Numerical Simulation of 3D EM Borehole Measurements Using an $hp$-Adaptive Goal-Oriented Finite Element Formulation

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SUMMARY

We describe two different algorithms for automatic generation (without any user interaction) of optimal goal-oriented grids within a $3D$ higher-order Finite Element (FE) package, in context of simulation of electromagnetic borehole resistivity measurements for the assessment of rock formation properties.

The first algorithm is based on transferring a two-dimensional optimal grid obtained from a $2D$ self-adaptive method. The resulting grid enables fast simulations (few seconds per logging position), although the accuracy of the final $3D$ results may be compromised in the case of highly deviated wells when combined with rapid spatial variations in electrical conductivity. For those cases, we developed a second algorithm based on a fully automatic three-dimensional goal-oriented strategy that, although it is considerably slower, it guarantees accurate results. A proper combination of both algorithms enables highly accurate numerical simulations of a variety of $3D$ resistivity logging measurements at different frequencies. This is illustrated in the manuscript with a Through Casing Resistivity (TCR) model problem.

Numerical results corresponding to laterolog measurements indicate that differential voltage measurements remain sensitive to the $3D$ effects of various $3D$ sources and receivers only in deviated wells. In vertical wells, measurements are insensitive to the $3D$ effects of the sources and receivers. Thus, the resulting $3D$ combined effect (due to the use of $3D$ sources and receivers and deviation of the well) is different from the superposition of individual $3D$ effects.

INTRODUCTION

During the last decades, a number of $3D$ simulators of resistivity logging measurements have been developed within the oil industry. These simulators have been successfully applied to study and analyze different physical effects occurring on three-dimensional problems (see, for example, (Wang and Signorelli, 2004), (Wang and Fang, 2001), (Newman and Alumbaugh, 2002), and (Davydycheva et al., 2003)).

Despite these recent advances, there are still a large number of three-dimensional effects for which reliable simulations are not available. An example of this is the effect of a dip angle in a cased well. Furthermore, in most of the existing results, only a partial validation of those results is reported, typically obtained by comparing solutions of simplified model problems against the corresponding solutions obtained with a lower dimensional ($2D$ or $1D$) numerical method.

This lack of $3D$ results (as opposed to $2D$ results) is due to major difficulties that are encountered when solving challenging $3D$ problems. Namely, for mesh-based methods (Finite Elements, Finite Differences, Boundary Elements, etc.), the size of the system of linear equations becomes excessively large to be solved within a few minutes/hours. These difficulties are magnified when considering cased wells. While steel casing provides mechanical competence that prevents the well from collapsing, especially across soft rock formations, presence of electrically conductive casing poses significant challenges to the probing of electrical properties behind casing. More precisely, obtaining either accurate physical measurements and/or reliable numerical simulations of resistivity measurements in the presence of steel-cased wells is of great difficulty, mainly because only the secondary field is proportional to the formation conductivity. The secondary field is several orders of magnitude smaller than the primary field induced by casing.

Despite the difficulties associated with the numerical modeling of Through Casing Resistivity (TCR) measurements, in this work we have developed a methodology that accurately simulates TCR measurements acquired in high-angle wells penetrating layered formations. Furthermore, the methodology is also suitable for simulations of different logging instruments, including laterolog and induction based instruments such as Logging-While-Drilling (LWD).

Our methodology is based on a $3D$ Finite Element Method (FEM) that supports elements of different size $h$, and different polynomial order of approximation $p$. Intuitively, small elements with low order of approximation are ideal to approximate singularities and rapid variations in the solution, while large elements with high $p$ are employed in areas where solution is smooth. Thus, the $hp$-FEM enables highly accurate (exponentially convergent) simulations — typical of spectral methods —, even in presence of singularities.

In addition, our methodology incorporates two different state-of-the-art self-adaptive goal-oriented $hp$-Finite Element strategies — for automatic generation of optimal grids —, a two-grid (multigrid) solver of linear equations — the fastest possible type of linear equation solver —, and a parallel implementation. We have also implemented an additional software that allows for transferring two-dimensional $hp$-refined optimal grids into the $3D$ code.

The resulting methodology has enabled us to study a large variety of $3D$ electromagnetic effects occurring in a borehole environment, such as the study of deviated wells, patched antennas, cased wells, laterolog and induction instruments, etc. In particular, we obtain $3D$ simulations of TCR measurements with relative errors below 5%. This level of accuracy is not possible with neither finite-difference formulations nor standard implementations of the finite-element method.

METHODOLOGY

Our methodology is suitable for arbitrary time-harmonic electromagnetic ($3D$) problems governed by Maxwell’s equations in the frequency domain. In the following, we outline the main ideas of our numerical methodology used for simulating $3D$ EM logging problems in a borehole environment. More precisely, we present two different algorithms for computer generation of optimal grids, as well as the iterative solver of linear equations.

To avoid geometry-induced numerical errors, our FEM combines the use of $hp$-FE discretizations with exact geometry elements.

Automatic computer-generation of optimal grids.

Algorithm I: $2D$ self-adaptive goal-oriented strategy for $3D$ problems.

This algorithm employs optimal $2D$ grids for axi-symmetric problems. These $2D$ grids are obtained using the self-adaptive goal-oriented $hp$-FEM described in (Pardo et al., 2006b,a) and verified in (Paszynski et al., 2005). After automatically generating an optimal $2D$ grid for the axi-symmetric problem associated to our $3D$ application of interest, we transfer the optimal $2D$ $hp$-grid to the $3D$ $hp$-FE software, by employing either four or eight second-order elements in the azimuthal direction. Then, the resulting $3D$ $hp$-FE grid is tilted to account for deviated wells. Also, three-dimensional sources may be considered at this point. Finally, our problem of interest is solved within the $3D$ $hp$-FE software setting. An analysis of the error of the final $3D$ results is available by considering a finer (globally $p$-refined) three-dimensional grid and comparing results obtained with both grids.
The total time required to simulate 80 logging positions using this algorithm is typically 10-60 minutes in a 3.0 Ghz Pentium IV processor using a direct solver of linear equations. Optimal grid generation is based on 2D algorithms, and therefore, it has a low computational cost. Optimal 2D grids contain typically 2K-10K unknowns, which translates into 20K-200K unknowns for the corresponding 3D after incorporating four or eight second-order elements in the azimuthal direction.

Algorithm II: 3D self-adaptive goal-oriented strategy.

This algorithm directly generates optimal 3D grids. More precisely, it produces a sequence of optimally hp-refined meshes that delivers exponential convergence rates in terms of a user prescribed quantity of interest against the size of the discrete problem or CPU time. The algorithm requires from the user to input a human-made conforming initial hp-grid. Then, the algorithm iterates along the following steps.

First, we solve our problem of interest in our current hp-grid that we shall denote as coarse-grid.

Second, we perform a global hp-refinement to obtain the corresponding \( h/2, \ p + 1 \)-grid, that is obtained from the coarse grid by breaking all elements into eight new elements, and raising the polynomial order of approximation, \( p \), uniformly by 1. We shall denote the resulting grid as the fine-grid. The fine-grid contains typically about 20 times more unknowns than the corresponding coarse grid.

Third, we solve our problem of interest on the fine-grid with the two-grid solver described below, and we estimate the coarse-grid error by computing the difference between fine-grid and coarse-grid solutions. If the computed coarse-grid error estimation is below the desired error tolerance, then we stop our iterations and provide the fine-grid solution as the ultimate result. The fine-grid solution error is expected to be about one or two orders of magnitude smaller than the estimated coarse-grid error.

Fourth, we utilize the fine grid solution to guide optimal refinements over the coarse grid. More precisely, we utilize the projection-based interpolation operator \( \Pi \) defined in Demkowicz and Buffa (2005) for various fine element space to pose the following optimization problem:

\[
\hat{h}p = \arg \max_k \sum_k \frac{\|E_k - \Pi_k E_k\|_2^2}{\|G_k - \Pi_k G_k\|_2^2},
\]

where \( E_k = E_{k+1}, G = G_{k+1} \) are the fine-grid solutions of the so-called direct and dual problems, respectively (see (Pardo et al., 2006) for details), \( K \) indicates an element, and \( \Delta N > 0 \) is the increment in the number of unknowns from the coarse grid \( h_p \) to the intermediate grid \( \hat{h}p \). This maximization problem is solved by executing an algorithm based on the sequential optimization of edges, faces, and interiors of elements. This algorithm, which is quite involved (Kurtz and Demkowicz, 2006), produces a list of elements to be refined, as well as their corresponding optimal refinements.

Finally, we execute optimal refinements as determined in the previous step in order to construct our next optimal coarse-grid. At this point, we discard the fine-grid.

A two-grid (iterative) solver of linear equations.

We have developed a two-grid solver of linear equations based on block-Jacobi smoother iterations on the fine grid combined with a global solution on a coarse grid (also called coarse grid correction). A fine grid edge-based overlapping block-Jacobi smoother has been employed to avoid degeneration of convergence properties due to the presence of elongated elements. This two-grid cycle is accelerated by using a goal-oriented steepest-descend method.

Typically, the iterative solver converges in less than fifteen iterations, even in presence of elongated elements with an aspect ratio up to 100000:1. Such a high aspect ratio is present in several of our TCR simulations, due to the elongated geometry of the casing.

NUMERICAL RESULTS

Model problems

Figure 1: 2D cross-section of the geometry of the laterolog (top panel) and TCR-type (bottom panel) problems. Each problem contains four different layers in the formation with varying conductivities, one transmitter electrode, and three receiver electrodes. The laterolog problem incorporates a resistive mandrel, and the TCR-problem a 1.27 cm thick conductive casing.

We consider two different resistivity logging instruments, as described in Figure 1: (a) a laterolog instrument, and (b) a TCR-type problem. In both applications we consider the Direct Current (DC) approximation at zero frequency, that is, \(-\nabla \sigma \nabla u = f\), where tensor \( \sigma \) represents the conductivity of the media, solution \( u \) is the scalar potential, and \( f \) is the load (divergence of the impressed electric current). We measure the second vertical difference of the potential, which is an approximation to the vertical derivative of the vertical component of the electric field.
For the laterolog problem, we consider three different types of electrodes:

- a) **Full electrode.** The electrode is axi-symmetric, like a solenoid.
- b) **Patch electrode.** The electrode contains four ten-degree patches, located at the following azimuthal angles: 0-10 degrees, 90-100 degrees, 180-190 degrees, and 270-280 degrees. To compare results with those obtained by the full electrode, we consider a load \(f\) with 9 times larger amplitude (per volume unit) than the load corresponding to the full electrode.
- c) **One patch electrode:** The electrode contains one ten-degree patch, located at the following azimuthal angle: 0-10 degrees (for the transmitter), and 180-190 degrees (for the receiver). To compare results with those obtained by the full electrode, we consider a load \(f\) with 36 times larger amplitude (per volume unit) than the load corresponding to the full electrode.

**Simulations of laterolog measurements**

First, we consider the laterolog model problem. Results have been obtained using ‘Algorithm I’. After comparing the final results delivered by ‘Algorithm I’ against those obtained in a globally \(p\)-refined grid, we have concluded that the error remains below 5%, which is sufficient for an accurate physical interpretation of the measurements.

Figure 2 displays the final logs for the three different types of electrodes, and two dip angles: 0-degree (top panel), and 60-degree (bottom panel). In the axisymmetric case (vertical well), results remain insensitive to the type of electrode. For the 60-degree deviated well, we observe the following physical effects: 1) Detection of the conductive layer is slightly shifted, as it typically occurs in deviated wells; 2) Readings corresponding to full-electrodes are lower in deviated wells than in vertical wells, due to the anisotropy on the highly conductive layer of the formation; 3) Measurements in deviated wells are sensitive to the different types of electrodes, demonstrating that the superposition of individual 3D effects is different from the combined effect.

In Figure 3 we display a 2D cross-section of the solution (scalar potential) in a 45-degree deviated well. We observe the grid-adaptation that Algorithm I performed towards the transmitter, receivers, and singularity points generated by three or more materials with different conductivities meeting at a point. Notice that the polynomial order of approximation \(p\) also varies (it is not displayed in the figure).

**Simulations of TCR-type measurements**

We consider the TCR-type model problem described above. We first employ ‘Algorithm I’. Results corresponding to 0-, 30-, and 60-degree deviated wells are displayed in Figure 4 — top panel —. Then, we perform a global \(p\)-refinement over the final grid delivered by ‘Algorithm I’, and we display the corresponding results in Figure 4 — bottom panel —. We observe a strong discrepancy in the results, which indicates that results produced by ‘Algorithm I’ are inaccurate for this problem. The relative error is above 1000% in the case of highly deviated wells, while it remains below 1% for the axi-symmetric case. This occurs because an optimal 2D axi-symmetric grid may not be adequate for highly deviated wells. We note that accurate simulations require from different grids in the \((\rho, z)\)-plane as we vary the dip angle.

As a remedy for the inaccurate results obtained with ‘Algorithm I’, we employ ‘Algorithm II’. Results are displayed in Figure 5. Accuracy of results is guaranteed in this case, since the relative error between the final \(hp\)-grid and the globally \(hp\)-refined grid, that is, the \(h/2, p + 1\)-grid, remains below 5%. Indeed, we observe that results are physically consistent.

As we increase the dip angle, we note a lower intensity reading in the highly conductive layer, which seems to be thicker than in the vertical
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well. This occurs because in deviated wells we measure an average of the conductivity of the different layers in the plane perpendicular to the well.

We also emphasize that for the TCR problem, a physical interpretation of results obtained with 'Algorithm I' is very dangerous, and will lead to unphysical conclusions.

CONCLUSIONS

We have described a methodology that provides accurate simulations of various 3D resistivity logging measurements. In particular, we have proposed two different algorithms for computer-aided automatic grid generation.

The first algorithm is based on employing optimal 2D grids. Similar ideas of employing 2D axi-symmetric grids for solving 3D problems are widely used within the oil-industry. We have demonstrated that these grids may be accurate and fast for a number of applications (for example, laterolog problems), but they may also provide highly inaccurate results in other applications (for example, TCR measurements). For the most challenging problems, we employ a different grid generation algorithm that, although it is computationlly more expensive, it guarantees accurate results.

Physically, we have also demonstrated with a laterolog example that it is necessary to consider all 3D effects in one simulation, since combination of several 3D effects is more complicated than the simple superposition of individual 3D effects.

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