**SUMMARY**

We develop a new finite-element method (FEM) to simulate wireline (WL) borehole acoustic waveforms. It works in the frequency domain, is fully adaptive, and enables accurate simulations in the presence of large contrasts of elastic properties. Coupling of acoustic and elastic wave propagation in the borehole and the formation, respectively, is achieved with a new, stable multi-physics formulation. In addition, the method includes a Perfectly Matched Layer (PML) technique for effective truncation of the spatial domain. Examples of applications are shown for the simulation of WL sonic waveforms acquired with monopole and dipole sources. Results indicate accurate and reliable reproduction of tool borehole, and formation propagation modes in the presence of fractures (we show this example thoroughly in this paper), layered formation, and a steel casing.

**INTRODUCTION**

Wireline sonic logging is one of the common techniques used by oil companies to measure elastic properties of rock formations. Numerical algorithms which can accurately model acoustic wave propagation in the borehole environment are an important tool to interpret sonic waveforms acquired in such adverse environments. In addition, it is imperative that such simulation algorithms concomitantly model tool and formation properties for reliable assessment of rock properties. To simulate efficiently this class of problems, we developed a new version of a fully automatic \( hp \)-adaptive multi-physics FEM code (Matuszyk et al., 2011). It optimally adapts the mesh for each frequency, both with respect to element sizes \( h \), and polynomial orders of approximation \( p \), delivering exponential convergence rates for all simulation problems. The use of high-order methods drastically reduces dispersion errors, delivering accurate and reliable simulation results. Furthermore, the \( hp \)-FEM is ideally suited to solve boundary layers that necessarily arise from the use of PML, which we use to truncate the spatial domain.

**THEORY AND METHOD**

We consider an axially symmetric geometry for the borehole environment with an arbitrary number of vertical concentric layers (e.g., tool, borehole, casing) followed by an arbitrary number of horizontal layers (formation composed of rocks and fluid filled fractures). Each of the components can be considered as an acoustic fluid (\( \Omega_A \)) or a linear elastic/viscoelastic solid (\( \Omega_E \)). Wave propagation phenomena for each of the components is defined through appropriate classical partial differential equations (Tang and Cheng, 2004). Acoustic/elastic coupling is achieved by imposing weak coupling conditions on the interface \( \Gamma_I = \Omega_A \cap \Omega_E \).

**Weak formulation for the coupled acoustic-elastic problem**

The problem is posed in the frequency domain and solved for a particular angular frequency \( \omega_n \). Accordingly, weak the form of the coupled problem reads as

\[
\begin{align*}
\text{Find } (u, p) & \in (\bar{u}, \bar{p}) + H^1_0(D) \times H^1_0(D) : \\
\{ & (\nabla p, \nabla q)_{\Omega_E} - (k_n^2 p, q)_{\Omega_A} - (\rho_f \omega_n^2 q, \n_f \cdot u)_{\Gamma_I} = (g_{ex}, q)_{\Gamma_{ex}} \\
& (C\nabla u, \nabla w)_{\Omega_E} - (p_n \omega_n^2 u \cdot w)_{\Omega_E} + (p_n u \cdot w)_{\Gamma_I} = 0,
\end{align*}
\]

where \( p(x, \omega_n) \) is a Fourier transform of the pressure in a fluid, \( u(x, \omega_n) \) is a Fourier transform of the displacement in a solid, \( \bar{p} \) and \( \bar{u} \) are appropriate Dirichlet data, \( k_n = \omega_n / c_f \), where \( c_f \) is the sound speed in the fluid, \( \rho_f \) and \( p_n \) are material densities, \( q \) and \( w \) are appropriate testing functions, and \( \n_f \) and \( \n_s \) are outward unit normal vectors defined for acoustic and elastic domains, respectively. For trial and test spaces, we take suitable subspaces of the scalar and vector Sobolev spaces \( H^1 \) and \( H^1 \) for acoustic and elastic domains, respectively. The source excitation term \( g_{ex} \) is defined on the boundary segment \( \Gamma_{ex} \subset \Gamma_A \). Components of the viscoelastic (complex) or elastic (real) tensor \( C(\lambda, \mu) \) are defined through solid density \( \rho_s \), reference speed for P- and S-waves, \( v_s^0 \) and \( v_p^0 \), respectively, as well as appropriate quality factors \( Q_p \) and \( Q_s \) (Aki and Richards, 2002), and are given by

\[
\begin{align*}
v_s &= v_s^0 \left(1 + \frac{1}{\pi Q_s} \ln \frac{f_n}{f_s} \right) \left(1 + \frac{i}{2Q_s} \right), \\
\lambda &= \rho_s (v_p^0)^2 - 2v_s^2), \\
\mu &= \rho_s v_s^2,
\end{align*}
\]

where \( f_n \) is frequency and \( x \) refers to either \( P \) or \( S \).

**Modeling of multipole acoustic sources**

A multipole source of order \( n \) can be constructed from the collection of \( 2n \) monopole point sources alternating in sign placed periodically along a circle of radius \( r_0 \) (Winbow, 1985). The resulting source radiation pattern exhibits \( \cos(n \theta) \) dependence, where \( \theta \) is the azimuthal angle. Thus, one can express such a source by generating a signal of initial amplitude \( p_0 \) given by

\[
g_{ex}^{(n)} = p_0 \cos(n \theta) = \frac{p_0 e^{-in \theta}}{2} + \frac{p_0 e^{in \theta}}{2} = g_n^+ + g_n^- n = 0, 1, \ldots
\]

It is important to notice that for \( n > 0 \) (e.g. dipole or quadrupole excitation) the axial symmetry of the problem is lost.

Due to the specific structure of the bilinear forms, having calculated the solution for excitation \( g_n^+ \), one can directly calculate the solution for excitation \( g_n^- \). Thus, the solution of the 3D problem (1) with an \( n \)-th order multipole source is reduced to the solution of the 2D problem on the trace domain \( (r, z) \) using solely the excitation \( g_n^+ \). Finally, the solution in 3D is given by

\[
p(x, \omega_n) = p^+ \cos(n \theta), \quad u(x, \omega_n) = \begin{bmatrix} u_x^+ \cos(n \theta) \\ u_y^+ \sin(n \theta) \\ u_z^+ \cos(n \theta) \end{bmatrix}.
\]
In the case of monopole excitation \((n = 0)\), the displacement vector exhibits only two components, \(u_r\) and \(u_z\).

Having calculated pressure \(p(x, \omega_0)\) in the Fourier domain for an appropriate range of frequencies \(\omega_0\), one transforms the solution into the time domain for arbitrary acoustic source via

\[
p(x, t) \approx \frac{\Delta \omega}{2\pi} \sum_{n=-c}^{c} S(\omega_n) p(x, \omega_0) e^{i \omega_0 t},
\]

where \(\Delta \omega\) is frequency spacing, \(c\) is the number of discrete frequencies used in the transformation, and \(S(\omega_n)\) is the spectrum of the acoustic source. We use the Ricker wavelet as acoustic pulse, which exhibits fast decay in the frequency domain, and thus enables a reduction in the number of frequencies needed to calculate accurately waveforms via the inverse Fourier transform.

**Truncating calculational domain using PML.**

Since the considered problem is posed in an unbounded spatial domain, its effective solution needs a properly truncated spatial domain. For this purpose, we use the Perfectly Matched Layer (PML) method (Michler et al., 2007, 2009).

In cylindrical coordinates \((r, \theta, z)\), the PML absorbing layer is modeled through complex stretching of the axial \((z)\) and radial \((r)\) coordinates for the given wavenumber \(k\), using the following transformations:

\[
x_j \rightarrow X_j(x_j, k), \quad \frac{\partial}{\partial x_j} = \frac{1}{X'_j} \frac{\partial}{\partial X'_j}, \quad \text{where} \quad X'_j = \frac{\partial X_j}{\partial x_j},
\]

and \(x_j\) and \(X_j\) represent unstretched and stretched coordinates, respectively. Then one obtains

\[
X_j = x_j + \frac{2r}{k} \frac{\xi'}{\xi} (1 - i)^m (z_j - x_j),
\]

where \(\xi' = \frac{x_j - x_j}{\delta r}\) and \(\xi = \frac{x_j - x_j}{\delta z}\).

The coordinates \(x_j\) and \(x'_j\) define the PML region for the coordinate \(x_j\). In the examples enclosed in this paper, we use \(p = 6\) and \(m = 3\). Due to exponential decay of the solution in the absorbing layer, homogenous Dirichlet boundary conditions are prescribed on the outermost boundary of the PML.

**Automatic \(hp\)-adaptivity.**

The self-adaptive \(hp\)-FEM (Demkowicz, 2007) is based on a two grid paradigm: a coarse mesh, and a fine mesh that is obtained from the former one by performing a global \(hp\)-refinement (each finite element of the coarse mesh is broken into four new son finite elements, and the polynomial order of approximation is uniformly increased by one). The fine mesh solution, \(u_{IF}\), is then used to guide optimal refinements of the coarse mesh to yield the next optimal coarse mesh. Next, \(u_{IF}\) is projected separately onto each coarse mesh element and onto a nested sequence of meshes that is locally embedded in the fine mesh. For each coarse mesh element, the sequence is built dynamically by testing selected types of local \(h\)-refinements and choosing the one that provides the maximum ratio of error decrease to the number of added degrees of freedom. This procedure is applied first for edges, and then for element interiors. Optimal refinement from the first step provides the minimal refinement for the next. Final optimal refinements for elements are possibly upgraded in order to maintain the 1-irregularity of the mesh and finally executed over the coarse mesh to generate the next optimal coarse mesh. The new version of the code enables simultaneous calculation for coupled multi-physics problems, where each physical field can be modeled in a different energy space.

**EXAMPLES**

As an example we chose the case of WL sonic logging in a non-cased borehole penetrating a homogenous formation with a fluid-filled fracture. We assume a WL tool (SLB Sonic Scanner) of diameter equal to 9.2 cm with an array of 13 equally-spaced receiver stations, and 6" inter-station spacing. Offset of the first station with respect to the acoustic source is equal to \(10.795^\circ\). The tool is positioned in the center of a borehole of diameter equal to \(8.625^\circ\). Outside, there is a fast formation with a fracture (see Figure 1). We consider several fracture thicknesses \((1, 3, 5\, \text{mm}, 1, 3, 5\, \text{cm})\), as well as a case without fracture for reference. Influence of fracture size on sonic waveforms is investigated for monopole and dipole sources. We use an acoustic pulse of central frequency equal to 8 kHz. Table 1 displays the physical parameters assumed for material properties.

![Figure 1: Geometry for WL logging with a fracture in the formation: (a) fracture below receivers, and (b) facing receivers.](image-url)
similar for each receiver. Presence of the fracture filled with a slower (in comparison to the formation) medium results in a very slight delay of the signal, proportional to fracture thickness, which is consistent with the underlying physics of wave propagation. The fracture has different impact on particular components of the sonic waveforms, corresponding to different acoustic modes present in the borehole.

Presence of the fracture has a small impact on the P-wave, decreasing slightly its amplitude with increased fracture thickness. There is no significant difference between the case with and without the fracture, which indicates insensitivity of the P-wave to presence and size of fracture. However, a fracture of any size significantly decreases the S-wave amplitude. The change in the amplitude corresponding to different fracture sizes is much smaller and does not change the phase of the signal as much as the presence of the fracture itself. This behavior indicates that the S-wave is sensitive to the presence of the fracture. In the case of the Stoneley mode, one observes that presence of the fracture changes the shape of the waveforms. Moreover, we observe a noticeable influence of fracture size on the Stoneley mode. Along with increased fracture size, the amplitude of the Stoneley mode gradually decreases. Additionally, the shift in the signal phase can be clearly observed, which suggests sensitivity of the Stoneley mode to the width of the fracture.

Figure 4 shows frequency dispersion curves obtained for the above cases. We show curves corresponding to the Stoneley and pseudo-Rayleigh modes (monopole excitation) and to the flexural modes (dipole excitation). Curves related to the reference case (no fracture) are identified with solid black lines. Presence of the fracture in front of the receiver array disturbs...
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significantly the calculated dispersion, which results in much less reliable dispersion curves. Curves fluctuate due to additional waveform variability induced by the fracture itself.

For the case of dipole excitation, we observe arrivals of the P-wave, S-waves, and the flexural mode. Because the physics of both propagation modes is strongly connected to the S-wave velocity in the formation, we observe a similar behavior. Presence of the fracture significantly changes the shape of the waveforms, primarily by decreasing their amplitudes. Fracture size has small influence on the S-wave packet, however it has a larger impact on the flexural wave. One observes larger changes in amplitude and phase of the signal for the latter case.

Figures 5 and 6 show waveforms obtained for monopole and dipole excitation, respectively, and a fracture facing the receivers. Here, the situation is different due to the distinct location of particular receivers with respect to the fracture. The front receiver is located below the fracture, whereby wave components corresponding to the borehole/formation modes remain unaffected. However, additional propagating modes arise at the final part of the waveforms. These are reflected waves from the fracture/formation interfaces. The size of the fracture drives the location of the upper interface and thus modifies the phase of the second incoming reflected wave. This behavior results in a large phase shift of the final wave packets.

![Figure 5: Waveforms: monopole, formation with fracture of different sizes facing the receiver array. (a) Front receiver, and (b) last receiver.](image)

Presence of a fracture noticeably influences the shape of acoustic waveforms. It has a large impact on shear-related wave modes, leaving compressional wave components nearly intact. The S-wave mode is solely sensitive to the presence of the fracture, which is manifested by a decrease in amplitude. On the other hand, the Stoneley (for monopole excitation) and flexural modes (for dipole excitation) exhibit the largest sensitivity to presence of the fracture. Both modes are sensitive also to fracture size: the larger the fracture, the larger the observed mode attenuation.

![Figure 6: Waveforms: dipole, formation with fracture of different sizes facing the receiver array. (a) Front receiver, and (b) last receiver.](image)

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