A Time-resolved Estimate of the Turbulence and Source Mechanisms in a Subsonic Jet Flow

Charles E. Tinney∗ Peter Jordan† and Joel Delville‡
Laboratoire d’Etudes Aérodynamiques, Université de Poitiers, France

André M. Hall§ and Mark N. Glauser¶
Syracuse University, Syracuse, NY, 13244, U.S.A.

The primary focus of this paper is to highlight the research activities that have been underway at two research institutions: (1) Syracuse University, in Syracuse, New York, USA, and (2) Laboratoire d’Etudes Aérodynamiques, in Poitiers, France. In recent years, a subset of the research activities at these institutions have been rooted in developing low-dimensional turbulence models to improve our existing understanding of the sound source mechanisms in high speed jet flows. The investigation discussed here involves a pressure-filtered volume reconstruction of the turbulence and source mechanisms of a Mach 0.60 jet. The model is time-resolved and is reconstructed using a purely experimental database.

I. Introduction

The difficulty of relating turbulent sources of sound to their respective signatures in the far field has been a challenging scientific problem since the advent of the second World War. Though many advances in aeroacoustics have provided new understandings of the sources mechanisms in high speed jet flows, many experimental and numerical limitations still remain; the intrusiveness of the instruments used to capture the turbulence dynamic, and the numerical accuracy necessary to model the acoustic source features. In an effort to circumvent these obstacles, some experimentalists have circuited towards studying the near field pressure region surrounding the turbulent jet, primarily because the features there have been found to relate the turbulent large-scale events of the flow to the sound that is observed in the acoustic field, far from the flow.

Briefly overviewing some highlights from a few of these studies, Petersen and Ko & Davies have demonstrated that the mixing layer and potential core regions of the jet are dominated by organized structures of concentrated vorticity that correlate well with the near field pressure. More recently, Arndt et al. decomposed the pressure field outside of the jet’s turbulent shear layer and found that it was dominated by the first few Fourier-azimuthal modes, suggesting that the near field pressure draws most of its energy from the first few turbulent velocity modes, in particular the axisymmetric and helical modes. However, in light of the decomposition of the jet’s turbulence [e.g. Glauser et al. and Citriniti & George], the Fourier-azimuthal structures have exhibited very rich behaviors with a preponderance of energy in the higher modes 4, 5, and 6, before the collapse of the potential core. These rich turbulent azimuthal modes have also been found in the higher Reynolds number and Mach number studies [see Ukeiley et al. and Tinney et al.] which have been shown to exhibit many other similarities with the lower Reynolds number and Mach number jet flows.

Where the jet’s sources of noise are concerned, Ukeiley & Ponton have recently confirmed that the overall sound pressure level is greatest after the collapse of the potential core, that high frequencies are

∗Post-Doctoral Fellow, LEA/CEAT, UMR CNRS 6609, 86036 Poitiers, France, Member AIAA.
†Research Scientist, LEA/CEAT, UMR CNRS 6609, 86036 Poitiers, France.
‡Senior Scientist, LEA/CEAT, UMR CNRS 6609, 86036 Poitiers, France.
§Ph.D. student, Dept. of Mech. & Aerospace. Eng., 151 Link Hall, Student Member AIAA.
¶Professor, Dept. of Mech. & Aerospace. Eng., 151 Link Hall, Associate Fellow AIAA.

Copyright © 2006 by Charles E. Tinney. Published by the American Institute of Aeronautics and Astronautics, Inc. with permission.

1 of 13
American Institute of Aeronautics and Astronautics Paper 2006-0621
emitted near the jet exit, and that the peak radiating acoustic frequency is emitted along a polar angle of 30\(^\circ\) with respect to the jet centerline. These investigations, as well as others, [Winant & Browand\textsuperscript{23}, Flowes Williams & Kempton\textsuperscript{14}, Citriniti & George\textsuperscript{13}, Jung et al\textsuperscript{26} and Ukeiley & Ponton\textsuperscript{27}, Michalke & Fuchs\textsuperscript{28}] support the idea that the primary source of the jet’s noise is generated by the low-order structures of the flow, whose time scales are surprisingly the smallest, and whose transformation near the end of the potential core (described by Citriniti & George\textsuperscript{13} as “volcano like”) creates a particular low dimensional event that remains fairly stable in form as it convects well into the far field regions of the jet.

In this paper, we will present an overview of an effort undertaken to reconstruct a dynamical estimate of the flow field using pressure signatures outside of the jet’s shear layer via \textit{Spectral Linear Stochastic Estimation}. A particular feature of this approach is the capacity of the pressure field surrounding the turbulent jet to filter out a certain quantity of information concerning the turbulence structure. In effect, the signatures left in the pressure field by the turbulence are a result of the low order structure, the higher azimuthal modes being inefficient in driving the hydrodynamic pressure, Jordan et al\textsuperscript{13}. The robustness of the technique depends on the coherence of the large-scale structures that are found to govern a moderate to large percentage of the turbulence field, and the strong hydrodynamic pressure signatures that they emit. Thus, by generating a pressure-filtered dynamical estimate of the turbulence field, a better understanding of the mechanisms driving the production of sound, may be possible.

\section{Spectral Based Estimation}

The technique that has been adopted in this paper has been recently described in Tinney et al\textsuperscript{21} and employs the dynamic pressure field surrounding the jet exit, in conjunction with spectral estimation coefficients, to temporally reconstruct the axial turbulent velocity along many positions in the flow. This Spectral Linear Stochastic Estimation (SLSE) was first demonstrated by Ewing & Citriniti\textsuperscript{14} who showed remarkably accurate estimates of the jet’s \(r, \theta\) topology, when compared to single time estimates. The procedure follows the conditional estimation techniques of Adrian\textsuperscript{1}, whereby the dynamic condition at \(u(x', t)\) can be determined from an unconditional source \(u(x, t)\) at the same instant. When differences exist between the spectral features of the unconditional and conditional fields however, a spectral estimator is necessary in order to resolve the temporal characteristics of the conditional events. This is generally the case when the unconditional grid is not a subset of the conditional grid, when the unconditional and conditional physical quantities are not the same, or when both of these situations arise, (as is the case here). The form for the conditional estimate is given as follows:

\[ \hat{u}_i(x', f) = \hat{B}_{ik}(x'; f)\hat{p}_k(f) \]  

whereby the spectral coefficients \(\hat{B}_{ik}(x'; f)\) are derived using a mean square average of the cross spectral densities in (2):

\[ \langle \hat{p}_j(f)\hat{p}_k^*(f) \rangle \hat{B}_{ij}(x'; f) = \langle \hat{u}_i(x', f)\hat{p}_k^*(f) \rangle \]  

The spectral densities, \(\langle \hat{p}_j(f)\hat{p}_k^*(f) \rangle\) and \(\langle \hat{u}_i(x,f)\hat{p}_k^*(f) \rangle\), are rewritten for simplicity as \(\hat{S}_{ij}(f)\) and \(\hat{S}_{ik}(x'; f)\), respectively, where \(j\) and \(k\) are the number of unconditional parameters \(p\) (pressure), and \(i\) is a component of the conditional event (velocity). A tilde \(\hat{u}\) is used to distinguish the estimate from the original, and the hat on \(\hat{\cdot}\) means that the variable \(\hat{\cdot}\) is complex. To distinguish the coherent structures in the cross spectra from the incoherent noise, the estimation coefficients are only determined when the coherence spectra \((G_{ij})\) exceed a predetermined threshold value \((\Upsilon)\). This is shown analytically in (3) and was demonstrated in more detail by Tinney et al\textsuperscript{21}:

\[ \hat{B}_{ij}(x'; f) = \begin{cases} 0, & \text{if } G_{ij}(x'; f) < \Upsilon \\ \frac{\hat{S}_{ik}(x'; f)}{\hat{S}_{ik}(f)}, & \text{if } G_{ij}(x'; f) > \Upsilon \end{cases} \]  

The advantages to this approach are twofold. The first is that unlike the \textit{Linear} and \textit{Quadratic Stochastic Estimation} techniques, the spectral characteristics of the conditional event are preserved if differences exist between the spectral densities of the conditional and unconditional parameters. This is imperative since the unconditional field encompasses pressure signatures, surveyed outside the jet’s entrainment region, which are used to estimate the dynamic conditional velocity at various positions within the potential core and mixing layer regions of the flow. The second point of interest, for fields where the estimated condition is separated...
in time (typically by a convection velocity: \( \tau \propto \delta x/U_{\text{conv}} \), is that the time lag between the unconditional source \( p_j(t - \tau) \) and the conditional event, \( u_i(x, t) \), is embedded in the spectral estimator. Therefore, by employing a spectral based estimation procedure, whose coefficients are filtered using criteria determined by the field itself (coherence spectra), a low-order dynamical estimate of the velocity field can be ascertained to a higher degree of accuracy. This may be vital where modelling of the source terms is concerned.

III. Facility and Instrumentation

Measurements were conducted at Syracuse University’s fully anechoic chamber (206m\(^3\) enclosure) whose highlights are discussed in Tinney et al.\(^{22}\) The experimental details used in this study are described in Hall et al.\(^{11}\) and comprise a Mach 0.60 axisymmetric jet, exiting from a 50.8mm diameter nozzle at a Reynolds number (based on nozzle diameter) of \( 6.4 \times 10^5 \). The flow’s total temperature and bypass air temperature were matched and held constant at 298°K. An illustration of the jet facility, as well as the instrumentation (described below), is shown in figure 1c.

A. Pressure field

Several experiments were performed in the \( r, x \) plane of the jet’s near-field pressure region to quantify the sensitivity of the measurement field outside of the shear layer, to turbulent structures in the flow. A subset of these findings are shown in figure 1a. Here, the spectral wave energy of the pressure manifests a change in slope at \( kr_s = 2 \). This is in concurrence with Arndt et al.\(^2\) and shows a clear dissemblance between the reactive and propagative components of the pressure field. The intensity decay rate in the inertial subrange and acoustic "far-field" regions are displayed and are of order \( kr_s^{-6.67} \) and \( kr_s^{-2} \), respectively. Based on these findings, fifteen near field dynamic pressure transducers (Kulite XCE-093 model, 35kPa, 2.9\( \mu \)V/Pa sensitivity, and a DC to 50kHz dynamic response range) were oriented analogous to the jet exit along an azimuthal array at \( \frac{r}{D} = \frac{x}{D} = 0.875 \), as shown in figures 1b & c. The pressure spectral densities from the azimuthal array (sampled at 30kHz) demonstrated essentially identical features to those shown in figure 1a, wherein the near field comprises a superposition of reactive and propagative components.\(^3\) The analysis of Arndt et al.\(^2\) has been revisited more recently by Coiffet et al.\(^5\) Furthermore, Jordan et al.\(^{14}\) have shown that the Fourier-azimuthal spectra were unchanging along all axial positions in the flow that they investigated (\( \frac{r}{D} = 1 \) to 6).

![Figure 1.](image)

Figure 1. (a)Pressure wave spectra measured at \( \frac{r}{D} = 0.75 \). (b)Orientation of Kulite transducers with respect to the LDA measurement plane. (c)Image of LDV and near-lip pressure transducer experimental arrangement in chamber.

B. Velocity field

Single-point, time-resolved measurements of the velocity field were acquired simultaneously with the azimuthal pressure array using a Dantec Dynamics two-component Laser Doppler Anemometer (LDA) system.
The system employs a 3W argon ion laser head, and measurements were performed using backward scattering.Seeder pressure was adjusted to achieve LDA sampling rates comparable with that of the near-field pressure (between 30kHz and 40kHz), and the system was arranged to capture only the streamwise ($u_1$) component of velocity. The spatial grid over which the LDA measurement volume was traversed is shown in figure (1) and was chosen to capture critical interfacial regions of the flow (to be discussed later).

Profile measurements of the mean and second order moments of the axial component of velocity are shown in figures (2a & b), respectively, using $\eta = (r - R)/x$, where $x$ is the downstream distance from the nozzle exit plane, $R$ is the radius of the nozzle at the exit, and $r$ is the distance from the jet center. Beyond the first diameter, both sides of the shear layer (outer and inner) exhibit a linear rate of growth, with a peak in the r.m.s around 17% of the jet exit velocity at $\eta(x) = 0$ and spreading angles relative to the jet axis of 0.2x and $-0.1x$ (11.3°, and $-5.7°$), respectively. These are consistent with many other investigations [Hussain & Clark(12) Jung et al.(13) and Kerhervé et al.(13)]. Preliminary measurements have also demonstrated turbulence intensities at the exit of the nozzle on the order of 1% in the potential core. Auto-spectral densities of the axial velocity are shown in figure (2c) using a 10% bandwidth moving filter and demonstrate typical characteristics of the potential core and mixing layer regions of a jet flow.

![Figure 2. Axial velocity ratios of the (a) mean and (b) second central moments. (c) Velocity spectral densities at Mach 0.60.](image)

C. Grid interpolation

Lighthill’s(14) theory involves a reformulation of the compressible equations of motion into an inhomogeneous wave equation (1) for the propagation of acoustic waves, using source terms (5) that comprise the stress tensor $T_{ij}$. Although the original formulations have been refined, the theory still remains central.

$$\frac{\partial^2 \rho}{\partial t^2} - a_\infty^2 \frac{\partial^2 \rho}{\partial x_i \partial x_j} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{5}$$

$$T_{ij} = \rho u_i u_j + (p - \rho a_\infty^2) \delta_{ij} - \tau_{ij} \tag{6}$$

In particular, equation (6) can be simplified as generation of jet noise is an inviscid process ($\tau_{ij} \approx 0$) and for unheated jets, $p - \rho a_\infty^2$ is negligible, thus reducing the source tensor to a single term $\rho u_i u_j$. Assuming that the density field inside the source region is constant, (5) can be rearranged to produce an expression for the sound pressure $p'(\vec{x}, t)$ in the far field at a distance $|\vec{x} - \vec{y}|$ from the source $T_{ij}$.

$$p'(\vec{x}, t) = \int_V \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}(\vec{y}, t) \left( \vec{x} - \vec{y} \right) \frac{d\vec{y}}{a_\infty} \frac{4\pi|\vec{x} - \vec{y}|}{4\pi|\vec{x} - \vec{y}|} \tag{7}$$

Therefore, since calculating the acoustic source terms involves a double spatial divergence of the Reynolds stress field, $\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j}$, the resolution and accuracy of the turbulence model is crucial. As illustrated figure (1b), the coarse spatial grid from which the LDA measurements were performed comprises six axial stations containing seven radial positions encompassing the potential core and mixing layer regions of the flow (42
total positions). Thus, where accuracy is concerned, it would be impossible to calculate source terms from a dynamical estimate using this grid. However, the seven radial grid points at each axial station for the velocity field were specifically chosen in order to quantify the pressure-velocity cross spectra \((S_{1k}(x, r; f))\) along critical interfacial regions of the flow comprising, (1) the center of the potential core at \(\frac{x}{R} = 0\), (2) the outer edge of the potential core near its interface with the high-speed side of the mixing layer, (3) the interface between the potential core and the high speed side of the mixing layer at \(\eta(x) = 0.10\), (4) the high speed side of the shear layer near its interface with the potential core at \(\eta(x) = 0.05\), (5) the center of the mixing layer at \(\eta(x) = 0\), (6) the low-speed side of the mixing layer at \(\eta(x) = 0.1\), and (7) the interface between the low-speed side of the mixing layer and the outer entrainment regions at \(\eta(x) = 0.2\).

Figure 3. Dash-dot-line shows the interpolation of the maximum and minimum peaks of the cross correlations (arbitrary scaling of the correlation's magnitude) at (a) \(\frac{x}{R} = 0\) and (b) \(\frac{x}{R} = 1\). (c) Frequency band \((StD(f))\) used for the threshold filtering.

This means that \(S_{1k}(x, r; f)\) can be interpolated between the measurement points of the aforementioned grid in order to improve the spatial resolution of the dynamical model (estimate) since the critical interfacial regions of the flow have been quantified by experiments. The interpolation is performed on the pressure-velocity cross-correlation, \(S_{1k}(x, r; f) \rightarrow \psi_{1k}(x, r, \tau)\), using a two step process. The first comprises a cubic spline of the form shown in (8), using coefficients \(A, B, C\) and \(D\) determined from (9).

\[
\psi = A\psi_j + B\psi_{j+1} + C\psi''_j + D\psi''_{j+1}
\]

(8)

\[
A = \frac{x_{j+1} - x}{x_{j+1} - x_j} \quad B = \frac{x - x_j}{x_{j+1} - x_j} \quad C = \frac{1}{6}(A^3 - A)(x_{j+1} - x_j)^2 \quad D = \frac{1}{6}(B^3 - B)(x_{j+1} - x_j)^2
\]

(9)

Differentiating (8) with respect to \(x\), results in (10) which can be considered continuous across the boundaries by evaluating \(\psi = \psi_j\) in the same manner at both intervals \((\psi_{j-1}, \psi_j)\) and \((\psi_j, \psi_{j+1})\).

\[
\frac{d\psi}{dx} = \frac{\psi_{j+1} - \psi_j}{x_{j+1} - x_j} - \frac{3A^2 - 1}{6}(x_{j+1} - x_j)\psi'' + \frac{3B^2 - 1}{6}(x_{j+1} - x_j)\psi''_{j+1}
\]

(10)

After some rearrangement of (10) using the interval conditions, (11) results as an \(N - 2\) series of equations for \(N\) points.

\[
\frac{x_{j+1} - x_j}{6}\psi_{j-1} + \frac{x_{j+1} - x_j}{3}\psi_j'' + \frac{x_{j+1} - x_j}{6}\psi_{j+1}'' = \frac{\psi_{j+1} - \psi_j}{x_{j+1} - x_j} = \frac{\psi_j - \psi_{j-1}}{x_j - x_{j-1}}
\]

(11)

To complete (11) as a linear system of \(N\) equations, the second derivatives at the boundaries are prescribed by setting them equal to zero, that is, \(\psi_1'' = \psi_N'' = 0\). Once more, the system of equations results in a tridiagonal matrix which can be easily solved using a back-substitution algorithm. In the second approach, the new polynomial is simply resampled at a higher rate using a low-pass symmetric filter. When this process was completed, the final cross correlation matrix consisted of 120 axial, and 70 radial positions (8,400 total points), encompassing the entire near field region of the turbulent jet. A sample of this is demonstrated in figures 3a and b along the maximum and minimum peaks of the cross correlation \(\psi(x, \tau)\) at \(\Delta \theta = 0\). In this figure, the result of the interpolation of the maximum and minimum peaks is drawn (dash-dot-line), along...
with the original cross correlations (colored lines). As one can see, the trends follow the peaks of the original data quite well with no apparent over/under estimates of the data in between. Though the amplitudes of the correlations are arbitrarily scaled, one can see the persistence of the pressure-velocity cross-correlation, even after the collapse of the potential core.\(^{11}\)

To further appreciate the resolution that has been afforded by this technique, the results of the interpolation scheme are shown in figure 4 using a full spectrum of time delays at \(\Delta \theta = 0^\circ\) (between the pressure array and velocity field). As one can see, the interpolated correlations of the space time structure enhance the turbulent features captured by the original sparse grid. In particular, the greater resolution captures many additional features on the low-speed side of the shear layer at \(\eta(x) = 0.10\) in figure 4, where the higher azimuthal modes are typically known to exist. The radial correlations in figure 5 demonstrate a demarcation between the potential core and mixing layer regions up until the core collapses. Also, the correlation of the pressure field with the velocity field in the potential core is shown to be in quadrature with the correlation in the mixing layer as was shown by Lau \textit{et al.}\(^{17}\) In figure 5b, the time between sensing the arriving and departing structure increases with axial distance from the nozzle exit. Once more, the topology at \(\frac{r}{D} = 1\), manifest shorter time scales (high frequency events) in the potential core, and larger time scales (low frequency events) in the shear layer and entrainment regions of the flow. After the collapse of the potential core at \(\frac{r}{D} = 5\), the space-time correlations assimilate many features across the entire shear layer.

Since the LDA measurement grid encompasses the radial and axial plane of the jet, and the pressure array is along an azimuthal array, the cross spectral densities employed in the \textit{SLSE} are capable of performing a full volume estimate of the jet turbulence. In order to ensure that only the coherent frequencies are included in the spectral estimate, the threshold filtering from (3) is incorporated. Figure 3 illustrates the threshold frequency bands that are included in the model estimate, as a function of spatial position \((x, r)\) in the flow. From this, one can see that the highest coherence threshold frequencies are near the lip of the nozzle. Though the peak frequencies decay slightly towards the center of the potential core, the lowest threshold frequencies are shown to occur along the entrainment regions of the mixing layer, as should be expected.

**IV. A pressure filtered dynamical estimate of the jet turbulence**

A dynamical estimate of the turbulence field is realized using the time resolved measurements of the near field pressure array. This is effected by estimating slices along the axial and radial plane of the jet at fifteen discrete azimuthal stations, using a symmetric succession of all \((k = 15)\) pressure signatures before and after the plane that is estimated \((\Delta \theta_{x} = -168^\circ, -144^\circ, \ldots, -24^\circ, 0^\circ, 24^\circ, \ldots, 144^\circ, 168^\circ)\).

In figure 5a, the model’s \textit{r.m.s} velocity profiles manifest a peak along the center of the shear layer at \(\eta(x) = 0\) and show very similar features to the \textit{LDA} measurements illustrated in figure 2b. The overall amplitude is shown to be roughly 30% of what was acquired with the \textit{LDA} system, which corresponds to correlation levels of that order. Spectral estimates are shown in figures 5b & c, calculated along the center of the potential core and mixing layer regions of the flow at \(\frac{r}{D} = 3\). Comparisons are made with the \textit{LDA} measurements and the results demonstrate how the characteristic frequencies of the jet have been captured, with a much more pronounced peak in the potential core region and a broader distribution of scales in the shear layer. Thus, the pressure field is acting to filter out many of the turbulent features stated earlier, while still preserving the characteristic large scale structure of the jet. The importance of these features where sound production is concerned, remains to be established experimentally, but there is evidence in the literature that they are less important in driving the far-field.

A time resolved reconstruction of the velocity field is shown in figure 6 using a continuous sequence of twelve time steps to demonstrate the dynamic features of the turbulence along a slice in the radial and axial plane of the flow. The time step increment is determined by the sampling frequency of the pressure array \((f_{s}^{-1} = \Delta t = 3.33 \times 10^{-5}s)\). In particular, the potential core region is clearly visible by the lack of any structures, and appears to collapse around five jet diameters from the exit. The shear layer, however comprises a series of compact, quasi-periodic structures that initiate near the nozzle lip (before \(\frac{r}{D} = 1\)) and grow in space as they convect downstream. At \(3\Delta t\), a series of counter rotating events are shown to occur on the upper half of the shear layer \((\frac{r}{D} = 0.5)\) between \(\frac{r}{D} = 1\ & 3\), and continue to move downstream as they transfer energy from the high speed irrotational core to the entrainment region.
Figure 4. (a) The axial space-time correlations employing a sub-grid interpolation at various radial positions in the flow and \( \Delta \theta = 0^\circ \), and (b) along the radial direction.
Figure 5. (a) The model’s r.m.s. profiles. Spectral estimates of the model (dashed line) juxtaposed surveys from the LDA system (solid line) at $\frac{x}{D} = 3$ and (b) $\frac{x}{D} = 0$, (c) $\frac{x}{D} = 1$.

Figure 6. Time series reconstruction of the streamwise component of velocity along a radial and axial slice in the flow from a Mach 0.60 jet where $\Delta t = 3.3 \times 10^{-5}$ s.
The acoustic source field is calculated and analyzed. The reconstruction is performed using Fourier azimuthal modes of the flow. Here, it is seen that most of the azimuthal energy is centered around the jet, is driven by the low-ordered structures in the flow.

In this section, a low-dimensional, pressure-filtered estimate of the velocity field is reconstructed from which the jet, is driven by the low-ordered structures in the flow. The reconstruction is performed using Fourier azimuthal modes of the first few POD modes, demonstrate a preponderance of energy in the axisymmetric and helical modes, with very little energy above the jet, is driven by the low-ordered structures in the flow. In light of the investigation’s objectives, it is conclusive from these findings that the pressure field immediately surrounding the jet, is driven by the low-ordered structures in the flow.

To further demonstrate this conclusion, the maximum correlation from the two-point spatial correlation \( \rho_{uu}(r, \theta, x) = \langle u(r, x, t) u(\theta + \Delta \theta, t + \tau) \rangle \) is plotted in figure 14 along various axial and radial positions in the flow using the first 6 azimuthal separations (\( \Delta \theta = 0^\circ \) to \( 120^\circ \)). It is clear from this illustration that the most influential region of the flow where the pressure field is concerned, is along the outer region of the potential core, and its interface with the high speed side of the mixing layer (\( \eta(x) \approx -0.1 \)), where the correlation coefficient is shown to be of order 30% [13].

The results of this are demonstrated in figure 5 using only the first POD mode to reconstruct the first four azimuthal modes of the flow. Here, it is seen that most of the azimuthal energy is centered around the jet shear layer (\( \theta = 1 \)), and is in contrast to the jet’s modal behaviors recently demonstrated by Jordan et al. [13].
number of eigenfunctions from the decomposition.

\[ \tilde{u}_n^m(r, m, x, t) = \sum_{n=1}^{\infty} \hat{a}_n(m, x, t) \phi_1^{(n)}(r, m, x) \]  

(14)

with

\[ \hat{a}_n(m, x, t) = \int_D \tilde{u}_n(r, m, x, t) \phi_1^{(n)}(r, m, x) r dr \]  

(15)

A sample volume reconstruction of four continuous time steps between \( \frac{x}{D} = 1 \) and 6 is shown in figure 8. At each time step (sub-figures a through d), the source field (right column) results from a direct calculation (6th order accurate compact finite difference scheme) of \( \frac{\partial^2 u_1}{\partial r \partial \theta} \) using the low-dimensional turbulence field (left column) which is reconstructed using a discrete number of modes. The mode numbers \( (n, m) \) are shown in the top left hand corner of the turbulence model and the time step increment is now twice the sampling frequency of the pressure array (\( \Delta t = 6.67 \times 10^{-5} \)s). Iso-surface values were determined using the peak r.m.s values of the velocity and source model (in the first row of each sub-figure), whose reconstruction employs a summation of the first five azimuthal modes \( (m = 0 \) to 4). The light and dark colors correspond to positive and negative motions, respectively. Only the first POD mode \( (n) \) has been used. Following these four sequential frames, the superposed estimate (first row) manifests a modicum of information pertaining to the low-order modes of the turbulence field (left) and its corresponding source field (right). To gain a better grasp of the dynamical characteristics of the individual building blocks of this low-dimensional model, the discrete modal reconstructions are shown using azimuthal modes 0,1 and 2. It can be seen how contributions

Figure 7. (a) Eigenspectra from the modal decomposition of the pressure-filtered turbulence model. (b) Surface contour of the peak pressure-velocity correlation along the axial and radial plane of the jet and the first 6 azimuthal separations.

Figure 8. Eigenvector reconstructions of the first few Fourier-azimuthal modes of POD mode 1.
from the different modes are intermittent, (spatially and temporally). An example of this is demonstrated by spatial gaps in the mode 0 reconstruction when compared to the superposed model, which comprises a contribution from other more energetic modes of the flow.

Figure 9. Time series reconstruction of the low-dimensional pressure-filtered axial component of the turbulence (column 1) and source (column 2) fields at (a) $1\Delta t$, (b) $2\Delta t$, (c) $3\Delta t$, (d) $4\Delta t$.

Looking at the behaviors of the source field, small wave number, high frequency sources are understandably seen to occur mostly near the jet nozzle, and grow as they convect downstream. Therefore, while the acoustic field near the jet exit is known to comprise high frequency motions, it is affected by the lowest dimensional events of the flow. Likewise, the source terms calculated from the discrete turbulent azimuthal modes demonstrate similar features to their respective turbulence counterparts. Since the arrangement of the LDA measurements captured the axial component of velocity, the source field could only be calculated for one of the nine total source terms, $\frac{\partial^2 u^2}{\partial x^2}$. The same model approach is currently being investigated using the measurements of Tinney who performed stereo ($u, v$ & $w$) PIV measurements of a Mach 0.85 axisymmetric jet. The results from this will allow a calculation of the full Lighthill source term, from which the acoustic far field can be estimated more accurately. This estimate will then be compared to the far field acoustic surveys that were performed synchronously with the PIV measurements.
The Proper Orthogonal and Fourier decompositions of the estimated velocity field permit an analysis of how energy is distributed amongst the various low-order modes of the flow. The characteristics of this distribution have come to be well known where the velocity field of the round jet is concerned [Citriniti & George[3]Timney et al.[23] and Ukeiley et al.[23]], but the structure of the corresponding low-order source field is at present not so well understood. The turbulence and source estimates presented in this work thus provide a valuable opportunity to compare this well-known turbulence structure with the source structure it induces, and to thereby better understand the relationship between the velocity and the source fields, whence useful clues can be obtained as to the nature of the mechanisms by which the turbulent kinetic energy of the flow is converted into sound energy. Of course it must be emphasized that for the moment there is an important limitation manifest in the fact that the source, as evaluated here, does not provide the radiating source structure, access to which will require further filtering operations which isolate the acoustically matched source wavenumber components [Crighton[6]]. This is an important objective for future work.

V. Conclusion

A detailed experimental investigation of a Mach 0.60 jet has been discussed in the context of acoustic source identification. The combination of methods that have been employed have afforded a 3-dimensional (full volume) time-resolved estimate of the axial component of the velocity field, using the pressure field surrounding the jet exit region as the unconditional event. This was performed using the spectral estimation techniques described in[24][25] and is possible due to the large coherence between the large scale features of the jet’s turbulence, and the hydrodynamic pressure field surrounding it. A decomposition of the model estimate was then performed to characterize the low-order turbulent kinetic features of the model. The results demonstrate a rapid convergence of the energy, where the radial POD structure is concerned (42% in the first POD mode) and a dominance of the axisymmetric mode, followed by a subsequent decay in energy with increasing azimuthal mode. When compared to the decomposition of the turbulent jet by Tinney et al.[23] the signatures left in the pressure field by the turbulence manifest the low order turbulent structure, the higher modes being inefficient in driving the near field pressure. From a low-order dynamical reconstruction of the turbulent jet, the source field is determined, and new interesting features are illustrated. This dynamical model of both the turbulence and its corresponding source field, can now be studied more extensively in order to better understand the sound source mechanisms.

References

14JORDAN, P., TINNEY, E., DELVILLE, J., COIFFET, F., GLAUSER, M.N. & HALL, A. 2005 Low-dimensional signatures of the
sound production mechanisms in subsonic jets: Towards their identification and control. 35th AIAA Fluid Dynamics Conference and Exhibit, Toronto, Canada 2005-4647.


16Ko, N.W.M. & Davies, P.O.A.L. 1971 The near field within the potential cone of subsonic cold jet. J. Fluid Mech. 50, part 1, 49-78.


20Petersen, R. A. 1978 Influence of wave dispersion on vortex pairing in a jet, J. Fluid Mech. 89, 469-495


