts \( j, o, \infty \) denote static and stagnation properties of the compressed gas and atmospheric properties, respectively, and the mass flow is determined from Eq. (1) using static properties of the gas so that \( \dot{m} = \rho_j A U_j \). For high enthalpy flows, molecular gas effects must be modeled, which for dry air, is taken to be composed of three elements, \( N_2, O_2 \) and \( Ar \) with mass fractions corresponding to \( \chi_1 = 0.7810, \chi_2 = 0.2096 \) and \( \chi_3 = 0.0094 \), respectively. The atomic weights of these elements, and others that are relevant to this analysis, are provided
Table 1: Atomic and molecular gas properties taken from Radzig and Smirnov\textsuperscript{12} and Cox et al.\textsuperscript{13}

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Structure</th>
<th>Atomic Weight [g/mol]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nitrogen</td>
<td>N</td>
<td>monatomic</td>
<td>14.007</td>
</tr>
<tr>
<td>Oxygen</td>
<td>O</td>
<td>monatomic</td>
<td>15.999</td>
</tr>
<tr>
<td>Argon</td>
<td>Ar</td>
<td>monatomic</td>
<td>39.948</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Molecular Mass [g/mol]</th>
<th>Vibr. Freq. [cm(^{-1})]</th>
<th>(h\nu^2) [kJ/kmol]</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(_2)</td>
<td></td>
<td>28.014</td>
<td>2358.6</td>
<td>0</td>
</tr>
<tr>
<td>O(_2)</td>
<td></td>
<td>31.998</td>
<td>1580.2</td>
<td>0</td>
</tr>
</tbody>
</table>

in Table 1. Solutions for \(\gamma_j(T_j)\) are obtained iteratively over a range of temperature ratios at each point in the flow until a converged solution, taken as a temperature value that is within 0.01% of its previous value, is satisfied.

At elevated temperatures, changes in specific heat capacities due to changes in vibrational energy must be accounted for.\textsuperscript{14} Internal energy is obtained by summing vibrational, rotational and translational energies, \(c_v = c_{tr} + c_{rot} + c_{vib}\). For monatomic and linear diatomic molecules \(c_{tr} = (3/2)\Lambda\), whereas \(c_{rot} = 0\) for monatomic molecules while \(c_{rot} = \Lambda\) for linear diatomic molecules. Vibrational energies are obtained to a good approximation using a harmonic oscillator model\textsuperscript{15} as follows,

\[
c_{vib} = \Lambda \left[ \frac{\Theta_v/2T}{\sinh(\Theta_v/2T)} \right]^2
\]  

where the characteristic temperature for vibration is \(\Theta_v = \hbar(2\pi\nu_i)/k\); the constants \(\hbar, c, k\) are provided in Table 2 while vibration frequencies \(\nu_i\) of relevant molecules are found in Table 1.

Table 2: Molecular gas constants taken from Vincenti and Kruger\textsuperscript{15}

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>reduced Plank constant</td>
<td>(\hbar)</td>
<td>(1.054572 \times 10^{-34})</td>
<td>J s</td>
</tr>
<tr>
<td>speed of light</td>
<td>(c)</td>
<td>(2.997925 \times 10^{10})</td>
<td>cm s(^{-1})</td>
</tr>
<tr>
<td>Boltzmann constant</td>
<td>(k)</td>
<td>(1.38054 \times 10^{-23})</td>
<td>J K(^{-1})</td>
</tr>
<tr>
<td>Avogadro’s number</td>
<td>(\hat{N})</td>
<td>(6.02252 \times 10^{23})</td>
<td>mol(^{-1})</td>
</tr>
<tr>
<td>Universal gas constant</td>
<td>(\Lambda)</td>
<td>(\hat{N} \cdot k)</td>
<td>8.3143 J mol(^{-1}) K(^{-1})</td>
</tr>
</tbody>
</table>

Sutherland’s Law is used to account for the effect of temperature on viscosity

\[
\frac{\mu}{\mu_0} = \left( \frac{T}{T_0} \right)^{3/2} \frac{T_0 + S}{T + S}
\]  

where \(S\) is an effective temperature, otherwise referred to as the Sutherland constant. For Air, \(T_0 = 273\) K, \(S = 111\) K, \(\mu_0 = 1.716 \times 10^{-5}\) Ns/m\(^2\) and is accurate to within a 2% error between 170 K and 1900 K.\textsuperscript{a} Heat transfer effects through the pipe walls are unavoidable when \(T_j \neq T_{\text{pipe}}\). The Prandtl number is thus inserted

\[
Pr = \frac{\mu c_p}{k}
\]  

where \(k\) is the thermal conductivity of the gas and \(c_p\) is the specific heat capacity at constant pressure. Values for \(k\) are obtained, to a good approximation, using the same Sutherland formula,\textsuperscript{16}

\[
\frac{k}{k_0} = \left( \frac{T}{T_0} \right)^{3/2} \frac{T_0 + S}{T + S}
\]  

\textsuperscript{a}These values are taken from Table 1-2 of White\textsuperscript{16}
which for air, \( k_0 \) is valued at 0.0241 W/m·K; solutions for \( k \) are accurate to within ±2% between 160 K and 2000 K \(^b\). The compressibility factor,

\[
Z = \frac{P}{\rho RT}
\]  

(11)

should have a value of 1.0 if the gas is perfect and is the case to within ±10% for air in the range 1.8 < \( T_r \) < 15 and 0 ≤ \( P_r \) ≤ 10. Dissociation effects are responsible for deviations from \( Z = 1 \) which are not modeled here for high enthalpy conditions. These equations are arranged to solve for input parameters that can be measured and monitored using standard laboratory sensors. That is, the nozzle pressure ratio (NPR; ratio of total pressure \( P_o \) to ambient pressure \( P_\infty \)) and the temperature ratio (TR; ratio of total temperature \( T_o \) to ambient temperature \( T_\infty \)). Findings from this model are presented in Fig. 2 and 3 for the range of nozzle diameters and plenum Mach numbers displayed in Table 3.

Table 3: List of nozzle throat diameters and theoretical exit Mach numbers being used to control the gas properties through the calibration stand.

<table>
<thead>
<tr>
<th>( D^* ) [in]</th>
<th>( M_e )</th>
<th>( D^* ) [in]</th>
<th>( M_e )</th>
<th>( D^* ) [in]</th>
<th>( M_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>1.0</td>
<td>2.5</td>
<td>2.0</td>
<td>2.5</td>
<td>3.0</td>
<td>2.5</td>
</tr>
<tr>
<td>1.0</td>
<td>3.0</td>
<td>2.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The effect of temperature ratio \( (T_o/T_\infty) \) on the Prandtl number is shown in Fig. 2 for a range of throat diameters. A dipping point at a temperature ratio of 1.75 is observed and is caused by the competing effects of viscosity and thermal conductivity of the gas. Different nozzle throat diameters are shown to have little effect on the trend. Albeit, \( D^* \) alone is a misleading identifier. Variations in \( D^* \) equate to different subsonic plenum Mach numbers, and therefore different total to static temperature ratios; the temperature ratio or plenum Mach number would be a more appropriate variable of interest. The findings are complementary to what is reported in the open literature and is presented in order to establish credibility with the model being developed here.

![Figure 2: Prandtl number for different throat diameters and temperature ratios \( (T_o/T_\infty) \).](image)

In order to gauge the kinds of Mach numbers that are expected to form inside this plenum, the diameter ratio \( (D_p/D^*) \) is plotted in Fig. 3a and for two different temperature ratios; the temperature ratio is shown to have no effect on the plenum Mach number. As expected though, when \( D_p = D^* \), the theoretical Mach number is 1.0, which decays with decreasing \( D^* \). To the contrary, the freestream velocity in the plenum is shown in Fig. 3b to depend significantly on temperature ratio \( (T_o/T_\infty) \), and so it is expected to play a pivotal role in the estimates for the wall shear stress.

\(^b\)These values are taken from Table 1-3 of White\(^16\)
The plenum pressure ratio and theoretical Mach numbers at the nozzle exit are shown in Fig. 3c,d alongside the gauge pressure that is expected to be measured at the wall. This is for $T_o/T_\infty = 1.0$ and is shown to be unaffected by the size of the nozzle throat. The findings are provided to help guide the selection of pressure transducers that will be required of this new test stand.

![Graphs](image_url)

Figure 3: (a) Diameter Ratio for a range of plenum Mach numbers. (b) Average velocity in the plenum for all diameter ratios and exit Mach numbers. (c) Plenum pressure ratio for a range of plenum Mach numbers. (d) Theoretical Mach number at the nozzle exit and total ($P_o$) and static ($P_s$) pressure in the plenum expressed as gauge pressure

B. Expressions for Estimating Entrance Length

Entrance length formulations are necessary for determining the axial location where the flow becomes fully developed and its profile unchanged. For turbulent pipe flow, this is estimated using

$$\frac{x_L}{D} \sim 4.4Re^{1/6}$$

and requires $Re_D \geq 4000$ to ensure that the flow is turbulent. Reynolds number and mass flow estimates are shown in Fig. 4a for three nozzle diameters ranging from $D^* = 1.0$ in. to 2.0 in. and for $T_o/T_\infty = 1.0$. Here we see the dependence of Reynolds number on the centerline velocity with values appearing greater than $1 \times 10^6$. Entrance lengths are shown in Fig. 4b (bottom) in dimensionless form (top) and based on a pipe diameter of $D_p = 2.9$ inch. Given the size of one’s research space, one must be mindful of the practical considerations when choosing the pipe diameter.
C. Expressions for Estimating Pipe Friction

With all relevant gas properties known, we must establish expressions for estimating the wall shear stress inside the plenum. In dimensionless form, the wall shear stress becomes the popular friction coefficient,

\[ C_f = \frac{\tau}{\rho u^2} \]  

which depends on a great number of factors including pressure gradient, surface roughness and flow state. If the flow state is turbulent, which occurs most often in pipes when \( \text{Re}_D \geq 4000 \), various curve fits are available for estimating \( C_f \) depending on whether the pipe wall is smooth or rough. All expressions assume that the flow is fully developed (expressions for estimating entrance lengths were provided earlier). One of the earliest formulations for smooth wall turbulent flow data between \( 4000 \leq \text{Re}_D \leq 10^5 \) is from Blasius and is expressed as

\[ C_f = \frac{0.0791}{\text{Re}_D^{0.25}} \]  

However, at higher Reynolds numbers where the law of the wall is an appropriate curve fit for the viscous sublayer and wake, the expression by Prandtl is more appropriate and is defined as follows

\[ \frac{1}{\sqrt{\Lambda}} = 2.0\log(\text{Re}_D\sqrt{\Lambda}) - 0.8 \]  
\[ \frac{1}{\sqrt{\Lambda}} \approx -2.0\log_{10}\left(\frac{\text{Re}_D\sqrt{\Lambda}}{3.7 + \frac{2.51}{\text{Re}_D\sqrt{\Lambda}}}\right) \]

where \( \Lambda = 4C_f \) and is valid for smooth wall turbulent pipe flow for any Reynolds number greater than \( \text{Re}_D \geq 4000 \).

Turbulent flow in rough pipes requires a different formulation which is based on the average roughness height \( k \). For sand grain pipe-friction, Prandtl-Schlichting developed the following classical relationship,

\[ \frac{1}{\sqrt{\Lambda}} \approx 2.0\log\left(\frac{\text{Re}_D\sqrt{\Lambda}}{1 + 0.1(k/D)\text{Re}_D\sqrt{\Lambda}}\right) - 0.8 \]

which should be good over the entire turbulent-flow regime. For \( (k/D)\text{Re}_D < 10 \), roughness is believed to be unimportant, whereas \( (k/D)\text{Re}_D > 1000 \), a fully rough flow ensues and is independent of Reynolds number. For commercial pipes where the roughness behavior is somewhat different, Colebrook developed a surrogate formula,

\[ \frac{1}{\sqrt{\Lambda}} \approx -2.0\log_{10}\left(\frac{k/D}{3.7} + \frac{2.51}{\text{Re}_D\sqrt{\Lambda}}\right) \]
which has become the foundation for the well known Moody charts. The findings from this model are
presented in Figs. 5 and 6 for three nozzle throat diameters $D^* = 1.0$ in., 1.5 in. and 2.0 in., over a range of
pressure ratios at $T_0/T_\infty = 1.0$. A roughness height of $k/D_p = 0.0673$ has been used for this exercise.

Figure 5: Commercial rough pipe (solid line), rough wall pipe (dashed line) and smooth wall pipe (dash-dot
line) calculations. (a) Pressure differential over $L_{12} = 1$ meter length of pipe expressed in psi and as a
percentage of static wall pressure. (b) Shear stress acting on the $L_{12} = 1$ meter length of pipe.

Figure 6: Commercial rough pipe (solid line), rough wall pipe (dashed line) and smooth wall pipe (dash-dot
line) calculations. (a) Shear force acting on the $L_{12} = 1$ meter length of pipe. (b) Hoop stress distribution
through 3-inch sch-80 pipe at 700 psig based on yield strength for 304 stainless steel of $f_y = 215$ MPa.

III. Facility Design & Construction

A. Overview

The new calibration stand was designed and fabricated in house at the Applied Research Laboratories at
The University of Texas at Austin (ARL-UT) where measurements were conducted. The facility is designed
to deliver pressurized gas to any one of the four independently operated test stands that are housed within
an anechoic structure. A plan view of this facility is shown in Fig. 7 with the new calibration stand shown
installed on Test Stand 4. The compressed air system functions as a blow down type with pressurized gas
being stored in four DOT-107A pressure vessels comprising 180 cubic feet of water volume storage at 2,100
psig. Compressed air is provided by way of a 4 stage, 100 horsepower Bauer compressor (model I280-100-E3)
that outputs 103 SCFM at 500 psig (86 SCFM at standard inlet conditions). The working fluid for these measurements is dry unheated air (total temperature, $T_o = 304$ K, ratio of specific heats, $\gamma_j = 1.4$, specific gas constant of air, $R = 287.05$ J kg$^{-1}$ K$^{-1}$), though a pair of auxiliary pressure vessels (20 cubic feet of water volume storage per vessel) are available to accommodate other gas types (helium, pure nitrogen, etc.). The operating conditions of each of the four test stands (nozzle pressure ratio and temperature ratio) are monitored and controlled using a National Instruments CompactRIO system capable of reducing variations in the set point to less than 1% over extended periods of time. A description of some of the components in this new facility are provided by Valdez and Tinney,$^{17}$ Baars and Tinney$^{18}$ and Donald et al.$^{19}$

![Figure 7: Plan view (to scale) of the high-speed fluid dynamics lab at the Applied Research Laboratories at The University of Texas at Austin (ARL-UT).](image)

The construction of the calibration stand is quite simple and comprises three short ($\sim 20$ in.) and two long ($\sim 119$ in.) sections of pipe that are fastened using class 600 flanges. The primary features that make up this new test stand are shown in Fig. 8. The settling chamber and test section are machined from 6061 aluminum while the piping (extruded schedule 80 seamless pipe) and flanges are composed of 304 stainless steel. A dense honeycomb flow straightener is installed in the settling chamber (400 cells per square inch) in order to remove swirling motions from the isolation valve and upstream piping. Flange faces are custom machined to have seamless junctions between the pipe and flange as well as provisions for male/female connections for accurate alignment. O-rings are used to seal the flange faces while all sections of pipe were assembled and then honed using a flexible silicon carbide precision-finish cylinder hone. While the stainless steel pipe wall has a smooth finish relative to black carbon steel pipe, the honing was performed to ensure that adjoining pipe seams were smooth. A exhaust nozzle is configured using a series of orifice plates that can be stacked together to either increase of decrease the nozzle throat diameter. The range of throat diameters are from $D^* = 1.00$ in. to 2.75 in. with 0.25 in. increments. Doing so, provides a quick and inexpensive way
of changing the throat diameter of the exhaust nozzle. A series of images of this new test stand are shown in Fig. 9.

![Diagram of the shear-stress calibration stand at ARL-UT](image1)

**Figure 8:** Schematic of the shear-stress calibration stand at ARL-UT. (a) Upstream flow conditioning section with isolation valve, flow straightener, settling chamber, subsonic contraction, and pipe plenum. (b) Downstream section with pipe plenum, test section, and stacked exhaust nozzles.

Two regions are of interest in this test stand. That is, the entrance length and fully developed regions where the static wall pressure is expected to decay nonlinearly and linearly, respectively. To gauge the locations of these two regions and to quantify wall friction effects, several wall pressure taps are inserted by welding thredolets at various positions long the length of the pipes (located upstream and downstream of the test section). The hole diameter that penetrates the pipe wall is 0.0625 inch while the locations of these holes are provided in Table 4. The length of the removable test section is 8 in. and is designed to accommodate an assortment of shear and pressure sensing instruments.
Table 4: Location of static pressure taps, flange faces and sensor ports relative to the origin at the end of the subsonic contraction.

<table>
<thead>
<tr>
<th>type</th>
<th>without instrument holder</th>
<th>with instrument holder (8.25 in. length)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x$ [in.]</td>
<td>$x/D_p$</td>
</tr>
<tr>
<td>flange face</td>
<td>0.00</td>
<td>0.00</td>
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<td>12.75</td>
<td>4.36</td>
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A preliminary series of measurements with this new tunnel were conducted in the spring of 2018 to quantify the range of shear stress values that it could produce. These measurements were performed using a total of five gauge pressure transducers and a barometric transducer. The gauge transducers, with a 0-69bars (0-1000 psig) range, are the GT16 series made by Steller (±0.1% of full-scale output accuracy). Given the output of these transducers, the accuracy is reported to be ± 1psi. These accuracies were verified by pressurizing all transducers several times using one of the high pressure gas lines in the ARL-UT lab. Findings from these preliminary tests are shown in Fig. 10. In Fig. 10a, the ramp up and ramp down process is shown whereby relative differences between transducers is within the line thickness of plot. Closer inspection, achieved by calculating the difference between all five pressure transducers and a standard reference transducer, is shown in Fig. 10b. Relative differences between all five gauge transducers is shown to be within ±0.1% over the entire 1000 psig range.

Several slow ramps of the new calibration tunnel to 500 psig (measured in the settling chamber) were then conducted using the larger diameter stackable nozzle \((D^* = 2.5)\). The larger nozzle equates to a larger plenum Mach number (around 0.5), according to Fig. 3a based on \(D_p/D^* = 1.170\). Therefore, a larger range of shear stress values are expected to form. In Fig. 11a, the drop in static wall pressure along the length of the plenum is illustrated for three different nozzle pressure ratios (NPR=10, 20 and 30). Each trend demonstrates how the static wall pressure decays linearly along the length of the plenum. Increasing nozzle pressure ratio is also shown to increase the rate at which the wall pressure drops, thereby confirming the findings from the predictions shown in Fig. 5a. The entrance length has not yet been verified. To demonstrate that the pressure decay is not entirely linear we illustrate the change in pressure \((dp/dx)\) in Fig. 11b between two points in the flow for a range of nozzle pressure ratios. The steeper trend corresponds to pressure differences between points located at pressure taps 5 and 1, \((\Delta P_{5,1})\) and then taps 5 and 2 \((\Delta P_{5,2})\). The trend is nearly identical for these two cases where the developing regions of the flow are expected to reside. On the contrary, the lower trend, corresponding to \(\Delta P_{10,6}\) and \(\Delta P_{10,7}\), exhibits a shallower slope, which is believed to be the effect of the growing boundary layer and the collapse of the potential core.

In Fig. 12a, the measured wall shear stresses are compared to the model predictions using Eq. (18)
Figure 10: Check of pressure transducer accuracy using closed pressure vessel. (a) Measured output pressure during ramp up and ramp down of pressure vessel. (b) Relative difference in pressure of all five transducers over 1000 psig range.

Figure 11: (a) Static wall pressure along the length of the plenum at three nozzle pressure ratios. (b) Change in pressure \( \frac{dp}{dx} \) at upstream and downstream locations in the plenum.
(Colebrook’s formula for commercial pipes) and a roughness height of $k/D_p = 2.17 \times 10^{-7}$. The measured shear stresses are calculated from $\Delta P_{10.6}$ and $\Delta P_{10.7}$ where the flow is assumed to be fully developed. The slope of this trend is found to be valued at 0.3 kPa/NPR. As for the shear force acting on this section of the plenum this is shown in Fig. 12b based on $\Delta P_{10.6}$ and $\Delta P_{10.7}$. Two trends are observed and are the result of the different lengths of pipe over which the shear stress is being integrated. That is, $\tau_{10.6}$ is acting on a longer section of the plenum denoted by $L_{10.6}$, whereas $\tau_{10.7}$ is acting on a shorter length of the plenum denoted by $L_{10.7}$. The slopes of these lines equate roughly to 0.47 lbf./ft./NPR.

Figure 12: (a) Shear stress based on $\Delta P_{5.1}$ and $\Delta P_{5.2}$ and then $\Delta P_{10.6}$ and $\Delta P_{10.7}$, compared to the model. (b) Shear stress based on pressure differences between pressure tap transducers located at pressure tap.

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References


