Synthetic and Field Examples of the Estimation of Capillary Pressure and Relative Permeability From Formation-Tester Measurements

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Abstract

Laboratory measurements of relative permeability and capillary pressure are seldom performed on core samples retrieved from petroleum-production wells. Reservoir engineers rely on a limited number of small core samples to characterize many of the large-scale multiphase flow petrophysical properties affecting the production and recovery of hydrocarbon fields. The question also remains whether laboratory measurements are truly representative of in-situ rock properties. Non-linear regression methods were recently proposed to estimate saturation-dependent petrophysical properties from fractional flow-rate measurements acquired with formation testers. However, such procedures are still unclear to many practicing analysts and to date have not been fully explored with both synthetic and field data. This paper develops and successfully tests a new method to estimate saturation-dependent rock properties on two field data sets.

Using in-house and commercial reservoir simulators, we model the processes of mud-filtrate invasion, acquisition of borehole resistivity measurements, and subsequent fluid withdrawal during sampling. In the examples considered, the formation tester consists of a dual-packer module acquiring pressure and fractional flow-rate measurements during the sampling operation. Based on the physics of water-base mud-filtrate invasion, borehole resistivity measurements and dual-packer measurements are first used to estimate both initial water saturation and permeability with initial estimates of capillary pressure and relative permeability. The latter are described with the Brooks-Corey model, which includes 6 independent unknown parameters. Subsequently, the measured pressure and fractional flow rates are used to estimate the 6 Brooks-Corey unknown parameters, thereby defining a new set of capillary pressure and relative permeability curves to refine the estimation of initial water saturation and permeability jointly from pressure and borehole resistivity measurements. This process repeats itself until borehole resistivity, pressure, and fractional flow-rate measurements are all honored within prescribed error bounds.

The estimation method satisfactorily reconstructs the relative permeability and capillary pressure curves with minimal a-priori information. Whereas the relative permeability end-points of water and oil can be readily estimated in a couple of non-linear iterations assuming that the remaining parameters are fixed, residual saturations add complexity to the inversion, especially for cases where the fractional flow rate exhibits a sharp decrease in contamination after oil breakthrough. We also investigate the use of Design of Experiment (DoE) tools to secure a reliable initial guess for nonlinear inversion and in understanding the separate contributions of the various measurements to specific inversion parameters. Such information is fundamental to designing a data-weighing scheme that selectively enhances the sensitivity of the measurements to unknown parameters during progressive steps of nonlinear inversion.

Introduction

The possibility of estimating relative permeability and capillary pressure curves from in-situ measurements has generated substantial interest in the past decades. Investigators have attempted to obtain such curves by matching long-time production data (Al-Khalifa, 1993; Kulkami and Datta-Gupta, 2000, Toth et al., 2006), well tests (Nanba and Horne, 1989; Puntel de Oliveira and Serra, 1995; Chen et al., 2008), and formation-tester measurements (Zeybek et al., 2004; Alpak et al., 2008). The advantages of using formation testers are (a) their flexibility to acquire bottomhole fractional flow-rate measurements (through optical sensors), (b) the ability to control the flow rate of fluid pumpout, and (c) the acquisition of pressures with different monitoring probes concomitant with fluid sampling on a relatively thin rock formation. Other efforts have
considered the inversion of relative permeability curves from resistivity logs (Ramakrishnan and Wilkinson, 1999; Zeybek et al., 2004; Alpak et al., 2004). Lately, Zeybek et al. (2004) used resistivity and formation-tester measurements to estimate end-point relative permeabilities for water \( (k_{rw}^\circ) \) and oil \( (k_{ro}^\circ) \), radius of invasion, and horizontal and vertical permeabilities. Resistivity logs were used mainly for estimating both radius of invasion and relative permeability end-points following the technique advanced by Ramakrishnan and Wilkinson (1999). In turn, initial guesses for horizontal and vertical permeability were obtained from nonlinear regression of single-phase pressure transients. Subsequently, the estimation method was validated with a field example by manually modifying unknown parameters from the visual inspection of the numerical match of water cut and pressure measurements acquired with the formation tester. Alpak et al. (2008) used a regularized Gauss-Newton (WRGN) nonlinear inversion technique to simultaneously estimate the modified Brooks-Corey (MBC) parametric coefficients of relative permeability and capillary pressure curves.

This paper develops a new method to estimate both relative permeability and capillary pressure curves from resistivity and formation-tester measurements (pressure and fractional flow rate). In contrast to previous approaches, we estimate the entire set of MBC parameters, radius of invasion, permeability, and fluid saturation by solving the nonlinear inversion problem through several iterative loops wherein different data weights are applied at each progressive stage to selectively increase the sensitivity of the measurements to a given unknown model parameter. Likewise, instead of the single-layer formation model examples invoked in past publications, the method introduced here is implemented on multi-layer rock-formation models and is applied to two field examples from wells in the North Sea. We validate the method on field measurements with realistic tool and formation properties. In addition, based on a synthetic model we implement Design-of-Experiment (DoE) procedures to define the location of initial-guess parameters thereby reducing non-uniqueness in the estimation process. DoE also proves an efficient method to describe the non-linearity of the estimation process, to quantify the contribution of each measurement to specific inverted parameters, and to explore the properties of the cost function used for nonlinear inversion.

**Description of Method**

**General Assumptions.** For the field and synthetic examples considered in this paper, we estimate the following parameters: formation permeability, fluid saturation, radius of invasion, and the set of MBC parameters corresponding to saturation-dependent capillary pressure and relative permeability. Input measurements are resistivity and formation-tester data (time records of pressure and fractional flow). In all cases, we assume water-based mud (WBM) invasion taking place in an oil-bearing formation; however, the method is not restricted by this last assumption. As it will be emphasized below, the method works with any generic reservoir simulator, thereby allowing a similar application to more general cases of fluid displacement that may involve oil-based mud (OBM) invasion or gas-bearing formations. We assume a vertical well penetrating horizontal formation layers with saturating fluids in capillary equilibrium. Layers are assumed isotropic and homogeneous.

**Modified Brooks-Corey Coefficients.** Saturation-dependent relative-permeability and capillary-pressure functions are defined with the modified Brooks-Corey (MBC) drainage equations (Brooks and Corey, 1964), given by

\[
\begin{align*}
    k_{rw}(S_w) &= k_{rw}^\circ (S_{wn})^{(1+2/\eta)} , \\
    k_{ro}(S_w) &= k_{ro}^\circ (1-S_{wn})^{(1+2/\eta)} , \\
    P_c(S_w) &= P_{ce} (S_{wn})^{1/\eta} , \\
    S_{wn} &= \frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}} ,
\end{align*}
\]

(1)

where \( S_w \) is water saturation, \( k_r \) is relative permeability, \( P_c \) is drainage capillary pressure, \( P_{ce} \) is capillary entry pressure, and \( \eta \) is as a measure of maximum pore size distribution. In the above equations, subscripts are used to identify water \( (w) \), oil \( (o) \), normalized \( (n) \), and residual saturations \( (r) \). End-point relative permeabilities are identified with the superscript symbol "\(^\circ\)". Alternative parametric representations could have been used in this paper to test the applicability of the inversion method (e.g. power-law models and B-splines); however, some of these representations include more independent parameters and hence are detrimental to the stability of nonlinear inversion. Moreover, MBC equations have been regarded as representative of water-oil drainage processes (Li, 2004), suitable to describe the formation-tester sampling examples considered in this paper. MBC equations have also been extensively used in the dynamic simulation of petroleum reservoirs.

**Simulation of Resistivity and Formation-Tester Measurements.** Numerical simulations of resistivity and formation-tester measurements were performed using a combination of UTFET, an in-house cylindrical two-phase simulator developed by the University of Texas at Austin (Chew et al., 1984; Zhang et al., 1999; Ramirez et al., 2006), and IMEX\(^\circ\), a three-phase black-oil commercial simulator developed by the Computer Modeling Group Ltd. (CMG). For resistivity modeling, the UTFET simulator is dynamically-coupled to a mud-filtrate invasion algorithm (Wu et al., 2002) that enforces explicit assumptions on capillary pressure and relative permeability properties. Once the process of mud-filtrate is simulated, we calculate the corresponding spatial distribution of electrical resistivity \( (R_e) \) using the simulated spatial distributions of water saturation \( (S_w) \)
and salt concentration ($C_w$). In turn, the spatial distribution of electrical resistivity is used to simulate borehole resistivity measurements ($\sigma_{\text{deep}}$ and $\sigma_{\text{shallow}}$) to be compared against field measurements. This method provides a direct link between formation permeability or MBC parameters and simulated apparent resistivity curves. For formation-tester modeling, the IMEX simulator uses the same earth model as in the resistivity simulations and corresponding distributions of rock/fluid properties to initialize the withdrawal of formation fluid. To account for skin, we first find the pressure drop due to skin ($\Delta P_{\text{skin}}$) from an analytical single-phase flow solution. Then we assume that the pressure drop due to skin is due only to damage and that such damage is present when two-phase flow occurs. In field units, the pressure drop due to skin is approximated with the equation

$$\Delta P_{\text{skin}} = 141.2 \left( \frac{q_0b}{kh} \right) S,$$

(2)

where $S$ is skin factor, $k$ is permeability, $h$ is formation thickness, $\mu$ is viscosity, $B$ is formation volume factor, and $q$ is production flowrate. In our examples, we found that the above equation was a good and reliable approximation.

**Multi-Layer Formations.** In dealing with multi-layer formations, we coupled the different inverted parameters to the intrinsic layer properties of the formation to construct a physically-consistent earth model. The generalized Tixier-Timur formula (Timur, 1968) was used to relate irreducible water saturation ($S_{\text{wr}}$) to layer values of permeability ($k$) and porosity ($\phi$):

$$S_{\text{wr}}(k,\phi) = \left[ a \left( \frac{\phi}{k} \right)^{1/c} \right],$$

(3)

where parameters $a$, $b$, and $c$ are found from regression analysis for a base layer (in our case, this is the layer that faces the formation tester). Once the value of $S_{\text{wr}}$ is assumed for such layer, the remaining $S_{\text{wr}}$ values for other layers are updated using the resulting $a$, $b$, and $c$ values. In contrast, given the complexity of residual oil saturation ($S_{\text{or}}$) relationships, we assumed that all layers contained the same $S_{\text{or}}$ value as the base layer.

Similarly, all MBC coefficients are assumed constant for the entire multi-layer rock formation except for the value of capillary entry pressure ($P_{\text{ce}}$) which is updated with the Leverett-J function, namely,

$$P_{\text{ce}}(k,\phi) = P_{\text{ce}}^{0} \left[ \left( \frac{\phi}{k} \right)^{0} \right],$$

(4)

where the superscript “0” identifies values of the base layer facing the formation tester.

**Inverse Problem.** The method requires a non-linear minimization technique to effectively match numerical simulations and measurements with a single set of model parameters. We achieve this objective with the Levenberg-Marquardt (LM) algorithm implemented by Alpak (2005). LM is extensively used in engineering problems because of its ability to control the stability and rate of convergence of the iterative nonlinear minimization. The vector of model parameters $x$ considered in this paper is given by

$$x = [k_j, \phi_j, k_{\text{res}}, k_{\text{ro}}, \eta, P_{\text{ce}}^0, S_{\text{or}}, S_{\text{wr}}, r_{\text{invasion}}]^T,$$

(5)

where $k_j$ is layer permeability, $\phi_j$ is layer porosity, $r_{\text{invasion}}$ is the radius of mud-filtrate invasion, and the superscript “T" designates the transpose. The quadratic cost function invoked for minimization is given by

$$C(x) = C_{\text{res}}(x) + C_p(x) + C_{\text{fw}}(x) + C_{\text{bt}}(x) = \frac{1}{2} \left[ W_D \cdot e(x) \right]^2 + \alpha \left[ x \right]^2,$$

(6)

where $e(x)$ is the vector of data residuals yet to be defined, and the subscripts res, $p$, $fw$, and bt identify the quadratic cost functions $C(x)$ associated with resistivity, pressure, fractional flow, and breakthrough-time measurements, respectively, $W_D$ is the data weighing matrix used to enforce relative importance to a given measurement type according to the model parameter being inverted during a given iteration loop (see “Workflow”). The Lagrange multiplier $\alpha$ included in Equation (1) is a scalar ($0<\alpha<\infty$) that assigns relative weight to the two additive terms included in Equation (6). In the above equation, the first additive term causes the minimization to honor the measurements, whereas the second additive term prevents instability in
the estimation due to non-uniqueness and insufficient or noisy measurements. The relative weight of either term is progressively adjusted by $\alpha$ as the inversion algorithm iterates toward the minimum of the cost function. We select the Lagrange multiplier based on the criteria given by

$$\alpha = \gamma \max \{\beta_n\}, \quad \text{provided that} \quad \frac{\min\{\beta_n\}}{\max\{\beta_n\}} < \gamma,$$

where $\gamma$ is a constant equal to $10^{-8}$ and $\beta_n$ are the eigenvalues of the real and symmetric matrix $J'(x)J(x)$. The entries of the Jacobian matrix, $J(x)$, are described as follows:

$$J(x) = \left[\frac{\partial e_{m}}{\partial x_{n}}, m = 1, \ldots, M; n = 1, \ldots, N\right],$$

where $M$ is the number of measurements, and $N$ is the number of unknown model parameters. In addition, because the estimation includes measurements associated with a wide variety of value ranges, the vector of data residuals, $e(x)$, is constructed to balance the contribution from all the available measurements, and is given by

$$e(x) = \left[\frac{S_1(x) - m_1}{m_1}, \ldots, \frac{S_M(x) - m_M}{m_M}\right]^T,$$

where $S$ and $m$ denote the simulated and measured responses, respectively.

**Design of Experiments (DoE) and Response Surfaces.** We use Design-of-Experiment tools to secure an initial guess for the nonlinear inversion algorithm. This is especially important when dealing with the complete set of unknowns described by Equation (5) because of the severe non-uniqueness associated with the estimation process. A good initial guess brings us to the neighborhood of the global minimum with reduced possibility of trapping about local minima. For the examples shown here, we use the commercial software Design-Expert® produced by Stat-Ease Inc.

In simple terms, Design of Experiment (DoE) is a statistical method that determines relationships between the factors affecting a process (unknown parameters) and the output of that process (measurements). Several algorithms underlie the statistical evaluation method (Montgomery, 2001). For the case of an experiment with $\lambda$ factors, each with two (low and high) levels, $2^\lambda$ forward (simulation) runs are needed to evaluate the quadratic cost function; as $\lambda$ and the number of levels increases, the number of runs becomes prohibitive. Our objective is to approximate the cost function with a minimum number of runs, enough to locate the initial guess for nonlinear inversion. To that end, we use D-optimal designs, where we specify a model (linear, quadratic, cubic, etc.) before the algorithm can generate specific search combinations. Possible combinations are designed such that we minimize the generalized variance of the parameter estimates.

Another analysis procedure is the response-surface method (RSM), which is a combination of DoE factorial design and interpolation functions to quantify the response variable. Let $x_1$ and $x_2$ be the experimental factors. We can then fit the response surface $R(x)$ with a second-order (quadratic) model, such as

$$R(x) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_1x_1x_2 + \beta_1x_1^2 + \beta_2x_2^2 + \text{experimental error},$$

where $\beta$ are constants found from regression analysis and adjusted so that $R(x)$ results in a smooth interpolated version of the actual cost function $C(x)$ used for inversion. Surface responses also allow us to:

- Systematically study the relationship between model parameters and output measurements with a relatively low number of forward simulations, and
- Define the parameters that control the experiment and their level of interaction with other parameters

**Workflow.** As emphasized earlier, the reliability of the proposed method hinges upon its flexibility to optimize the sensitivity of each parameter within certain subsets of the measurements. Fig. 1 shows a schematic of the method used when interpreting field and synthetic examples. There are two main loops: The first loop (Loop 1) is used for the estimation of layer permeability ($k$) and water saturation ($S_w$) assuming a fixed set of relative permeability ($k_r$) and capillary pressure ($P_c$) curves. The second loop (Loop 2) is used to refine the previous estimates of relative permeability and capillary pressure by assuming fixed values of layer permeability and water saturation. Refinement iterations continue until convergence is achieved by the inversion algorithm.
Fig. 1. Flowchart describing the two main iteration loops included in the estimation method. Loop 1 assumes a fixed set of relative permeability and capillary pressure curves to estimate permeability and water saturation. Loop 2 assumes that the latter parameters are fixed and refines the parameters involved in the representation of relative permeability and capillary pressure curves.

In this paper, Loop 1 was originally performed manually and reported in a separate paper (Angeles et al., 2008). Results obtained from that initial work were input as initial guess for Loop 2 for further refinement of the $k_r$ and $P_c$ parameters. This section (Loop 2) is fully automated with the inversion engine. Once a set of $k_r$ and $P_c$ parameters is found that fit the formation-tester measurements (pressure $P(t)$ and fractional flow rate $F_w(t)$ measurements), we return to Loop 1 and Loop 2 and continue the iterations until the numerical simulations match both resistivity and formation-tester measurements and hence convergence is achieved. In our examples, the method converged after two iterations between Loop 1 and Loop 2. However, each individual loop required at least 12 and 200 iterations for Loop 1 and Loop 2, respectively. Technical details about the workflow are better understood with the synthetic and field examples described in the next section.

Proof of Concept with a Synthetic Example

General Description. Fig. 2 describes the transient pressure and fractional flow-rate measurements associated with the synthetic model studied in this section. Even though the model is synthetic, it is constructed in close similarity to that of one of the field examples (Field Example A). It also allows us to appraise the accuracy and reliability of the inversion method under controlled conditions, and provides insight to the influence of different subsets of the measurements on inverted parameters. In addition, since we know the exact solution, we can use the synthetic model to appraise the RSM used to secure an initial guess of unknown parameters.
to increase the values of entire multiphase flow in the tool flowline that connects the sandface and the spectroscopy sensor. Thus, rather than matching the fully-automated nonlinear inversion. In the method will yield high values of krw and kro as this is the region where the minimum difference is achieved. Notice that the same region is where the actual values of krw and kro (0.553 and 1.0, respectively) lie. Such a behavior indicates that pressure measurements constrain well the solution for this example. On the other hand, if we used fractional flow instead of pressure, the method will yield high values of kro but low values of krw. This would result in a poor inversion of krw. The same situation occurs when we use only oil-breakthrough time (B.T.) as input: krw is poorly defined and biased toward low values. One explanation for the biasing in both cases is that krw affects both the process of mud-filtrate invasion and the fluid sampling operation. Low values of krw result in low invasion rates, which imply less mud-filtrate to withdraw during fluid sampling causing a decrease in the oil-breakthrough time and the level of contamination (fw) at late times. However, low values of krw also delay the withdrawal of fluids during sampling, thereby increasing the aforementioned quantities. In consequence, there is a wide range of krw values, especially in the low range, which provide a similar match of synthetic measurements (non-uniqueness). Finally, we calculate the desirability function by adding an extra weight (double, for this example) to pressure measurements. As observed from the figures, the weighing scheme is satisfactory and we secure an initial guess that achieves maximum desirability at parameter values that are not far from those of the actual solution. Fig. 4 describes a similar analysis for η and Pce. Whereas pressure surface responses show marginal sensitivity to both parameters, the remaining surface responses of oil-breakthrough time and fw at late times successfully yield the initial guess for η (with actual value of 2). Conversely, Pce suffers from the same biasing toward low values previously observed for krw; this behavior is more detrimental to Pce than to krw for the case of field data. As emphasized earlier, field measurements of

DoE Results. We start by performing the DoE simulations required to construct the corresponding surface responses for this model. A total of 25 runs were performed under the D-optimal design instead of the 3^4 = 81 runs required for a full factorial design with 3 levels and 4 factors (krw, kro, η, and Pce). The latter number of runs is minimal compared to what we would need to run to obtain the actual cost function (or an approximation, or surrogate of it). For instance, increasing the number of levels to 6 per factor would require 6^4 = 1296 forward simulations. Table 1 summarizes the most likely solution calculated after the construction of the surface response and compares it to the actual set of parameters used to generate the synthetic model. Fig. 3 and Fig. 4 help to further explain the results described in Table 1. We considered three responses during the analysis based on pressure differentials, fractional flow, and oil-breakthrough time. Note that to perform the calculations we used only the late-time portion of the fw curve (after 10,000 seconds) because field measurements usually contains sharper decay rates due to physical factors that are not explicitly taken into account by our simulator (e.g., fluid segregation and multiphase flow in the tool flowline that connects the sandface and the spectroscopy sensor). Thus, rather than matching the entire fw curve, we concentrate on matching the late-time portion of the fw curve related to level of contamination and oil-breakthrough time (the time at which oil enters the formation). In all cases, we applied a power-law model to ensure a good fit with the analytical surface response (thus why the exponents 0.51, 0.91, and 0.59, respectively). In addition, the plot of “Desirability” is a weighed linear combination of the three responses that, in this case, are varied according to the specific parameter under study.

Table 1. Comparison of MBC parameters obtained from the RSM method and the actual parameters used for the synthetic model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actual Value</th>
<th>RSM Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>krw^0</td>
<td>0.55</td>
<td>0.31</td>
</tr>
<tr>
<td>kro^0</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>η</td>
<td>2.00</td>
<td>2.29</td>
</tr>
<tr>
<td>Pce</td>
<td>1.50</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Fig. 2. Time records of pressure (left panel) and fractional flow (right panel) numerically simulated for the synthetic model.

Fig. 3 and Fig. 4 convey interesting features of each individual surface response that can be used to our advantage during a fully-automated nonlinear inversion. In Fig. 3, the pressure difference between simulated and measured responses will tend to increase the values of kro and krw as this is the region where the minimum difference is achieved. Notice that the same region is where the actual values of kro and krw (0.553 and 1.0, respectively) lie. Such a behavior indicates that pressure measurements constrain well the solution for this example. On the other hand, if we used fractional flow instead of pressure, the method will yield high values of kro but low values of krw. This would result in a poor inversion of krw. The same situation occurs when we use only oil-breakthrough time (B.T.) as input: krw is poorly defined and biased toward low values. One explanation for the biasing in both cases is that krw affects both the process of mud-filtrate invasion and the fluid sampling operation. Low values of krw result in low invasion rates, which imply less mud-filtrate to withdraw during fluid sampling causing a decrease in the oil-breakthrough time and the level of contamination (fw) at late times. However, low values of krw also delay the withdrawal of fluids during sampling, thereby increasing the aforementioned quantities. In consequence, there is a wide range of krw values, especially in the low range, which provide a similar match of synthetic measurements (non-uniqueness). Finally, we calculate the desirability function by adding an extra weight (double, for this example) to pressure measurements. As observed from the figures, the weighing scheme is satisfactory and we secure an initial guess that achieves maximum desirability at parameter values that are not far from those of the actual solution. Fig. 4 describes a similar analysis for η and Pce. Whereas pressure surface responses show marginal sensitivity to both parameters, the remaining surface responses of oil-breakthrough time and fw at late times successfully yield the initial guess for η (with actual value of 2). Conversely, Pce suffers from the same biasing toward low values previously observed for krw; this behavior is more detrimental to Pce than to krw for the case of field data. As emphasized earlier, field measurements of
Fig. 3. Surface responses associated with the estimation of end-point relative permeabilities ($k_{rw}$ and $k_{ro}$) from pressure differentials (upper left panel), fractional flow (upper right panel), oil breakthrough time (lower left panel), and the combination of the three, referred to as “desirability” (lower right panel).

Fig. 4. Surface responses associated with the estimation of the pore-size distribution coefficient and capillary entry pressure ($\eta$ and $P_{ce}$, respectively) using pressure differentials (upper left panel), fractional flow (upper right panel), oil breakthrough time (lower left panel), and the combination of the three, referred to as “desirability” (lower right panel).
fractional flow exhibit more drastic changes than synthetic measurements and hence we may not be able to match the entire $f_w$ curve, thereby missing valuable dynamic information to improve the estimation. This loss of information worsens the inversion results for the case of $P_{ce}$. Thus, to construct the desirability function and to improve inversion results, it becomes clear that we have to (1) place more weight on both $f_w$ and oil-breakthrough time, and (2) reduce the influence of transient pressure measurements on $\eta$ and $P_{ce}$. Fig. 5 illustrates the corresponding Box-Cox and Predicted vs. Actual plots for fractional flow rate measurements. The Box-Cox plot confirms the appropriateness of the power transform used in the regression analysis, whereas the Predicted vs. Actual plot shows a straight unit-slope line, thereby confirming the goodness of fit during the transformation. Alternatively, we also obtained analytical expressions for the 3 surface responses. The specific analytical expression obtained for pressure difference is given by

$$R(x)^{0.51} = +0.88099 \times -0.47688k_{rw}^0 -1.00746k_{ro}^0 -0.13619\eta -0.034892P_{ce} +0.095702k_{rw}^0 \times k_{ro}^0 +0.023324k_{rw}^0 \times \eta +0.011531k_{ro}^0 \times P_{ce} +0.080594k_{ro}^0 \times \eta +0.011325k_{ro}^0 \times P_{ce} +4.76927E-04 \times \eta^2 +0.23933(k_{ro}^0)^2 +0.43436(k_{ro}^0)^2 +4.56383E-003(\eta)^2 +6.72469E-003(P_{ce})^2. \tag{11}$$

One could continue constructing new surface responses close to the initial estimates and refining the previous results. In theory, after several iterative refinements we should be able to obtain the true parameters associated with this synthetic model. Albeit simple and straightforward, such iterative refinements converge slowly when the number of unknown parameters increases. Hence, we only use the DoE approach to secure a good initial guess for nonlinear inversion.

**Inversion Results.** As described in Table 2, we implemented several inversion strategies with the synthetic example to estimate anywhere from 1 to 6 unknown parameters. Notice that, unlike the case of surface responses, inversion is posed to honor the complete curve of fractional flow and not only its late-time component. This strategy proves critical when we perform inversions with field data. Notice that a new RSM with 6 unknowns was performed for the exercises considered in this section that rendered slightly different initial guesses from those reported in Table 1. We observe that the estimation of either one or both relative permeability end-points ($k_{rw}^0$ and $k_{ro}^0$) is well-constrained when the inversion is performed with accurate knowledge of the remaining parameters included in Equation (5). In fact, the inversion remains stable when posed to simultaneously estimate the two parameters. That is not the case when we add $\eta$ and $P_{cc}$ as unknown parameters in the inversion: the minimization is trapped in some local minima away from the correct solution. These observations confirm what was previously reported by Alpak et al. (2008) and Zeybek et al. (2004), who found that the simultaneous inversion of the remaining parameters was ill-constrained and highly dependent on the initial guess and a-priori knowledge of unknown parameter. The central technical feature of our estimation method is that we split the inversion into several inner loops to progressively increase the number of unknowns without losing stability and by reducing adverse convergence effects due to non-uniqueness. This is achieved by attaching different weights to the various measurements in order to progressively enhance the sensitivity of the data to specific inverted parameters.

Consider, for instance, the estimation of 4 unknown parameters via simultaneous inversion with 2 and 3 inner loops. The 3-loop example inverts first for $k_{rw}^0$ and $k_{ro}^0$ (loop 1) then for $\eta$ (loop 2), then for $P_{cc}$ (loop 3), and repeats the process until convergence is achieved. Likewise, the corresponding sets of weights attached to the cost functions of pressure, fractional flow, and oil-breakthrough time (in that order) change as follows: [0.15E-01, 0.37E+02, 0.11E+01] for inner loop 1, [0.63E+03, 0.23E-02, 0.11E-02] for inner loop 2, and [0.15E-02, 0.15E-01, 0.46E+04] for inner loop 3, respectively. Evidently, such a strategy places more weight on $f_w$ measurements when estimating $k_{rw}^0$ and $k_{ro}^0$, more weight on pressure
Table 2. Summary of results obtained with the synthetic example to assess the performance of different inversion techniques and their relationship to the number of unknown parameters included in the estimation process.

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of Unknowns</th>
<th>Parameter</th>
<th>Initial Guess</th>
<th>Actual Value</th>
<th>Final Result</th>
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</thead>
<tbody>
<tr>
<td>Simultaneous Inversion</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td></td>
<td>$k_r$</td>
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<tr>
<td>2</td>
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<td>0.55</td>
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<tr>
<td>3</td>
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<td>0.60</td>
</tr>
<tr>
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<td>$k_r$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$\eta$</td>
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<td>2.00</td>
<td>1.89</td>
</tr>
<tr>
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<td>$k_r$</td>
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<tr>
<td>6</td>
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<td>$k_r$</td>
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measurements when estimating $\eta$, and more weight on oil-breakthrough time when estimating $P_{ce}$. Results are more stable and accurate when we use this weighing scheme than when performing the 2-loop and simultaneous inversion strategies. Furthermore, it can be observed that for the case of 6 unknown parameters (now including $S_{w}$ and $S_{o}$), the 4-loop inversion strategy yields satisfactory results although it requires a larger number of iterations to achieve convergence. Fig. 6 describes the convergence rate for the latter case. Although not shown here, the 6-unknown inversion is significantly affected by the presence of several local minima in the cost function; however, by securing a good initial guess with the RSM, the inversion algorithm is stable and converges to values close the actual MBC parameters used to construct the synthetic example.

Field Examples

General Description. We tested the inversion method with field data acquired in two wells located in a Norwegian field. In both field examples (here referred to as A and B), fluid sampling was performed with a dual-packer module and resistivity measurements were acquired with laterolog tools. Initial estimates of $k$, $S_{w}$, and $r_{invasion}$ after main loop 1 (see Fig. 1) were reported and discussed in a separate publication (Angeles et al., 2008). For the examples considered in this section, we continued with main loop 2 (the refinement of $k_r$ and $P_{ce}$ parameters), and performed additional iterations between main loops 1 and 2 until achieving final convergence. Perhaps the most significant differences between the inversions for Field Examples A and B are in the approach to relate the different values of layer permeability and in the construction of the multi-layer formation model. Field Example A takes advantage of available NMR electrofacies processing to choose the location of the numerical layers and, at the same time, to relate the different layers to a much smaller number of electrofacies based on the premise that they exhibit similar pore-size distributions. This strategy proved adequate to perform the inversion because instead of dealing with 18 unknown values of layer permeability, only 6 unknowns were included in the estimation. On the other hand, due to lack of NMR electrofacies processing, Field Example B uses the following porosity-permeability correlation formula:

$$
\log k_i = b_i \phi_i + a_i,
$$

where $k_i$ is layer permeability, $\phi_i$ is layer porosity, and $a_i$ and $b_i$ are correlation parameters. The above transformation was validated in several nearby wells and it allowed us to reduce a set of 21 unknown parameters (21 numerical layers) into 2 correlation parameters ($a_i$ and $b_i$). In both field examples, we obtained satisfactory values of $k$, $S_{w}$, and $r_{invasion}$ with the estimation strategies described earlier.
Fig. 6. Convergence of MBC parameters as a function of nonlinear iteration number for the case of a 4-loop, 6-unknown inversion strategy implemented on the synthetic example. As indicated by the plots, the estimation of most of the unknown parameters converges after 200 iterations.

Table 3. Summary of rock, fluid, and petrophysical properties for Field Example A

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
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<tr>
<td>Oil density</td>
<td>g/cc</td>
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<tr>
<td>Oil viscosity</td>
<td>cp</td>
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<td>Mud-filtrate salt concentration</td>
<td>ppm</td>
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</table>

Field Example A. Fluid sampling was performed at X456.0 ft MD in a marine Jurassic reservoir consisting of shaly sandstone with thin siltstone beds and dolomite stringers. Fig. 7 shows the location of the formation test and the petrophysical evaluation for the studied area. Fig. 8 describes the corresponding numerical layering and associated NMR electrofacies. Table 3 summarizes the assumed rock, fluid, and petrophysical properties in the numerical simulations for this example. Upon completing the initial estimates of loop 1, we applied the same inversion strategy for 4 loops and 6 unknowns as in the synthetic example. As initial guess we used the one found from RSM analysis performed for the synthetic example given the semblance of that example with Field Example A.

Contrary to what was expected from the synthetic example, the same weight strategy was not successful and hence required a few modifications. The estimate for \( k_{rw}^0 \) increased dramatically to 1, values of \( \eta \) and \( P_{ce} \) remained in the vicinity of the initial guess, and values of \( Sw_r \) and \( Sor \) drastically decreased below 0.01 (clearly unrealistic values). An explanation for this behavior was found in the fractional flow measurements. Previously, for the inversion of measurements we used the complete set of available \( f_c \) measurements (including early- and late-time intervals). Field measurements, however, exhibit a drastic decay in mud-filtrate contamination immediately after the oil-breakthrough time. In addition, we performed several sensitivity analyses and found that, regardless of the combination of MBC parameters considered for inversion, the simulator could not match the low values of filtrate contamination observed in the late-time input measurements. This behavior suggested that there were physical phenomena that were not taken into account by the simulations (e.g. different slip velocities and buoyancy effects at the sandface, within the wellbore section limited by the packers, and in the flow line between the borehole and the optical sensor). Consequently, we discarded time samples acquired between the oil-breakthrough time and the late-time asymptotic response, although at the expense of losing information that proved valuable for the inversion of
Fig. 7. Petrophysical analysis and location of the formation tester for Field Example A (left panel) and corresponding numerical layering constructed via NMR electrofacies analysis (right panel).

Fig. 8. Numerical multi-layer model constructed in the depth-segment of study and corresponding NMR electrofacies for Field Example A. The model consists of 18 layers whereas the estimation considers only 6 unknown parameters.

Fig. 9. Comparison of initial-guess and inverted relative permeability and capillary pressure curves for Field Example A.
synthetic measurements (previous inversion case). This also explains why in Figs. 3 and 4 we only show RSM surfaces constructed exclusively with the late-time portion of the input \( f_w \) transient measurements.

After modifying the weight scheme as suggested by the constructed RSM surfaces, we achieved good convergence for the estimation of the 6 MBC parameters. One additional iteration was then required between main loops 1 and 2 to obtain the results shown in Table 4. Whereas Fig. 9 describes the estimated relative permeability and capillary pressure curves, Fig. 10 illustrates the good match between simulations and measurements of transient pressure and fractional flow. In addition to the final estimated parameters, Table 4 reports the associated uncertainty bounds, obtained by adding a small perturbation to each inverted parameter and calculating the corresponding variation of one of the cost functions (in this case, the quadratic cost function related to transient pressure measurements). The perturbed cost function was then compared to a base value above which the parameter was deemed uncertain, such as the value of noise in pressure measurements (±3 psi). Parameters that lead to large perturbations in the cost function exhibit short uncertainty brackets. In this field example, even though the largest uncertainty appears to be in \( \eta \), parameters \( S_{wr} \) and \( S_{or} \) were strongly dependent on the weight scheme adopted during inversion, which could indirectly increase their corresponding uncertainties.

Field Example B. The next example is derived from a well located in the same hydrocarbon field as Field Example A. It includes a sandy section extending from X095 ft to X160 ft and considers a rock formation similar to that of the previous field example, comprising sandstones with thin claystone beds grading into siltstones. Fig. 11 shows the location of the formation test. Unlike the previous example, Field Example B includes formation-tester measurements acquired with an interval pressure transient test (IPTT), and with one monitoring probe in addition to the dual-packer pressure probe. The monitoring probe was located 6.5 ft above the center of the packer interval (3 ft) and the test was performed at X117.8 ft, which included a final buildup stage after 1.5 hours of fluid sampling. Table 5 summarizes the rock, fluid, and petrophysical properties for this example. There were no NMR electrofacies available and hence we designated the numerical layers based on the calculated permeability log, which was previously correlated to core data with the constants \( a_k=-4.07 \) and \( b_k=20.932 \). The latter parameters were used in an initial set of iterations for main loop 1 until the numerical simulations of resistivity and formation-tester data matched the corresponding field measurements (Angeles et al., 2008). As done with Field Example A, we used the resulting layer values of \( k \), \( S_{wr} \), and \( r_{invasion} \) to further refine the relative permeability and capillary pressure curves.

Fig. 12 shows the final match between measured and simulated values of pressure and fractional flow. The match is very good, especially at 6000 seconds after the inception of the test. Measurements acquired at early times were not adequately matched (as in Field Example A), even with single-phase analytical solutions, possibly because of physical phenomena not accounted for by the multi-phase numerical simulations, including fines migration, tool and wellbore storage, and/or formation damage. Fig. 13 compares the inverted \( k_r \) and \( P_c \) curves to actual core measurements acquired in the same well at X115.5 ft. Table 6 summarizes the final inverted MBC parameters along with their corresponding uncertainty bounds. We observe that uncertainty bounds are similar to those of Field Example A.
Fig. 11. Petrophysical description of the depth segment considered for Field Example B, indicating the location of both the formation tester packer (X117.8 ft) and the monitoring probe (X111.3 ft).

Fig. 12. Comparison of time records of packer pressure (upper left panel), probe pressure (upper right panel), and fractional-flow measurements (lower panel) against the corresponding simulations obtained with inverted MBC parameters for Field Example B.

Discussion
Our estimation method is based on the use of a generic reservoir simulator and hence could be readily adapted for its application to the analysis of general fluid-displacement phenomena, such as cases of OBM mud-filtrate invasion or gas/gas condensate displacement. One limitation is the forward modeling of borehole resistivity measurements, which should be performed for the specific resistivity tool used to acquire the measurements. A salient question is whether transient pressure measurements alone are sufficient to reliably and accurately estimate the entire set of unknown parameters included in Equation (5). This question was partially answered by Angeles et al. (2008), where it was shown that both resistivity and formation-tester measurements contribute to the reduction of pervasive non-uniqueness in the estimation of $k$, $S_w$, and $r_{invasion}$. 
As observed in Table 2, the inversion of 6 unknown parameters is challenging in itself; inverted estimates would be rendered uncertain if $k$, $S_w$, and $r_{invasion}$ were estimated from transient pressure measurements alone. The synthetic and field examples of estimation described in this paper indicate that it is only possible to reliably and accurately estimate the MBC parameters when $k$, $S_w$, and $r_{invasion}$ are calculated from measurements other than those acquired with formation testers. Uncertain and biased estimates of $k$, $S_w$, and $r_{invasion}$ necessarily lead to biased estimates of MBC parameters.

Another key step included in the inversion of the 6 MBC parameters is the data weighing scheme adopted in this paper, which allowed us to overcome the difficulties reported in similar and previously documented estimation projects (Zeybek et al., 2004; Alpak et al., 2008). Our inversion examples conclusively indicate that the stability of the estimation process improves by splitting the contributions from different measurements during progressive stages of inversion focused on a given unknown parameter. However, we also reported that a different data weighing scheme was necessary for the inversion of synthetic and field measurements depending on whether the inversion was performed with either the full set of transient measurements of $f_w$ or a subset of them (e.g. the late-time response). This is a matter of concern which inevitably leads one to inquire whether the inversion should be performed with all or part of the $f_w$ transient measurements, and whether a given time window of the $f_w$ transient measurements could be reliably simulated with the physics model assumed in this paper. As emphasized earlier, transient optical/spectroscopy measurements are not acquired at the sandface but rather some distance away inside the tool flow line. If significant fluid redistribution occurs through the flow line, fractional flow-rate measurements will no longer be representative and hence biases will arise in the estimation of parameters such as $S_{wr}$ and $S_{or}$, which heavily depend on those measurements. We could expect similar difficulties in cases of deeply invaded low-permeability formations where the duration of fluid sampling may not be sufficient to achieve fluid cleanup. Further studies are necessary to avert, and possible correct transient measurements of fractional flow rate affected by tool-storage and flow line effects prior to using them for the inversion of MBC parameters. Ironically, inversion studies such as the ones reported in this paper could be used to assess the quality, reliability, and internal consistency of transient measurements of pressure and fractional flow rate acquired with formation testers. Data inconsistencies will be detected by the inability of the inversion method to simultaneously match all the available measurements. In the examples considered, we implicitly assumed that fluid rates driven by the fluid pump were accurately measured. In practice, however, those measurements will also be affected by
electronic and mechanical malfunctioning that, when unaccounted for, could bias the estimation of unknown MBC parameters.

Even though we performed the estimation of $P_e$ and $k_r$ curves based on the modified Brooks-Corey equation, alternative representations exist to describe this saturation-dependent functions, including Burdine-consistent B-spline and power-law models. From an inversion point of view, power-law models are appealing because of their relatively small number of free parameters. Unfortunately, often the flexibility of B-spline models is needed to accurately represent the convexity of relative permeability curves. It was observed by Watson et al. (1988) that relative permeability of core samples could be adequately described with B-spline models of dimension 6. This represents a total of 12 unknown parameters in addition to $k_r$, $S_{or}$, $S_{or}$, and $r_{invasion}$ which would severely complicate the estimation process if not make it unfeasible with the use of resistivity and formation-tester measurements alone. Additional independent measurements would be necessary for the accurate and reliable estimation of the extended set of unknown parameters. Similarly, the estimation of non-equal models of imbibition and drainage to describe relative permeability and capillary pressure curves would be possible only with additional independent measurements. In closing, we conjecture that a miniaturized version of the acquisition and inversion procedure considered in this paper could be possible for the estimation of relative permeability and capillary pressure properties of core samples.

Conclusions

Synthetic and field examples indicate that it is possible to reliably estimate $k_r$, $S_{or}$, $r_{invasion}$, together with the entire set of modified Brooks-Corey parameters ($k_{rw}^0$, $k_{ro}^0$, $\eta$, $P_c^0$, $S_{or}$, and $S_{or}$) using a combination of resistivity, pressure, and fractional flow rate measurements. The estimation method advanced in this paper is based on splitting the inversion into several sequential iterative loops that enhance the sensitivity of the measurements to a given unknown parameter by applying differential weights to measurement subsets. Overall, we found that $S_{or}$ and $S_{or}$ are difficult to estimate when transient measurements of $f_w$ are biased and/or inaccurate. Using only the late-time asymptote of $f_w$ measurements (at end of fluid sampling) removes important dynamic information about the saturation-dependent fluid-displacement process, thereby increasing the non-uniqueness of the estimation. On the other hand, end-point relative permeabilities can be reliably estimated if the remaining MBC parameters are assumed known. Parameters $\eta$ and $P_c^0$ can also be reliably estimated with the proposed inversion method although with relatively larger uncertainties than $k_{rw}^0$ and $k_{ro}^0$.

The estimation method proposed in this paper includes the use of DoE surface responses to secure a good initial guess of unknown parameters and to diagnose insufficient measurements with poor sensitivity to unknown parameters. The DoE approach also permits to quantify a priori the differential sensitivity of the various measurements to specific unknown parameters, thereby reducing potential biases and instabilities in the inversion. The same information could be used to select differential data-weighing schemes for nonlinear inversion.

Experience with the inversion of field data sets indicates that inversion itself is a valuable procedure to verify the accuracy, reliability, and internal consistency of measurements acquired with formation testers. Biases in the measurements due to storage, plugging, pump, and flowline effects are often difficult to detect and quantify without numerical simulations. Data inconsistencies will be detected by the inability of the inversion procedure to simultaneously honor all the available measurements under the assumed model conditions.

Nomenclature

- $B$ : Formation volume factor, [RB/STB]
- $C_{mf}$ : Conductivity of mud-filtrate, [mho/m]
- $C_w$ : Conductivity of connate water, [mho/m]
- $C(x)$ : Cost function, [ ]
- $e(x)$ : Vector of data residuals, [ ]
- $F_w$ : Fractional flow rate, [fraction]
- $h$ : Layer/formation thickness, [ft]
- $J(x)$ : Jacobian matrix, [ ]
- $k$ : Formation permeability, [md]
- $k_{rw}$ : Relative permeability of water, [ ]
- $k_{rw}^0$ : End-point relative permeability of water, [ ]
- $k_{ro}$ : Relative permeability of oil, [ ]
- $k_{ro}^0$ : End-point relative permeability of oil, [ ]
- $M$ : Number of measurements, [ ]
- $N$ : Number of model parameters, [ ]
- $P$ : Pressure, [psi]
- $P_c$ : Capillary pressure, [psi]
- $P_{ce}$ : Capillary entry pressure, [psi]
- $\Delta P_{skin}$ : Pressure drop due to skin, [psi]
- $q$ : Production flow rate, [STB/day]
- $R(x)$ : Response surface, [ ]
- $R_w$ : Water resistivity, [$\Omega$-m]
- $R_t$ : Total resistivity, [$\Omega$-m]
- $r_{invasion}$ : Radius of invasion, [ft]
- $S$ : Skin factor, [ ]
- $S_{or}$ : Residual oil saturation, [fraction]
- $S_v$ : Water saturation, [fraction]
- $S_{or}$ : Irreducible water saturation, [fraction]
- $S_{wn}$ : Normalized water saturation, [fraction]
- $\mathbf{W}_d$ : Data weight matrix, [ ]
- $\mathbf{x}$ : Vector of model parameters, [ ]
- $\alpha$ : Lagrange multiplier, [ ]
- $\beta$ : Eigenvalue, [ ]
- $\eta$ : Pore size distribution index, [ ]
- $\phi$ : Porosity, [fraction]
- $\mu$ : Viscosity, [cp]
- $\sigma_{deep}$ : Deep apparent conductivity, [mho/m]
- $\sigma_{shallow}$ : Shallow apparent conductivity, [mho/m]
Acronyms

CMG : Computer Modeling Group Ltd.  
DoE : Design of Experiment  
IMEX : Black Oil Simulator from CMG  
IPTT : Interval Pressure Transient Test  
LM : Levenberg-Marquardt  
MBC : Modified Brooks-Corey  
MD : Measured Depth  
NMR : Nuclear Magnetic Resonance  
OBM : Oil-Based Mud  
RSM : Response Surface Method  
UT : The University of Texas at Austin  
UTFET : UT Formation Evaluation Toolbox  
WBM : Water-Based Mud  
WRGN : Weighted Regularized Gauss Newton

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References


