Investigating the ‘modified’ Complementary Technique Using Pressure-Velocity Correlations of an Axisymmetric Jet

André M. Hall* and Charles E. Tinney†

Syracuse University, Syracuse, NY, 13244, U.S.A.

Mark N. Glauser‡

Syracuse University, Syracuse, NY, 13244, U.S.A.

To investigate the validity of the ‘modified’ Complementary Technique\(^\text{10}\) as applied to the axisymmetric jet, a quantitative measure of the strength of the cross-correlation between the pressure field near the nozzle exit \((z/D=0.875)\) and the velocity field is examined. The jet nozzle used in these experiments has an exit diameter of 5.08 cm, and is operated in the compressible flow regime at \(M=0.6\), at an exit temperature of \(25\) °C. The fluctuating pressure field is sampled by an azimuthal array of 15 dynamic pressure transducers, evenly spaced at 24 degrees apart. The velocity field is simultaneously captured using a single point LDA system, with a 3-Watt argon ion laser head, traversed along several streamwise and radial locations. The cross-correlation between the pressure and streamwise component of the velocity field is examined as a function of radial, streamwise, & azimuthal \((r,z, \theta)\) location. The greatest magnitudes, on the order of 25% correlation, are observed within the potential core region just outside the centerline. Streamwise convection velocities of \(0.77U_j\) and \(0.73U_j\) are calculated in the potential core and shear layer, respectively. In the future, a spectral comparison can then be employed to validate Tinney et al’s\(^\text{12}\) application of the technique to the turbulent sound source regions of the axisymmetric jet.

I. Introduction

The Complementary Technique introduced by Bonnet \textit{et al.}\(^\text{4}\) is a powerful low-dimensional mathematical modeling tool. It combines the strengths of both the Proper Orthogonal Decomposition (POD), Lumley,\(^\text{9}\) and Linear Stochastic Estimation (LSE), Adrian.\(^\text{1}\) The technique has recently been modified to incorporate knowledge of fluctuating surface pressure in the estimation of the velocity field, Taylor & Glauser.\(^\text{10}\) The idea stems from the use of Particle Image Velocimetry (PIV) to measure turbulent flow fields. These systems are multi-point in space, but single point in time. They are able to capture 3 components of velocity in an entire plane, however, due to low sampling rates (less than 20Hz), they are unable to resolve the time scales associated with high Re flows. To overcome this, the Complementary Technique is modified from it’s original form in the sense that the LSE is now used to build up a conditional average between the POD coefficients (acquired at discrete time step dictated by the PIV sampling rate) and the fluctuating pressure field. A time resolved estimate of the POD coefficients (having global knowledge of the flow field) can then be used to reconstruct a time resolved, low-dimensional, estimate of the flow field.

It is clear that this technique relies greatly on the premise that a strong correlation exists between the fluctuating pressure field and the velocity field. The work discussed herein is aimed at quantifying the strength of that correlation in the axisymmetric jet. An azimuthal array of pressure transducers will be
cross-correlated with the streamwise velocity component of a high Reynolds number flow, Re=6.4E5, and magnitudes examined. These experiments also provide a data set, which can be used to substantiate the 'modified' Complementary Technique. Velocity measurements are captured with the use of Laser Doppler Anemometry (LDA), a single point in space, but multi-point in time system. A continuous time series can be attained, and accurate spectral characteristics of the flow field can be resolved. This can then be compared to estimated spectra, evaluated in separate experiments in the same facility by Tinney et al.\textsuperscript{12} using the POD/LSE based 'modified' Complementary Technique.

II. Modified Complementary Technique

The use of the term 'modified' Complementary Technique is closely linked to the use of PIV to sample velocity. In the original context of the technique, the POD is applied to velocity data acquired by a limited number of hot wire probes (at least two) to resolve a basis set of eigenfunctions. The measured velocity field can then be projected onto these eigenfunctions to create the POD coefficients that now have global knowledge of the flow field. The term global here is however limited to that region over which measurements are acquired. To resolve the entire flow field, the LSE is first used to estimate velocity information at locations which have not been measured using a conditional averaging scheme applied to the measured locations. Once an estimated flow field is attained, this can then be projected onto the basis set of eigenfunctions obtained from measured data, containing the true physics of the flow field. The estimated POD coefficients obtained are then used to create a low-dimensional reconstruction of the flow field.

The more widespread use of PIV in the field of turbulence research in recent years has led to the development of a 'modified' form of the Complementary Technique. These systems have considerable spatial resolution, as they are able to capture all 3 components of the velocity field in an entire plane. However, they are limited in their sampling frequency, and therefore cannot produce time series to resolve phenomenon associated with high Re flows. Data of the form \( \vec{u}(\vec{x}, t_s) \), where \( t_s \) denotes a discrete time step dictated by the low sampling frequency of the PIV system is obtained. The integral eigenvalue problem that is POD (1) is then applied to this velocity data set to resolve a basis set of eigenfunctions \( \phi_i(\vec{x}) \),

\[
\int R_{i,j}(\vec{x}, \vec{x}') \phi_j(\vec{x}') d\vec{x}' = \lambda \phi_i(\vec{x})
\]  

(1)

The kernel of the equation, \( R_{i,j} \) is the spatial two-point cross-correlation tensor of the measured velocity field (ensemble averaged),

\[
R_{i,j}(\vec{x}, \vec{x}') = \langle u_i(\vec{x}, t_s) u_j(\vec{x}', t_s) \rangle
\]  

(2)

The projection of the measured velocity field onto this set of eigenfunctions yields the POD coefficients \( (a_n) \),

\[
a_n(t_s) = \int u_i(\vec{x}, t_s) \phi^{(n)}_i(\vec{x}) d\vec{x}
\]  

(3)

Since these are obtained through the integration over all space \( d\vec{x} \), these coefficient have global knowledge of the flow field. However, they are also a function of the discrete time step, \( t_s \), of the measured velocity data used to attain them.

To temporally resolve these coefficients, LSE is used. Its use is 'modified' from the original context described above in the sense that, it is no longer used to estimate the velocity field (the entire flow field can be sampled by PIV). The conditional averaging scheme is applied to the POD coefficients, obtained at discrete time steps, and the simultaneously sampled fluctuating pressure field\( p(t) \). The pressure field is chosen, as it can be sampled at frequencies high enough to produce a continuous time series, and is therefore able to resolve the time scales of high Re flows. The POD coefficients have global knowledge of the flow field, and is also a scalar quantity, and can therefore be expected to demonstrate a higher correlation with the pressure field.

\[
\langle a_n(t_s) | p(t) \rangle = a_n^{est}(t)
\]  

(4)

The new time resolved estimated POD coefficients, can be expressed as a power series expansion truncated at the linear term,

\[
a_n^{est}(t) = B_n p(t)
\]  

(5)
Expanding to higher order terms has been shown to yield similar results, Tung & Adrian. The linear term $B_{nk}$ is defined by,

$$\langle p_j(t_s)p_k(t_s) \rangle B_{nk} = \langle p_j(t_s)a_n(t_s) \rangle$$

Once the time resolved estimated POD coefficients have been determined, they can then be projected onto the eigenfunctions obtained from the discretely measured flow field to reconstruct a time resolved estimate of the flow field.

$$u_{est}^i(\vec{x}, t) = \sum_{n=1}^{N} a_{est}^n(t)\phi_i^{(n)}(\vec{x})$$

This estimate of the velocity field can then used to estimate power spectra, as well as cross-spectra; (where $u_{est}^{*}$ denotes the complex conjugate). This step is an essential part in the validation of the technique, as comparisons can then be made to measured LDA power spectra and cross-spectra.

$$u_{est}^i(\vec{x}, f) = \int u_{est}^i(\vec{x}, t)e^{-jft}dt$$

$$S_{uu}^{est}(f) = \frac{1}{T}\langle u_{est}^i(\vec{x}, f)u_{est}^{*}(\vec{x}, f) \rangle$$

The 'modified' Complementary Technique has been demonstrated in numerous experiments, and proven to give excellent results. The backward facing ramp, Taylor & Glauser, a NACA-4412 airfoil, Glauser et al and Glauser et al, and the axisymmetric jet, Tinney et al. The technique relies greatly on the basis that a strong correlation exists between the fluctuating pressure field and the velocity field. That correlation is now examined here.

### III. Experiment

These experiments were conducted in the Syracuse University Skytop Anechoic Chamber. Tinney et al refurbished the chamber, with the purpose of performing experiments to study the far-field acoustics of a high Reynolds number compressible jet. The nozzle exit is 5.08 cm in diameter, with a matched 5th order
polynomial interior profile contraction. The flow temperature was held constant at 25°C, at a Reynolds number of 6.4E5, and Mach number of 0.60. The nozzle exhibits turbulence intensities on the order of 1% in the region of the potential core.

The fluctuating pressure field was sampled by an azimuthal array of fifteen Kulite XCE-093 model, 0-34kPa gauge transducers; these have a frequency response range from DC to 50kHz (Fig. 1). Five Endevco, model 136 signal conditioners provide excitation for the transducers, with three channels per unit. These are positioned near the outside edge of the shear layer, well within the hydrodynamic region, at 0.875D downstream of the jet exit, and 1.75R from the centerline.

To capture the velocity field, we utilized the optical based measurement technique of Laser Doppler Anemometry for its high accuracy and non-intrusive nature (Fig. 2). The system is a Dantec Dynamics LDA, powered by a 3-Watt argon-ion laser head. It incorporates a burst spectrum analyzer (BSA) processor. This allows for multi-bit sampling of Doppler bursts and true real time 8-bit FFT to give fast and accurate velocity data. The system is only able to capture a single point in space, and therefore has to be traversed throughout the flow field. The traverse was fixed in the plane corresponding to an azimuthal location of 0 degrees with the horizontal, and shifted for several radial and streamwise positions (Fig. 1). Each radial location corresponds to a position, extending out from the center of the jet axis, to the outside edge of the shear layer. Locations were chosen along the centerline, the edge of the potential core, along the edge of the jet lip, and the outside edge of the shear layer. To resolve a more dense measurement grid, positions directly centered between each of the previous locations, were also sampled.

Imperative in these experiments is the ability to simultaneously sample both signals. A National Instruments PXI system equipped with two NI-4472 boards is used to fulfill this requirement. Each board contains 8 single and differential channels with 24-bit resolution, capable of sampling up to 102.4kHz per channel. In addition, each channel contains an independent Delta-sigma A/D converter with built in low pass filter. The signals from all fifteen transducers are digitized and sampled at 20kHz. The last channel is used to sample the LDA velocity measurement duration signal form the BSA processor, indicating beginning and end. Once start and stop are indicated, this is then used to section off the corresponding pressure data.

IV. Velocity Analysis

To characterize the jet’s aerodynamic properties, mean streamwise and radial velocity profiles were obtained, along with turbulence intensities (Fig. 3). A fairly dense grid was used to acquire the profiles. It incorporated over thirty-five measurement locations, spanning from beyond the outer edge of the shear layer, to this similar location on the opposite side of the flow field; traversed vertically from top to bottom. Although known to be symmetric in nature, the entire profile was sampled as a means of quantifying the facility. This was done for eight separate streamwise locations, z/D=0.5 through z/D=7. Mean values were calculated using an average of at least 20,000 samples for each location (average sampling rate was 2kHz). The turbulence intensities observed are on the order of 1% across the potential core region near the jet exit, and 15-17% along the center of the shear layer. The spreading velocity and angle were calculated to be 5m/s and 11°, respectively.

The use of Laser Doppler Anemometry results in the need for additional post processing, depending on the final purpose of the data. The LDA produces an irregularly sampled data set, dependent only upon the rate at which particles randomly pass through the control volume. The raw LDA data must be manipulated to obtain an accurate spectral density estimate, as well as autocorrelation function; both require input of equidistantly spaced data, or a continuously sampled signal. A ‘zeroeth’ order interpolation scheme was used to rebuild a continuous time series from the randomly sampled data set acquired in these experiments.
This scheme is a widely used method of analyzing LDA data sets, and has been shown to produce an accurate spectral estimate, as well as autocorrelation function, when high average sampling frequencies are achieved, Benedict.\(^3\) The raw data was re-sampled at a frequency equal to that used to sample the pressure field (20kHz). The average sampling frequency of the LDA system was maintained above 15-20 kHz to limit the number of repeated values in the zeroth order scheme, and thereby improving the spectral and autocorrelation function estimates.

![Figure 4](image4.png)  
**Figure 4.** Normalized autocorrelation coefficient at \(z/D=1\)

![Figure 5](image5.png)  
**Figure 5.** Normalized autocorrelation coefficient at \(z/D=5\)

![Figure 6](image6.png)  
**Figure 6.** Normalized autocorrelation coefficient along centerline

![Figure 7](image7.png)  
**Figure 7.** Normalized autocorrelation coefficient along center of shear layer

The normalized autocorrelation coefficient of the streamwise velocity component,

\[
\rho_{uu}(\tau) = \frac{\langle u(t)u(t + \tau) \rangle}{\langle u^2(t) \rangle} \tag{10}
\]

is plotted as a function of radial location for streamwise locations \(z/D=1\) and \(z/D=5\), (Figs. 4 & 5). This stands as a direct relation to the scales of event within a flow field. It is noticed that the correlation length increases as radial location is increased. This is evident as the location at which the plots level off to zero is shifted further outward with increasing \(r\). The scales of the structures within the flow field are known to increase as we move outward from centerline, which coincides with the observed trend.

This is also seen in autocorrelation plots as a function of streamwise location, \(z\), along the fixed grid defined earlier (Figs. 6 & 7). It is interesting to note also, that structures within the potential core region
also exhibit an increase in scale as they evolve downstream. The curvature of the autocorrelation coefficient at the origin also serves as a measure of the micro-scales. In each plot, the curvature at the origin increases with increasing radial location, as well as with increasing streamwise location. This is again evidence of the evolving scales of structures within the flow field, both in $r$ and $z$, as they travel from the jet exit.

Power spectral estimates calculated using 8192 samples averaged over approximately 70 blocks, are plotted for streamwise locations, $z/D=1$ and $z/D=3$ as a function of increasing radial location (Figs. 8 & 9). Within the shear layer, the region of energy roll off denoted as the inertial subrange, exhibits a slope of $f^{-5/3}$, indicative of a turbulent flow field. The peaks observed, signify the dominant frequency of the events passing through that region. There is a dominant peak at approximately $2kHz$ within the potential core. It is noticed that the peak shifts toward lower frequencies as radial location is increased, indicating an increase in the scales of the dominant events. The energy content also increases with increasing radial location. It is also interesting to note that the spectra along the centerline shows no pronounced peak until $z/D=2$, suggesting that the significant development of instability may begin here. Experiments conducted at $M=0.3$, show a pronounced peak developing at $z/D=1$. This agrees with work by Ko & Davies, who proposed that the development of the axisymmetric jet is a function of exit velocity.
A spectral analysis is also performed as a function of streamwise location, extending from $z/D = 1$ to $z/D = 5$, the end of the potential core (Figs. 10 & 11). A similar trend results, as an increase in energy content is noticed with increasing streamwise location. Within the potential core region of the flow, peaks in the spectra all occur at the similar frequency; there is a slight shift towards lower frequencies. This implies that the scales within this region are of the same order, but do show some increase in size as they move downstream. Again we note, as discussed earlier, the development of the first peak at $z/D = 2$. As we move to the center of the shear layer, we start to see a more pronounced trend, where the peak frequency falls toward lower values with increasing streamwise location. This new trend implies that the scales of the structures outside of the potential core region increase more rapidly with increasing streamwise location. This occurrence is clearly justifiable, as the vortical structures generated by the interaction between the high velocities of the potential core and the slower moving surrounding air are known to coalesce as they move downstream. The shift in the spectral peak clearly denotes the growth of these structures.

V. Pressure Analysis

The positioning of the transducers proved to be the key element in resolving true correlations. There has been much analysis and discussion about the near-field pressure region of the axisymmetric jet. A great deal of attention has been aimed at quantifying the different regimes of the hydrodynamic versus acoustic pressure field and the transition between.

At the onset of these experiments, the array of transducers were naively placed along the outer surface of the jet nozzle, and positioned such that the measurement membrane were just nearly impacted by the flow field. In this region, the power spectrum was observed to be flat, and completely broadband. There was no evidence of a pronounced peak. It is suspected that the instabilities caused by the vortex interaction, which create the hydrodynamic pressure field, have not yet developed this near to the jet exit. The velocity power spectrum also verifies this assumption, as it shows no significant peak until $2D$ downstream, again indicating that there is a period of development. The experiment was then redesigned, and the transducers were moved further downstream to a location of $z/D = 0.875$ (4.45 cm). The array was positioned outside the edge of the shear layer, $r/R = 1.75$ (4.45 cm), based on the natural spreading angle of 11° that was previously calculated.

Arndt et al evaluated this transition region by modeling the spectral variation from near-field to far-field, based on the unsteady Bernoulli equation. The model suggests that the regions dominated by hydrodynamic pressure exhibit a spectral decay in the range from $k y^{-6}$ to $k y^{-6.67}$, where $y$ denotes outward radial distance from the center of the shear layer. However, regions dominated by acoustic pressure, i.e. the far-field, exhibit a $k y^{-2}$ roll off. The slope of the spectra calculated in our experiments demonstrates a similar decay (Fig. 12). It is clearly however, not that of an acoustic pressure field. The transition between the two regions observed in the Arndt et al investigation of $k y = 2$ is also noticed here, demonstrating the influence of the radiated acoustic field even at a location this close ($y = 19.05$ mm, $z/D = 0.875$) to the jet exit.

VI. Results

To evaluate the two-point cross-correlation between the fluctuating pressure and velocity fields, the product of the two signals is averaged and normalized by the product of the RMS of each.

$$\rho_{p,u}(\tau) = \frac{\langle p_i(t)u(t + \tau) \rangle}{\sqrt{\langle p_i^2(t) \rangle} \sqrt{\langle u^2(t) \rangle}}$$  \hspace{1cm} (11)
The pressure signal is held fixed, and velocity shifted for increasing \( \tau \), as events exiting the jet lip would be sensed first by the transducers, then later by the LDA, as it moves downstream.

The use of an azimuthal array of transducers gives an added domain, \( \theta \), over which the correlation can be evaluated. It can be expected however, since the velocity measurements were acquired in the plane of the 0 degree transducer, that this would demonstrate the strongest correlation. Surface plots of the cross-correlation between the velocity measured at a radial location of \( r/R = 1 \), and \( z/D = 2 \) & \( z/D = 4 \), clearly shows this to be the case (Figs. 13 & 14). In both figures, the amplitude of the correlation peaks at 0 degrees, then begins to decrease. A slight shift in phase is also noticed as the peak moves to higher \( \tau \), with increasing theta. This is to be expected, as the distance between the two signals is increasing. The magnitude of these peaks however, is considerably less than that of the peak noticed with the 0 degree transducer, making this trend hard to distinguish. The profile is nearly symmetric, as this trend is mirrored on the opposite side, denoted as the 'second' 180 degrees of separation. For this reason, correlations are only evaluated as a function of the 0 degree transducer.

![Figure 13. Cross-correlation surface plot at \( z/D = 2 \), \( r/R = 1 \) depicting the symmetric nature of the function (with 0 degree transducer).](image1)

![Figure 14. Cross-correlation surface plot at \( z/D = 4 \), \( r/R = 1 \) depicting the symmetric nature of the function (with 0 degree transducer).](image2)

In the cross-correlation plots, the peak is not at the origin as seen in the autocorrelation plots; there is now a phase lag, \( \tau \), between the two signals. With each measured location, the magnitude goes high as the signals move into phase, drops below zero (out of phase), and then begins to level off to zero as the signals no longer correlate. As we now evaluate the cross-correlation at several streamwise locations, \( z/D = 2 \) through \( z/D = 5 \) (Figs. 15, 16, 17, & 18) as a function of \( r \), it is seen that the magnitude increases with increasing radial location at first. This occurs until a location along the edge of the potential core, where it then begins to fall off with increasing radial location. Positions above \( r/R = 1 \) demonstrate almost no correlation. The peak value are consistently noticed at the locations measured just outside the centerline, within the potential core region, approximately 25%.

The time lag, \( \tau \), associated with each peak, is also seen to decrease with increasing radial location. This shift is consistent with increases in radial location. One explanation for this is that as the LDA is traversed outward in the radial direction, towards the transducer, the phase lag between the signals decreases slightly. The width of the peaks can also be very revealing, about the growth of the structures as we extend radial, from the potential core into the shear layer. They illustrate the length over which the structures are correlated. Within the potential core, the peaks are sharp and indicate a correlation with a consistent length scale. In the shear layer, the peaks observed are more broadband, and indicate larger distribution of structures resulting in lower magnitudes, and longer correlation lengths.

At the end of the potential core, \( z/D = 5 \), the magnitude of the correlation at the centerline has become the dominant signal at 16%, relative to the magnitude at other radial locations (Fig. 13). The peak correlation again begins to fall off with increasing \( r \), to nearly zero above \( r/R = 1 \). This can be expected as the collapse of the potential core signifies the growth of the shear layer. Therefore, all positions except the centerline are now within the shear layer, and demonstrate a weaker correlation (lower magnitude) as compared to those
Figure 15. Normalized pressure-velocity cross-correlation for all radial locations at $z/D=2$.

Figure 16. Normalized pressure-velocity cross-correlation for all radial locations at $z/D=3$.

Figure 17. Normalized pressure-velocity cross-correlation for all radial locations at $z/D=4$.

Figure 18. Normalized pressure-velocity cross-correlation for all radial locations at $z/D=5$. 
at the centerline.

The magnitude of the cross-correlation is now more closely examined as a function of streamwise location. The values are plotted for a fixed radial location corresponding to the line marking the center of the potential core (Fig. 19), the edge of the potential core (Fig. 20), and the center of the shear layer (Fig. 21). These plots clearly demonstrate the phase shift associated with increasing downstream location. Each peak experiences a noticeable increase in time lag at larger z/D separation. The magnitude of each peak is also observed to decrease with increasing z/D. This indicates that as the flow develops and becomes more random at downstream locations, the correlation between the signals starts to decay. The changes in the width of the peaks, indicating the changes in consistency of length scales is again also seen. Within the potential core region, which is dominated by a consistent length scale, the width of each curve is nearly constant as we move downstream. However, it is observed along the edge of the potential core (high shear region), that there is an increase in the width of each successive peak. Again indicating a randomness of length scales. Along the center of the shear layer, this the trend is again noticed, as a pronounced difference in the width of each peak is seen. The structures in this region of highest fluctuation, are known to roll up and interact as they propagate downstream, causing an increased randomness of scales. This is thought to account for the spreading of the shear layer.

![Normalized pressure-velocity cross-correlation along edge of potential core](image1)

The magnitude of the cross-correlation is now more closely examined as a function of streamwise location. The values are plotted for a fixed radial location corresponding to the line marking the center of the potential core (Fig. 19), the edge of the potential core (Fig. 20), and the center of the shear layer (Fig. 21). These plots clearly demonstrate the phase shift associated with increasing downstream location. Each peak experiences a noticeable increase in time lag at larger z/D separation. The magnitude of each peak is also observed to decrease with increasing z/D. This indicates that as the flow develops and becomes more random at downstream locations, the correlation between the signals starts to decay. The changes in the width of the peaks, indicating the changes in consistency of length scales is again also seen. Within the potential core region, which is dominated by a consistent length scale, the width of each curve is nearly constant as we move downstream. However, it is observed along the edge of the potential core (high shear region), that there is an increase in the width of each successive peak. Again indicating a randomness of length scales. Along the center of the shear layer, this the trend is again noticed, as a pronounced difference in the width of each peak is seen. The structures in this region of highest fluctuation, are known to roll up and interact as they propagate downstream, causing an increased randomness of scales. This is thought to account for the spreading of the shear layer.

![Normalized pressure-velocity cross-correlation along centerline](image2)

![Normalized pressure-velocity cross-correlation along center of shear layer](image3)

The slope of the curve connecting the peaks of the correlation plots serves as a good measure of the convection velocity of the structures within the flow field, as it represents the ratio of change in distance over change in time. Space-time surface plots make this slope more easily visible. The streamwise convection velocity is evaluated along the centerline, (Fig. 22), the edge of the potential core, (Fig. 23), and the center of the shear layer, (Fig. 24). The slope is noticeably steeper within the potential core region. The value of the convection velocity at each z/D increment is charted in Table 1. Along both lines within the region of the potential core, an average convection velocity of 0.77Uj is calculated, and 0.73Uj at the center of the shear layer. The value of 0.73Uj in the shear layer is within the range observed by other investigators at varying Reynolds numbers; (0.71Uj) Crow & Champagne,5 (0.72Uj) Williams,14 and (0.65Uj) Ko & Davies.8

The value of 0.77Uj within the potential core region requires more consideration. The radial space-time...
Figure 22. Pressure-velocity cross-correlation surface plot along centerline.

Figure 23. Pressure-velocity cross-correlation surface plot along edge of potential core.

Figure 24. Pressure-velocity cross-correlation surface plot along center of shear layer.

Table 1. Estimated convection velocity between each streamwise location from \( z/D = 2 \) to \( z/D = 5 \).

<table>
<thead>
<tr>
<th>( z/D^* )</th>
<th>CL</th>
<th>EPC</th>
<th>CSL</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-3</td>
<td>152.4 m/s</td>
<td>127.1 m/s</td>
<td>138.5 m/s</td>
</tr>
<tr>
<td>3-4</td>
<td>152.4 m/s</td>
<td>169.3 m/s</td>
<td>152.4 m/s</td>
</tr>
<tr>
<td>4-5</td>
<td>169.3 m/s</td>
<td>169.3 m/s</td>
<td>152.4 m/s</td>
</tr>
</tbody>
</table>

* Denotes a 1D change in streamwise location.
correlation plots showed earlier, demonstrate that the distance between the two signals plays a role in the phase lag observed. The increase in phase as the distance between the two signals increases suggests the direction of propagation is into the potential core, slowing its convection and causing its collapse. This is consistent with the results found by Ko and Davies.\(^8\) It also suggests, as they observed, that the convection velocity within the potential core is not equal to that of the shear layer.

The evaluation of the two-point correlation is further validated by cross-spectral analysis. The cross-spectral density function is the Fourier transform of the cross-correlation function,

\[
S_{pu}(r, z_0, \omega, \theta) = \int R_{pu}(r, z_0, \tau, \theta) e^{-i\omega\tau} d\tau
\]  

(12)

Just as the cross-correlation function is a measure of the coherence between two signals, the cross-spectral density function also gives a measure of coherence, in frequency space. This function is in general complex, with the real and complex parts termed as the coincident and quadrature spectral density functions, respectively. The magnitude of the cross-spectral function demonstrates how the energy is distributed to various frequencies. The coincident spectral density is evaluated at \(z/D=3\) and \(z/D=5\). Both the velocity and pressure power spectrum are shown to demonstrate the effect each has on the signal. It is seen that the
curve lies directly between the two signals, and shows influence of both. At $z/D=3, r/R=0$ where there is a pronounced peak in the velocity power spectrum, the cross-spectrum also demonstrates a more pronounced peak (Fig. 25). At $r/R=1$ where the velocity spectrum is flat and broadband, the cross-spectrum becomes more broadband (Fig. 26). A similar trend is noticed at $z/D=5$ (Figs. 27 & 28). The peak encompasses a range of frequencies that appear to be a morph of the velocity and pressure spectra. This indicates the range of frequency over which the strongest correlation can be expected. When the velocity spectrum is more broadband, as is the case within the shear layer, this range increases. Evidence that the random scales of structures within the shear layer cause the correlation to occur over a wider range of frequencies related to those structures is shown in the cross-correlation plots as a reduction in the magnitude of the correlations. However, within the potential core region where scales are observed to be consistently of the same order, the correlation will steadily occur over a small range of frequencies, at higher magnitudes.

VII. Conclusions and Future Work

The fluctuating pressure field of the $M=0.6$ (Re=6.4E5) axisymmetric jet, sampled at $z/D=0.875, r/R=1.75$ by an azimuthal array of transducers, 24 degrees apart, has been shown quantitatively to exhibit a strong cross-correlation with the fluctuating velocity field. The largest correlation is exhibited between the 0 degree transducer, positioned in the plane of LDA velocity measurements, and regions within the potential core just outside the centerline. Magnitudes of the order of 25% are seen. Within the shear layer, the level of correlation exhibited differs greatly within the inner region than that of the outer, the entrainment region; from 15% to less than 5%. The cross-correlation follows a consistent trend in $r$, extending from the centerline. The magnitude increases from the centerline through the edge of the potential core, where it then begins to fall off until the center of the shear layer. Outside this position, the magnitude falls to considerably smaller values. The phase lag, $\tau$, observed between the signals is shown to increase with increasing radial separation as well. The shortest lag is exhibited at locations within the outer shear layer (entrainment region), where the separation of the LDA and transducer is smallest. This suggests that the direction of propagation of the structures is toward the potential core, as observed by other investigators. Along the streamwise axis, the cross-correlation is seen to decrease in magnitude with increasing $z/D$. This trend is exhibited in along all line traversed, with the exception of the centerline. Here, the magnitude increase from $z/D=1$ to $z/D=3$, then begin to fall off to the end of the potential core, $z/D=5$. Spectral analysis also shows that the centerline’s dominant characteristic also follows this same trend, increasing from $z/D=1$ and peaking at $z/D=3$, then falling off.

Evaluation of the streamwise convection velocity by the slope of the curve connecting the peaks of the cross-correlation shows a value of $0.73U_j$ within the shear layer, and $0.77U_j$ within the potential core. The notion discussed earlier, that structures propagate into the potential core region slowing its convection and thereby causing its collapse would explain this result.

Cross-spectral analysis shows that the signals are coherent and do produce a coincident spectral density function. The frequency range over which the maximum correlation can be expected is seen to change with the characteristics of the velocity spectrum. Where the spectrum is more broadband, such as in the shear layer, the coincident spectrum exhibits a broadband spectrum. On the other hand, when the velocity spectrum exhibits a pronounced peak, as in the potential core, the coincident spectrum is seen to also exhibit a more pronounced peak. Suggesting that consistent scales of the structures within the potential core result in a higher cross-correlation, than that of the more random shear layer.

These experiments provide a unique data set that can be used to validate the ‘modified’ Complementary Technique. Spectral characteristics measured here, can be compared to estimated spectra acquired through use of this technique. Also, knowledge of time lags and convection velocities determined can be used to refine the technique. If this information is known a priori, the application of the LSE can be manipulated to produce the strongest possible correlations. This in turn, would provide a more accurate final estimate of the reconstructed velocity field.

VIII. Acknowledgements

We gratefully acknowledge the support of the National Science Foundation through the CNY-PR AGEP, the Air Force Office of Scientific Research, and the New York State Science, Technology, and Academic Research Office.
References


