The Evolution of the Most Energetic Modes in a High Subsonic Mach Number Turbulent Jet

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Low dimensional techniques are applied to the compressible turbulent mixing layer in the sound source regions of the flow from a cold (75°F) Mach 0.85 jet (z/D=3 to 8) using POD and Fourier decomposition. Measurements are acquired along the streamwise cross plane (r, θ) using a multi-component PIV system with an azimuthal grid spacing of 10° to prevent aliasing of the Fourier-azimuthal modes. The decomposition is performed using single, two and three component forms of the POD (u-streamwise, v-radial, w-azimuthal) applied in radius. Fourier decomposition is applied along the azimuthal direction because of the mean periodic nature of axisymmetric flows. The relative distribution of energy from this joint technique is shown to be consistent with Glauser & George5 and Jung et al.8 who used the scalar (u) form in the incompressible axisymmetric mixing layer, and Ukeiley et al.18 who used a vector form (streamwise and radial component) in the compressible Mach 0.30 & 0.60 axisymmetric jet. The dominant Fourier-azimuthal modes in the current investigation at z/D=3 and z/D=8 are $m = 5$ and $m = 2$, respectively, and is similar to the previous findings whereby the mean energy shifts to lower modes with the growth of the mixing layer.

Using the findings from this low-dimensional analysis, a Modified form of Bonnet et al.2’s Complementary Technique is employed to reconstruct temporally, the evolution of the joint technique’s expansion coefficients via Adrian’s1 Linear Stochastic Estimation.

I. Introduction

Proper Orthogonal Decomposition (Lumley9) is an effective technique for decomposing a stationary random field of vectors into a characteristic basis set from which the maximization of the ordered basis, defines the most probable vector from that set, in the mean square sense. This maximization relies on selecting a candidate event with the largest mean square projection on the vector field $\vec{u}$ (via the calculus of variations), and the normalization follows from $\langle |\alpha |^2 \rangle = \langle |\vec{u}, \phi \rangle ^2 / |\phi, \phi |^2$, where $<>$ denotes ensemble averaging. The kernel used in the maximization is constructed using Hilbert-Schmidt’s theory of integral equations with symmetric kernels, and the solution from the integral eigenvalue problem of (1) yields the orthonormal basis functions of this problem,

$$\int \int R_{ij}(\vec{x}, \vec{x}')\phi_j(\vec{x}')d\vec{x}' = \lambda \phi_i(\vec{x}) \quad (1)$$

where the kernel $R_{ij}$ is the velocity (ensemble averaged) two-point cross correlation tensor created from a series of discrete stationary turbulent flow events, $R_{ij}(\vec{x}, \vec{x}') = \langle u_i(\vec{x}, t)u_j(\vec{x}', t) \rangle$, and the eigenvalue $\lambda$ is equal to $|\alpha|^2$.

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For flows that are axisymmetric, or have mean azimuthal invariance, the kernel from \(_{(1)}\) can be constructed using Fourier decomposition in azimuth. Thus the solution from \(_{(2)}\) yields a set of basis functions with ordered POD modes (n) and azimuthal modes (m).

\[
\int_D B_{ij}(r, r', m, z_o) \phi_i^{(m)}(r', m, z_o) r'dr' = \lambda^{(m)}(m, z_o) \phi_i^{(m)}(r, m, z_o)
\]  

\(_{(2)}\)

Because the POD decomposition is performed independently for each Fourier-azimuthal mode, the azimuthal modes do not follow the order of the POD modes \((\lambda^{(1)}(m) > \lambda^{(2)}(m) > \lambda^{(3)}(m) ... > 0)\), and this “joint” technique has been shown to be most insightful for characterizing the relative kinetic energy of the various turbulent azimuthal structures in the flows from axisymmetric geometries. Tinney et al has recently shown this joint decomposition technique using the flow through an axisymmetric sudden expansion.

The use of these low-dimensional techniques has been shown to be a useful and effective tool for extracting the large-scale events in turbulent shear flows. The pioneering work of Glauser & George first applied POD in radius, and Fourier decomposition in azimuth to the axisymmetric mixing layer of an incompressible jet, \(Re=1e^6\). Citriniti & George and Jung et al later applied joint POD-Fourier-azimuthal decomposition and found similarities under a range of Reynolds numbers (incompressible) using the streamwise component only (scalar form). Ukeiley et al illustrated the dominant low-dimensional behavior of the flow from a Mach 0.3 and 0.6 axisymmetric jet along the sound producing regions of the flow (\(x/D=4\) to 8) using these joint techniques.

For the present investigation, the compressible turbulent mixing layer from an axisymmetric, cold (75°F) Mach 0.85 jet will be investigated using joint decomposition. The technique will utilize multi-component forms of the POD and although it is still two dimensional, whereby Fourier-azimuthal decomposition is applied along the second dimension, it should not be confused with the former (2-d scalar POD). Measurements will be confined to the sound source regions of the flow (\(x/D=3\) to 8) because of the relationship of this investigation to sound source identification described by Tinney et al. Optics based measurement tools, (PIV) are used to capture multi-point simultaneous measurements of the instantaneous velocity field. The Reynolds number of this flow at Mach 0.85 is of order 1e\(^6\), based on nozzle exit diameter.

### II. Modified Complementary Technique

The application of the Modified Complementary Technique (MCT) to turbulent flows is not new. Taylor & Glauser used surface pressure signatures to estimate the large scale events over an inclined back step geometry. Glauser et al like the investigations of Taylor & Glauser extended the investigation to estimate the turbulent events shedding from a NACA-4412 airfoil. These two investigations used an unconditional event (surface pressure signatures) to estimate the conditions in the flow based on a physical and quantifiable relationship between the two scalar events. The conditional estimate in these studies employed Adrian’s Linear Stochastic Estimation (LSE) whereby the conditional average \(\bar{u}(x', t) = <u(x', t)u(x, t)>\), created from a field of vectors, implied that the fluctuation at position \(x'\) could be estimated knowing the fluctuation at position \(x\) at the same instant. This LSE procedure yields the following estimate for \(\bar{u}(x', t)\),

\[
\bar{u}_i(x', t) = A_{ij}(x') u_j(x, t).
\]  

\(_{(3)}\)

Values for the estimation coefficients, \(A_{ij}(x')\), are chosen such that the mean square error is minimized: \(e_i = <[\bar{u}_i(x', t) - u_i(x', t)]^2>\) for \(i = 1, 2, 3\). This minimization requires that \(\frac{\partial e_i}{\partial A_{ij}(x')} = 0\), and leads to an equation of the form:

\[
\langle u_j(x)u_k(x)\rangle A_{jk}(x') = \langle u_j(x)u_i(x')\rangle
\]  

\(_{(4)}\)

where the estimated components are found from the expansion of \(_{(3)}\).

The use of MCT assigns the basis functions of the POD as the conditional events in the LSE and is a powerful approach for two reasons. The first is the use of the joint POD-Fourier-azimuthal based expansion coefficients which, by definition, represent a spatial field in the flow, as opposed to the single point estimates conventionally used with the stochastic techniques. The second is of particular interest to the current

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\(^4\)We will use the term “joint” to distinguish the low dimensional techniques that couple Fourier decomposition (applied to the homogeneous field) and POD (applied to the inhomogeneous field) from the stand-alone POD technique (applied to the entire field of interest).
investigation where the conditional event is not a property of the original function being estimated, but rather an unconditional event that has a suitable coherence with the condition. Naguib et al[11] and Murray & Ukeiley[13] like the work of Glauzer et al[6] used surface pressure measurements to estimate the conditional events of the flow above a flat plate and in a cavity, respectively. However, unlike Glauzer et al[6] these later investigations used a higher order Quadratic technique to improve the estimate’s accuracy.

In the present investigation, the pressure signatures in the near field hydrodynamic regions surrounding the flow from a compressible axisymmetric jet will be used as the unconditional events. This combines the scalar LSE with the joint decomposition so that the time-resolved instantaneous fluctuating pressure can be used to estimate the joint POD-Fourier-azimuthal expansion coefficients as are necessary for performing a low-dimensional estimate of the velocity field. Most importantly, the pressure signatures from these events can be captured outside of the sound source regions of the flow so that the natural near field acoustic characteristics from these sources (which propagate to the acoustic far field) are not disturbed! This will allow us to compare a low-dimensional dynamical reconstruction of the sound source events to the simultaneously sampled far field noise signatures as is the eventual motivation of this study. The modified technique, using the form of the joint decomposition, is broken down as follows.

- The random expansion coefficients from the solution of (12) are calculated from the basis functions at each statistically independent time step, \((t_s)\) using multi-component forms of the joint decomposition technique. The kernel is generated using select components \((c)\) of the velocity field via the PIV snapshots discussed in §III

\[
\hat{a}_n(m, z_o, t_s) = \int_D u_c(r, m, z_o, t_s)\phi_c^{(n)}(r, m, z_o)dr
\]

- From the simultaneous sampling of the hydrodynamic near field pressure (an azimuthal array of \(j = 15\) transducers near the jet’s lip) with the joint expansion coefficients calculated from the PIV measurements, a cross correlation matrix is formed (over all \(t_s\) instances for all \(n\) and \(m\)) and the cross spectral densities between these two scalar terms are computed.

\[
\hat{S}_j(n, m, z_o, f) = \int (p_j(t_s - \tau)\hat{a}_n(m, z_o, t_s))e^{-i(2\pi f \tau)}d\tau
\]

- Using the enormous time series data of the near field pressure, the cross spectral densities between all \(j\) and \(k\) transducers are determined, and the symmetric cross spectral correlation matrix is formed.

\[
\hat{S}_{jk}(f) = |\hat{p}_j^*(f)|\hat{p}_k(f)
\]

- Following the form of the LSE technique in [11], such that \([A][B] = [C]\), the linear estimation coefficients are determined, where \([B] = \hat{b}_j(n, m, z_o, f)\) and are complex.

\[
\hat{b}_j(n, m, z_o, f) = \frac{[C]}{[A]} = \frac{\hat{S}_j(n, m, z_o, f)}{\hat{S}_{jk}(f)}
\]

- Expanding the estimation coefficients with the spectral form of the pressure data (these transducers are sampled at bandwidths capable of resolving the spectral characteristics of the sound source events), an estimate of the joint expansion coefficients are realized at continuous time intervals.

\[
\hat{a}_n(m, z_o, t) = \frac{1}{2\pi} \int \{\sum_j \hat{b}_j(n, m, z_o, f)\hat{p}_j(f)\}e^{i(2\pi ft)}df
\]

- In the final step of the procedure, select modes \((n)\) are projected onto the joint expansion coefficients and the transformation from Fourier-azimuthal mode to azimuth provides a dynamical reconstruction of the more energetic events of the flow in the sound source regions.

\[
\hat{u}_c(r, m, z_o, t) = \sum_n \hat{a}_n(m, z_o, t)\phi_c^{(n)}(r, m, z_o)
\]

Coherence between the near field lip pressure and the joint expansion coefficients (velocity field) requires a crucial phasing coefficient \(\tau\) based on the transport speed of the turbulent events. By incorporating a full measure of the cross spectral relationship between the joint expansion coefficients and the near field pressure, the necessary spatial and temporal phase characteristics of the final event estimate are greatly improved.
\[ \bar{u}_c(r, \theta, z_o, t) = \frac{1}{2\pi} \int \bar{u}_c(r, m, z_o, t) e^{i(2\pi m\theta)} dm \]  

(11)

### III. Experimental Arrangement

Experiments were conducted at Syracuse University’s newly refurbished 206m³ fully anechoic chamber, shown in figure\[\text{1}\]. A detailed overview of the S.U. chamber and its characteristics can be found in Tinney et al.\[\text{16}\] In short, the measurements were acquired from a 2\text{inch} diameter axisymmetric nozzle at Mach 0.85 with a 2\% co-flow air at 75\text{o}F (jet and co-flow air temperatures). The stereo (3 component) PIV system used in the investigation was orientation to capture 1,250 statistically independent slices of the streamwise cross plane (originally a cartesian grid of 66 (x) by 51 (y) points, where \Delta x and \Delta y are 5.0e-2D and 6.6e-2D, respectively) along several streamwise positions in z (\Delta z = 0.25D) via. a single degree of freedom traverse. The laser sheet thickness is 5\text{mm} and the \Delta t between successive image pairs is 2\mu\text{s} when the particle’s speed (jet speed) is of the order of 300\text{ms}^{-1}.

![Figure 1. S.U. anechoic chamber with co-flow duct and experimental jet rig. Jet rig is shown instrumented with stereo PIV system.](image)

![Figure 2. Layout of jet and instrumentation in the chamber.](image)

The cylindrical grid used in the joint decomposition incorporated a total of 972 spatial points constructed from 27 radial (\Delta r = 5.8e^{-2}D) and 36 azimuthal (between 0 &2\pi) locations. The timing between PIV snap shots was 0.25s. Since the dominant time scales in the flow are of order 3e^{-3}s (St\text{D} \sim 0.6), the sampling rate of the PIV system was clearly shown to preserve statistical independence. This is important since we are dealing with a limited number of samples. An illustration of the experimental arrangement is shown in figure\[\text{1}\].

The near field pressure signatures necessary for performing the MCT are captured using an azimuthal array of fifteen Kulite XCE-093 model, 5psig pressure transducers. The diameter of the transducer’s body is 0.093\text{inches}, and are rear vented gauge transducers. The instrument’s response characteristics (20mV/psi sensitivity, 150kHz natural frequency, DC to 50kHz dynamic response range) were chosen specifically for this investigation, and for future investigations at elevated jet exit temperatures because of their operability at temperatures up to 525\text{o}F. Excitation to these instruments was provided by five model-136 Endevco signal conditioners (three channels per unit) and were digitized using a National Instruments PXI system (two NI-4472 boards for sixteen total channels). The probe holder’s design shown in figure\[\text{3}\] illustrates the transducer’s azimuthal distribution with \Delta \theta = 24\text{o}, and was motivated by the Fourier-azimuthal decomposition of the velocity field.

Typical mean and second order exit profiles of the streamwise velocity at Mach 0.85 are shown in figures\[\text{1}\].
and Mollo-Christensen et al. The similarity variable $\eta_s = 1 - (2r/D)/z$, where $z$ is the downstream distance from the nozzle exit plane and $r$ is the distance from the center of the mixing layer, has collapsed the profiles for all streamwise positions. A study of the grid’s sensitivity was performed using the original grid density ($\Delta \theta = 4^\circ$) and suggested no aliasing at this larger separation ($\Delta \theta = 10^\circ$). Higher azimuthal separations ($\Delta \theta = 15^\circ & 18^\circ$) were found to slightly alias the Fourier-azimuthal decomposition as suggested by Glauser & George by shifting energy to lower azimuthal modes. The investigations by Ukeiley et al. and Taylor et al. employed 10° azimuthal spacing increments for the Mach 0.3 and 0.6 jet study. This sensitivity study is illustrated in figures 6 and 7 for the first three POD ($n$) modes using (13) from § IV.

IV. Discussion of Results

Since the two point correlations form the basis of the decomposition technique, it would be appropriate to illustrate their characteristics under these flow conditions for completeness. Hence we will use this opportunity to display the Reynolds stresses (in cylindrical coordinates) from which the kernel in (2) is
created, and from which the basis functions \( \phi_k^{(n)}(r, m, z) \) are solved. These correlations contain the necessary spatial relationships, in a mean square sense, between the turbulent motions of the flow, and is what makes the POD and the joint decomposition techniques powerful tools in turbulence. Therefore, the cross correlation relation between these points can inscribe essential characteristics about the flow that is distinguishable from other large eddy identification techniques. These results were first presented by Ukeiley et al. using measurements from the Syracuse University facility (the jet’s r, θ plane) and from the Jamie Whitten National Center for Physical Acoustics (the jet’s r, z plane). The strength of the azimuthal and radial structure can be seen from these illustrations. Along the center of the jet’s mixing layer (r/R=1) at 5D in figures 8 and 9, the normal stresses are greatest. Towards the center of the jet, the azimuthal structure is shown to extend over the entire range between 0 and 2\( \pi \).

![Figure 8](image1.png)  
**Figure 8.** The \( R_{uu} \) correlation function, (a.) surface mesh of \( R_{uu}(r, r' = r, \theta_o + \Delta \theta, 5) \), (b.) of (a.) normalized by max. value in \( R_{uu} \), (c.) surface mesh of \( R_{uu}(r = 0.5, r', \theta_o + \Delta \theta, 5) \), (d.) of (c.) normalized by max. value in \( R_{uu} \).

In order to best illustrate the relative energy of the basis set, the eigenvalue solutions are normalized by the total energy at a given streamwise position in the flow using (12) and (13). The total number of modes in the joint scalar problem consists of \( n = 27 \) POD and \( m = 18 \) azimuthal modes (because of symmetry, the cosine transform is performed over \( M/2=m \) modes).

\[
\zeta(z) = \sum_n \sum_m \Lambda^{(n)}(m, z) \quad (12)
\]

\[
\Lambda^{(n)}(m, z) = \frac{\lambda^{(n)}(m, z)}{\zeta(z)} \times 100 \quad (13)
\]

Using solution variables just described, results of the scalar joint decomposition at the first few streamwise positions in the flow are shown in figure 12 and clearly articulates the dominance of the first POD mode. Figures 13 through 15 characterize the distribution of azimuthal modal energy (using the dominant first POD mode only). It is clear from these.

![Figure 3](image2.png)  
**Figure 3.** Image of final kulite arrangement for sampling the near field pressure region.
adds more energy to the dominant eigenmodes, as opposed to shifting energy to lower azimuthal modes. The findings are similar to those of Ukeiley et al., who studied the Mach 0.30 and 0.60 axisymmetric jet using two-component joint technique with streamwise ($\rho u$) and radial ($\rho v$) components of mass flux. From the investigations at the Syracuse facility, the dominant mean turbulent energy transforms from a higher mode number ($m = 5\&6$) near the jet lip, to a lower Fourier azimuthal mode number ($m = 2$) as events move past the potential core regions of the flow. It should be emphasized that the additional component of velocity (for instance, the radial component in figure 14 and the radial and azimuthal component in figure 15) merely adds more energy to the dominant eigenmodes, as opposed to shifting energy to lower azimuthal modes. The solutions to the two and three component vector joint-decomposition are shown in figures 14 and 15. The eigenspectra from $z/D=3$ to 3.75 of $n=1:3$ and $m=0:12$ using scalar joint decomposition, ($u$-component). 

Figure 10. The $R_{uv}$ correlation function, (a.) surface mesh of $R_{uv}(r = r', \theta_o + \Delta\theta, 5)$, (b.) of (a.) normalized by max. value in $R_{uv}$, (c.) surface mesh of $R_{uv}(r = 0.5, r', \theta_o + \Delta\theta, 5)$, (d.) of (c.) normalized by max. value in $R_{uv}$.

Figure 11. The $R_{uv}$ correlation function, (a.) surface mesh of $R_{uv}(r = 1.0, r', \theta_o + \Delta\theta, 5)$, (b.) of (a.) normalized by max. value in $R_{uv}$, (c.) surface mesh of $R_{uv}(r = 1.5, r', \theta_o + \Delta\theta, 5)$, (d.) of (c.) normalized by max. value in $R_{uv}$.

Figure 12. Eigenspectra from $z/D=3$ to 3.75 of $n=1:3$ and $m=0:12$ using scalar joint decomposition, ($u$-component).

Figure 13. Eigenspectra distribution, $z/D=3$ to 8, $n=1:3$ and $m=0:14$, scalar joint decomposition, ($u$-component).
similarity to Ukeiley, et al is impressive if one appreciates that the results are obtained using two different facilities, under slightly different conditions, using a completely different arrangement of instrumentation. These findings also suggest a remarkable similarity in the structure from the incompressible jet studies of Jung et al and Glauser et al.

Figure 14. Eigenspectra distribution, \(z/D=3\) to 8, \(n = 1 : 3\) and \(m = 0 : 14\), joint decomposition (u&v components).

Figure 15. Eigenspectra distribution, \(z/D=3\) to 8, \(n = 1 : 3\) and \(m = 0 : 14\), joint decomposition (u,v&w components).

Figure 16. Relative energy distribution, \(z/D=3\), Mach 0.85, joint decomposition (u,v&w-components).

Figure 17. Relative rate of convergence for the first POD and azimuthal modes, \(z/D=3\), Mach 0.85.

V. Data Convergence

The coefficient matrix \(\Lambda^{(n)}(m, z)\) from (13), if observed for all individual POD modes (during the summation of each POD mode over all azimuthal modes) and all individual azimuthal modes (during the summation of each azimuthal mode over all POD modes), is shown schematically in figure 16 at \(z/D=3\) using the three-component form (u,v&w) of the joint technique. Consistency in the convergence was found along all positions in the flow in \(z\). In particular, the convergence to nearly 100% of the total turbulent energy is achieved with only 20% of the azimuthal modes \((m)\), and about 7% of the total POD modes \((n)\), which constitutes only 2% of the total number of modes. These findings are similar to those discussed by Citriniti & George, Tinney et al, and Glauser & George.

An individual characterization of the modal convergence using the joint techniques is shown (figure 17) similarly to figure 16. The convergence of the 2-d scalar POD is also shown along side. The 2-d scalar

\[\Lambda^{(n)}(m, z)\]
POD assumes that the entire flow is inhomogeneous and the POD is applied over the entire 2-d cross plane absent of the Fourier methods which have been typically applied to flows with mean azimuthal invariance. From this, we can see that the Fourier solution does not converge as rapidly as the POD solutions. This is most likely a result of the order of the eigenvalues \((\lambda^{(1)}(m) > \lambda^{(2)}(m) > \lambda^{(3)}(m)... > 0)\) whereby the more energetic azimuthal mode is not necessarily the first. In fact as the solution moves downstream to \(7D\) (not shown), the convergence of the Fourier-azimuthal modes becomes more reflective of the POD modes because of the shift in Fourier energy.

The magnitude of the cross correlation \(\hat{\rho}(n,m,\tau,z,j)\) used to create the cross spectral density function in (1) is shown. This is normalized by the product between \(\sigma_p(\tau,j)\) and \(\sigma_p(n,m,z)\), where the modulus of the expansion coefficient \(\sqrt{Re\hat{a}(n,m,z)^2 + Im\hat{a}(n,m,z)^2}\) and of the cross correlation function \(\sqrt{Re\hat{p}(\tau,j)^2 + Im\hat{p}(\tau,j)^2}\) are used. Results are shown averaged over all fifteen transducers and clearly articulates a decent correlation of 40% with the zeroeth azimuthal mode in figure 13 and 20% with the second azimuthal mode in figure 19. The first POD mode \((n = 1)\) is used in both instances. The convection speeds of these events, based on the space-time locations of the first peaks, is \(0.77U_{cl}\) and \(0.67U_{cl}\) for \(m = 0\) and \(m = 2\) modes, respectively. The convection speed for the \(m = 1\) mode (not shown) is \(0.73U_{cl}\). The trend suggests that the higher (smaller) Fourier-azimuthal modes convect at slower speeds, as oppose to the larger modes that move much more rapidly. Glauser et al and Taylor et al were able to perform the decomposition using a cross-spectral tensor because of the CTA instruments employed in their investigation. From this they were able to illustrate that at Mach 0.6, the peak frequency of the \(m = 0\) mode was \(1.6kHz\) with a peak in the \(m = 8\) mode around \(200Hz\) and is similar to the trends observed with the space time correlation function in the current investigation. These are complementary findings that are difficult to interpret under the classical theory of aerodynamically generated sound and will be investigated more thoroughly in the future.

VI. An Evolutionary Estimate of the Most Energetic Modes

From the basis functions of the decomposition, a low-dimensional reconstruction of select PIV images is performed. Recall that the time difference between images exceeds several time scales of the flow, as is the motivation for employing the MCT techniques. Using the 2-d scaler POD, we are still capable of gaining some insight into the original characteristics of the turbulent events from this flow.

Since the stereo PIV system is capable of capturing all three components of the velocity field along several streamwise cross-plane positions in the flow, we can illustrate the normal streamwise components \((v(x)\) and \(w(y))\) using the original cartesian grid. This is shown in figures 20 and 21 at \(z/D=3\) and 8, respectively and is similar to the next series of illustrations, including the time evolution estimate of the most energetic modes at the end of this section.

In figure 22 original PIV cross-plane snap shots of the fluctuating velocity field \((u\text{-component})\) at Mach
Figure 20. Original PIV cross-plane snap shot of $v(x)$ and $w(y)$ velocity components at 4 statistically independent time steps at $z/D=3$ and Mach 0.85.

Figure 21. Original PIV cross-plane snap shot of $v(x)$ and $w(y)$ velocity components at 4 statistically independent time steps at $z/D=8$ and Mach 0.85.

Figure 22. Original PIV cross-plane snap shot at 4 statistically independent time steps at $z/D = 3$ and Mach 0.85.

Figure 23. Low-dimensional reconstruction of figure 22 using 2-d scaler POD at $z/D = 3$ with 100 POD modes (3% energy).
0.85 and z/D = 3 are shown at four discrete time steps. Next to this is figure 23 which uses the raw images from figure 22 to reconstruct a corresponding low-dimensional picture of the flow. A striking resemblance between the two figures illustrates the capabilities of the low-dimensional technique to capture the key features of the flow. The events are shown to possess the azimuthal behaviors found in flows from axisymmetric geometries. At this streamwise position, the relatively small turbulence levels in the core regions of the jet are visible. Further downstream, at z/D=8 in figures 24 and 25, similar behaviors are seen. Here the shear layer is shown to have grown noticeably, thus engulfing the potential core. The turbulent scales also cover a much broader region of the flow.

An estimate of the most energetic events in the sound source regions of the Mach 0.85 jet are shown using the scalar (u-component only) form of the joint technique in (11). Because the near field pressure is sampled at rates (fs = 30kHz) comparable to the most energetic events of the flow (∼ 3kHz), the timescales are easily realized. Keeping in mind that the near field pressure field has one single spectral characteristic, thus, the linear estimation coefficients from (5) contain the necessary cross spectral information to preserve the natural spectral characteristics of the modal events. Figure 26 is a reconstruction using the first POD mode and Fourier-azimuthal modes 0 through 9 at an initial time step t0. Two time steps later at 2Δt = 6.7e−5s in figure 27, the same events are shown and have evolved.

In these figures, we can see that there is a multitude of modal events at 3D and that their relative size is small, similar to the raw and low-dimensional images shown in figures 20 through 25. As the events convect downstream towards 8D, they are shown to grow azimuthally and radially by shifting energy to the lower more energetic azimuthal modes at 8D. Comparing the events at 4D and 4.5D at t0, to the events at t0 + 2Δt, the structure in the lower quadrant of the image has rapidly erupted from a single characteristic event to two characteristic events. This small segment of the evolutionary model presented here illustrates an instant when energy is transferred from lower modal events to higher modal events. This instantaneous transfer of energy among the more energetic modes is thought to possess the signature features responsible for much of the local sources of acoustic energy.

To validate the spectral characteristics of the estimation technique, single point velocity measurements have been acquired using a stereo LDV system. The results from these measurements are presented by Hall et al (7) at slightly lower Mach numbers, 0.3 and 0.6 along several streamwise positions in the flow. The future objective will be to compare the spectral characteristics of the estimated events presented here, to the spectral measurements of Hall et al (7) at relative positions in the flow.
Figure 26. Low dimensional estimate of the streamwise velocity component at $t_0$ using $n=1$ and $m=0:9$ from near field pressure via MCT.

Figure 27. Low dimensional estimate of the streamwise velocity component at $t_0 + 2\Delta t$ using $n=1$ and $m=0:9$ from near field pressure via MCT.
VII. Conclusion

The most energetic modes of a Mach 0.85 jet were examined using multi-component forms of the joint POD-Fourier decomposition techniques. Measurements of the velocity field (r, θ cross-plane) were performed using a stereo PIV system traversed along the sound source regions of the flow (z/D=3 to 8, Δz = 0.25D). The results indicated a dominance in the m = 5 Fourier-azimuthal mode at 3D, with a shift to the m = 3 Fourier-azimuthal mode at 5D. A grid sensitivity indicated that the inclusion of additional components in the decomposition were not shown to shift the energy amongst modes, but rather change the relative energy in each mode, thus supporting previous investigations by Citriniti & George, Jung et al and Glauser & George who used the scalar decomposition in the axisymmetric mixing layer (incompressible) and Ukeiley et al who used a vector form (streamwise and radial component) in the compressible Mach 0.3 & 0.6 axisymmetric jet. By employing a cross-spectral-based modified form of Bonnet et al Complementary Technique, a dynamical estimate of the sound source events were realized. This first order low-dimensional estimate used the near lip (hydrodynamic) pressure signatures as the unconditional events because of their small bandwidth capabilities, (sampled at 30kHz) in order to overcome low sampling speeds with conventional PIV systems. More importantly, the application of this technique has been shown to greatly reduce, if not eliminate, the intrusiveness on the acoustic characteristics of the sound source events.

Future investigations will use this low-dimensional model to evaluate the modal evolution of the Lighthill source terms in order to estimate the far field noise, and will be compared to a simultaneous survey (already performed) of the acoustic far field regions. From this model, we expect to determine the key signature events near the jet’s lip that are responsible, in an evolutionary sense, for the more energetic sources of noise. Eventual applications of these exciting findings will extract the necessary information for controlling these signature events in order to reduce their radiated noise.

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