JOINT INVERSION OF DENSITY AND RESISTIVITY LOGS FOR THE IMPROVED PETROPHYSICAL ASSESSMENT OF THINLY-BEDDED CLASTIC ROCK FORMATIONS

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ABSTRACT

Limited vertical resolution of logging tools, particularly deep resistivity tools, often causes ambiguity in log interpretation across interbedded rock formations. This situation routinely gives rise to inaccurate assessments of in-place hydrocarbon reserves. To gain better understanding of measurement behavior in interbedded sequences due to limitations of vertical resolution (resolution approximately equal to 1 foot), we introduce a new joint inversion procedure that effectively combines borehole measurements of density and induction resistivity. The objective of the inversion is to reduce shoulder-bed effects on the interpretation of porosity and hydrocarbon saturation across interbedded rock formations and to reduce non-uniqueness in the estimation of porosity and hydrocarbon saturation.

We undertake the assessment of interbedded clastic formations with two radically different procedures depending on whether the beds are thinner or thicker than 1 foot. For the case of beds thicker than 1 foot, bed boundaries are defined from the inflection point of a high vertical-resolution log. Instead of inverting for formation resistivity and density separately, we introduce a new method to jointly estimate the porosity and water saturation of the flushed and virgin zones.

For the case of beds thinner than 1 foot (thin beds), we introduce a new Bayesian statistical inversion approach that delivers global statistical properties of the laminated sequence, including probability density functions of net-to-gross ratio, porosity, hydrocarbon saturation, and hydrocarbon reserves.

The joint inversion methodology has been successfully applied to synthetic and field data, thereby considerably improving the petrophysical estimates obtained with traditional log interpretation methods. We appraise the advantages of the combined inversion of density and resistivity measurements with respect to standard log interpretation procedures.

INTRODUCTION

Formation evaluation practitioners commonly use the term “thin beds” to refer to beds with a thickness that is below the resolution of conventional logging tools. Experience shows that one foot is a practical limit for thin beds (Passey et al., 2004). Limited vertical resolution of logging tools often creates ambiguity in the petrophysical assessment of thinly-bedded rock formations. Figure 1 shows an outcrop example of a bed sequence with these characteristics. In some cases, even a very small proportion of shale beds can substantially affect the resistivity measurements and hence result in inaccurate quantitative estimations of hydrocarbon reserves.

Figure 2 compares apparent resistivity logs simulated for an Array Induction Imager (AIT<sup>1</sup>) tool across a clean hydrocarbon-bearing sand bed and a similar zone with 20% thinly-bedded shale laminations. The resistivity response decreases from over 100 ohm-m to less than 3 ohm-m with the inclusion of shale laminations. The decrease in resistivity will likely result in the incorrect interpretation of the bed sequence as a wet zone.

<sup>1</sup> Mark of Schlumberger
Figure 2: Simulated AIT response across a hydrocarbon-bearing zone of pure sand (left panel) and a similar zone with 20% thinly-bedded layers of shale (right panel).

Similar results have been documented by Anderson (1986), Anderson et al., (1988), Walsgrove et al., (1999), and Hagiwara et al., (1995). Some solutions to a similar problem were proposed by Passey et al. (2004, 2006).

The objective of this paper is to improve the estimation of petrophysical properties in thinly-bedded formations by modeling the density and induction logging tool responses and using inversion methods to estimate true formation properties. In so doing, we assume a vertical well penetrating horizontal beds subject to axial-symmetric mud-filtrate invasion. Also, for simplicity and in an effort to generalize the results to measurements acquired with standard logging suites, we choose not to consider the possibility of measuring anisotropy of electrical resistivity as a way to diagnose presence of bed laminations thinner than the vertical resolution of standard array-induction measurements.

Due to physical limitations, the potential improvement of vertical resolution of logging tools is limited. Numerical modeling and inversion are good candidates to partially solve this problem. Wang et al., (2007), Zhang et al., (1995), Zhang et al., (1999), proposed a fast simulation and inversion method to interpret AIT measurements. Mendoza et al. (2007) proposed a fast method to simulate density logs based on tool-specific spatial flux-scattering functions. The flux-scattering functions were computed with Monte Carlo simulations of particle-level transport in homogenous media.

Based on Mendoza et al.’s (2007) results, in this paper we develop a one-dimensional (1D, vertical direction) procedure to invert density logs into layer-by-layer values of bulk density using 1D flux-scattering functions constructed specifically for a general density tool (Mendoza et al., 2007). In addition, we perform inversion of raw AIT resistivity measurements to estimate electrical resistivity values for the same layered sequence. The joint interpretation of the two sets of measurements is approach with a combined inversion method in an effort to reduce non-uniqueness in the estimation and hence improve the accuracy and reliability of the results. Moreover, the joint inversion of density and resistivity measurements is posed to directly render layer-by-layer estimates of porosity and water saturation with the implementation of a specific resistivity-porosity-saturation model.

Given that density and induction logs together provide strong constraints to the resistivity—porosity-saturation model, non-uniqueness of the estimation is substantially reduced by the simultaneous use of the two measurements. For the inversion of density and resistivity logs, we make use of raw measurements (mono-sensor densities, 2 curves, and conductivity measured at each receiver coil, real part, 8 curves) rather than the commonly used post-processed curves, e.g. a compensated density curve and formation resistivity curves that exhibit different vertical resolutions and radial lengths of investigation.

We test the joint inversion methodology with one set of synthetic measurements and one set of field measurements. Appendices A and B provide specific details about the implementation of the inversion algorithm.

The deterministic inversion method described in this paper is intended for beds thicker than 1 foot. Bed boundaries can be selected using high-resolution logs such as the high-resolution density log. However, for bed thicknesses below 1 foot, selecting beds boundaries is very difficult, if not impossible, even with very high resolution logs. Due to the increased number of variables, the model is insufficiently constrained and non-uniqueness often prevents obtaining physically consistent results for individual bed properties in the presence of thin beds. Instead of inverting for properties of individual beds, a statistical method is introduced to estimate the global petrophysical properties of a thinly-bedded sequence. Compared to the properties of an individual bed, the global statistical properties of a thinly-bedded sequence have more significance for formation evaluation purposes. Passey et al. (2004) proposed a statistical technique using the Monte Carlo method to compute the overall distribution and uncertainty of bed properties. With the same spirit, we introduce a statistical method that uses Bayesian inversion to estimate the global statistical distribution of properties, such as net-to-gross ratio, sand porosity,
and sand water saturation, of a thinly-bedded sequence using density and induction logs. In addition, the Bayesian inversion procedure delivers estimates of uncertainty of sand-bed petrophysical properties.

**INVERSION METHOD**

Figure 3 shows the formation model assumed in the joint inversion procedure. It consists of a vertical well penetrating horizontal beds. The latter beds are invaded such that the invasion is axial-symmetric and is described with a single piston-like saturation front of known radial location. The bed boundaries are predefined by selecting the inflection points on any high resolution log available. In this case, we use a high-resolution density log to select bed boundaries. The porosity in the sand beds ($\phi_{sd}$), the flushed-zone free-water saturation ($S_{xof}$), and the true formation free-water saturation ($S_{wf}$) of each layer are variables to be estimated.

Electrical conductivity of the flushed-zone ($\sigma_{xof}$) and true conductivity ($\sigma_t$) of each layer are computed using a specific interpretation equation. In this paper, we use the dual-water saturation-resistivity model (Dewan, 1983). Specific equations for the virgin- and flushed-zone electrical conductivities are

\[
\sigma_t = \frac{\phi_t}{a} \left[ S_{wt}^{-m} S_{bs} \left( \frac{1}{R_b} - \frac{1}{R_w} \right) + S_{wt}^{m} \right],
\]

and

\[
\sigma_{xof} = \frac{\phi_{xof}}{a} \left[ S_{xoft}^{-m} S_{bs} \left( \frac{1}{R_b} - \frac{1}{R_{xof}} \right) + S_{xof}^{m} \right],
\]

respectively, where $S_{wt}$ and $S_{xof}$ are the total water saturation in the virgin and flushed-zones, respectively, $\phi_t$ is total porosity, $R_{xof}$ and $R_w$ are the electrical resistivity of mud filtrate and connate water, respectively, $S_b$ is the fraction of pore water which is bound to the clay, and $R_b$ is the electrical resistivity of clay-bound water. The parameter $a$ is the tortuosity factor, $m$ is the cementation exponent, and $n$ is the saturation exponent. Table 1 lists the specific parameters that were used to construct the synthetic example described in this paper.

The density of each layer is computed using the density composition equation, given by

\[
\rho_t = C_{sh} \rho_{sh\_wet} + \phi_{sd} \left[ S_{sf} \rho_{sf} + (1 - S_{wf}) \rho_h \right],
\]

\[
+(1 - \phi_{sd}) \rho_m
\]

where $\rho_t$ is bulk density, $C_{sh}$ is volumetric shale concentration estimated separately from the gamma-ray log, $\rho_{sh\_wet}$ is shale density including clay-bound water, $\phi_{sd}$ and $S_{wf}$ are porosity and free-water saturation in the sand, respectively, $\rho_w$ and $\rho_h$ are water and hydrocarbon density, respectively, and $\rho_m$ is matrix sand density.

The equations used to compute $S_{wt}$, $\phi_t$ and $\phi_{sd}$ are given by

\[
S_{wt} = S_{wf} \phi_{sd} + C_{sh} \phi_{sh} \phi_{sd} + C_{sh} \phi_{sh} \phi_{sd},
\]

\[
\phi_t = \phi_{sd} + C_{sh} \phi_{sh},
\]

and

\[
S_{sh} = \phi_{sh} \phi_{sd} + C_{sh} \phi_{sh},
\]

where $\phi_{sh}$ is shale porosity.

The quadratic cost function being minimized in the joint inversion process is given by

\[
C(\phi_{sd}, S_{wf}, S_{xof}) = \left\| \sigma_{log} - \sigma_c \right\|^2 + \alpha \left\| \rho_w - \rho_t \right\|^2
\]

\[
+ \left\| W_1 \left( \phi_{sd} - \phi_{sd0} \right) \right\|^2 + \left\| W_2 \left( S_{wf} - S_{wf0} \right) \right\|^2
\]

\[
+ \left\| W_3 \left( S_{xof} - S_{xof0} \right) \right\|^2
\]

Figure 3: Formation model assumed for the joint inversion of density and resistivity measurements. Each bed is described with specific values of porosity $\phi$, flushed-zone water saturation $S_{xof}$ and virgin-zone water saturation $S_{wt}$. The radial length of invasion is assumed known from a-priori information.
where $\sigma_c$ and $\rho_c$ are simulated tool responses of the current model, respectively, and $\sigma_{\text{log}}$ and $\rho_{\text{log}}$ are the induction and density raw-measurement curves, respectively. The variables $\phi_{\text{sd}}$, $S_{\text{wf}}$, and $S_{\text{xxof}}$ are the current model and $\phi_{\text{sd0}}$, $S_{\text{wf0}}$, and $S_{\text{xxof0}}$ are the reference model (in our case the reference model is obtained from the separate inversion of density and electrical resistivity measurements). The parameter $\alpha$ is a weight factor that is used to balance the fitting of conductivity logs and density logs. We assume $\alpha=1$. The variables $W_1$, $W_2$, and $W_3$ are parameter-regularization matrices which should be adjusted based on layer thickness and data sampling rate. In our case, the regularization parameter is proportional to the inverse of the layer thickness in feet and this value is further multiplied by a larger number, e.g. 0.15 for $W_1$ and a smaller number, e.g. 0.05 for $W_2$ and $W_3$. The purpose of these model-weighting matrices is to give more regularization to the initial porosity guess obtained from the separate inversion, which is usually closer to the true value than the initial estimate of water saturation. We note that the inversion is performed over a pre-defined depth segment of variable length as opposed to depth sample. Therefore, the data vectors $\sigma_{\text{log}}$ and $\rho_{\text{log}}$ included in eq. (7) contain measurements acquired for all samples contained within the inversion segment. For the case of array-induction resistivity data, the vector $\sigma_{\text{log}}$ contains raw electrical conductivity measurements for all possible frequencies and source-receiver combinations, while for the case of density data, the vector $\rho_{\text{log}}$ contains density measurements acquired with both short- and long-spaced detectors.

Table 1- Summary of dual-water parameters and rock and fluid properties used to estimate water saturation and porosity for the example of inversion with synthetic data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
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<tbody>
<tr>
<td>Tortuosity factor $a$</td>
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</tr>
<tr>
<td>Cementation exponent $m$</td>
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<td>2</td>
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<tr>
<td>Saturation exponent $n$</td>
<td>-</td>
<td>2</td>
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<tr>
<td>Connate water resistivity</td>
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<td>Mud filtrate resistivity</td>
<td>Ohm-m</td>
<td>0.15</td>
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<tr>
<td>Clay bound water resistivity</td>
<td>Ohm-m</td>
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<tr>
<td>Sand matrix density</td>
<td>g/cm$^3$</td>
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<tr>
<td>Shale density</td>
<td>g/cm$^3$</td>
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<td>Water density</td>
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<td>Hydrocarbon density (mix.)</td>
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<td>Shale Resistivity</td>
<td>Ohm-m</td>
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<td>Shale Porosity</td>
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Figure 4 shows a flow chart that describes the various algorithmic components of the joint inversion method. The inversion begins with an initial guess of $\phi_{\text{sd}}$, $S_{\text{wf}}$, and $S_{\text{xxof}}$ computed from the separate inversion of density and induction measurements. Moreover, the inversion radius is obtained from the inversion of AIT raw measurements. The initial guess is also used as the reference model. We make use of a nonlinear iterative method to minimize the quadratic cost function given by eq. (7). At each iteration, the numerically simulated measurements of density and resistivity are compared to the logs and a search direction is calculated using the Gauss-Newton method (Habashy and Abubakar, 2004), wherein the entries of the Jacobian matrix are calculated by a combination of finite differences and analytical expressions (see Appendix A). In addition, we use a line search to calculate the step size that provides a monotonic decrease of data misfit toward the minimum of the cost function from iteration to iteration.

**Example of Inversion with Noisy Synthetic Data:** A synthetic model was constructed to verify the joint inversion method under controlled conditions. The synthetic model is intended to describe a complex sand-shale sequence consisting of 28 beds with their thickness ranging from 0.2 ft to 2 ft. Bed thicknesses were designed to include a wide range of inter-bed separations as well as a wide range of density and electrical resistivity contrasts between adjacent beds. In addition, sand-layer resistivities were assigned to reproduce a complete capillary-transition zone from irreducible to full water saturation. Table 1 describes the specific petrophysical, fluid, and dual-water parameters used in the construction of the synthetic model. Mono-sensor density measurements were simulated using the Monte Carlo particle-level transport code MCNP (X-5 Monte Carlo Team, 2003) with at sampling interval of 3 inches. Raw AIT conductivity measurements were simulated with a sampling interval of 3 inches. In addition, 5% zero-mean random Gaussian noise was added to the simulated induction and density measurements.

Figure 5 compares the input and simulated density measurements rendered by the joint inversion method. Panels in that figure show that the input short-spacing (SS) and long-spacing (LS) densities compare favorably to the SS and LS densities simulated from the inverted layer-by-layer values of density. Moreover, the inverted layer-by-layer values of density are consistent with the true density values. Figure 6 shows that conductivity induction logs simulated from the inverted resistivity model agree well with the 8 input AIT raw conductivity measurements.
logs (for clarity only the AIBC1 and AIBC7 are plotted in Figure 6). Likewise, we observe that inverted layer-by-layer values of electrical resistivity are consistent with the true values of formation resistivity.

Figure 7 shows that the inverted layer-by-layer values of $\phi_{sd}$, $S_{wf}$, and $S_{xof}$. The inversion of these properties was performed under the assumption of accurate knowledge of the petrophysical, fluid, and dual-water parameters listed in Table 1. We observe that the inverted layer properties are in close agreement with true layer properties, thereby confirming the accuracy and reliability of the joint inversion algorithm in the presence of moderate amounts of measurement noise.

Figure 8 compares the pay zones estimated from the available logs (using standard interpretation procedures) and from the joint inversion method. The inversion method identifies more pay zones in the upper and lower parts of the inversion segment. We also note that the inversion method removed some thin pay zones which were ambiguous. The net-to-gross ratio estimated from the inversion is 39%, compared to 15% estimated directly from the available logs. If we assume horizontal continuity of reservoir layers, the calculated reserves are 4970 STB/acre from the inversion method and 3160 STB/acre from log data. In both cases, hydrocarbon reserves are calculated using the equation

$$N_r = \frac{7758Ah\phi_{sd}(1-S_{wf})R_f}{B_o},$$

where $N_r$ is recoverable reserves, $A$ is area in acres, $h$ is thickness in ft, $\phi_{sd}$ is sand porosity, $S_{wf}$ is free-water saturation, $R_f$ is recovery factor, and $B_o$ is oil formation volume factor. The calculation of reserves is performed by summing the right-hand side of eq. (8) over all layers (or log sampling points for the case of log data). Table 2 describes the specific values for the parameters included in eq. (8) that were used to calculate hydrocarbon reserves.

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<th>Variable</th>
<th>Units</th>
<th>Value</th>
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<td>Area $A$</td>
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<tr>
<td>Recovery factor $R_f$</td>
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</tr>
<tr>
<td>Oil formation volume factor $B_o$</td>
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</tr>
<tr>
<td>Water saturation cutoff</td>
<td>fraction</td>
<td>0.4</td>
</tr>
<tr>
<td>Shale volume cutoff</td>
<td>fraction</td>
<td>0.8</td>
</tr>
<tr>
<td>Porosity cutoff</td>
<td>fraction</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 2- Parameters used in the estimation of hydrocarbon reserves.

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2 AIBC stands for Array-Induction Borehole Corrected Conductivity curve.
Figure 6: Comparison of measured and simulated AIT raw measurements: AIBC1 (Track1) and AIBC7 (Track 2). Track 3 compares the inverted and true values of formation resistivity.

Figure 7: Track 1: true porosity (dotted red line) and inverted porosity (red line). Track 2: true $S_{o}$ (black dots) and inverted $S_{o}$ (black line). Track 3: true $S_{water}$ (blue dots) and inverted $S_{water}$ (blue line).

Figure 8: Comparison of the pay zones (blue) estimated with the joint inversion method (Track 1) and pay zones estimated from logs (Track 2). Track 3 compares porosity from logs (black line), porosity from the joint inversion method (red line), and true porosity (red dots). Track 4 compares water saturation from logs (black line), water saturation from the joint inversion method (red line), and true water saturation (red dots).

Example of Inversion with Field Measurements: The formation under consideration is a deepwater Gulf-of-Mexico unconsolidated shaly-sand sequence in a turbidite system which is formed mainly by channel levees. The sedimentary structure includes ripple stratification, clay laminations, and massive intervals with moderate to good grain sorting (Malik et al., 2007). The well was drilled with oil based mud and has a complete suite of logs including raw density monosensor measurements acquired with the HRCC tool and induction conductivity measurements acquired with the AIT-H tool. Sampling intervals of density and conductivity logs are 2 inches and 3 inches, respectively. Also, core measurements and core images were available for verification. We defined 213 beds using the high-resolution density log. Average bed thickness is approximately 1.5 ft. Figure 9 is a core slab image that indicates significant presence of thin shale laminations. The well penetrating the beds at a small dip angle and hence can be assumed to be vertical. Table 3 describes the petrophysical, fluid, and dual-water properties assumed in the joint inversion of density and resistivity measurements. These properties

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are derived from core laboratory measurements and hence are not subject to uncertainty analysis in the inversion. The particular depth segment chosen for inversion contains a wide range of hydrocarbon-bearing sand properties as well as a significant amount of shale layers, hence constitutes a good testing example.

Figure 10 compares the density and induction-resistivity logs simulated from the inverted layer-by-layer properties against the corresponding input logs. (Track 1 for SS density, Track 2 for LS density, Track 3 for AIBC1 raw electrical conductivity, and Track 4 for AIBC7 raw electrical conductivity; for clarity only 2 out of 8 raw electrical conductivity logs are shown in the figure). We observe a very good agreement between the simulated and input logs.

Figure 11 compares the pay zones estimated with the joint inversion to pay zones estimated directly with standard well-log interpretation methods. This comparison indicates that additional pay zones were identified with the inversion method (Track 1 shows pay zones identified with the inversion method and Track 2 shows pay zones identifies with the deep apparent resistivity log, AIT90, and the high-resolution density log, RHO8). The same figure compares layer-by-layer porosity (Track 3) and water saturation (Track 4) values estimated with the joint inversion method to the corresponding values estimated with standard well-log interpretation methods and core data. Porosity values estimated from both log and inversion methods are consistent with those of core data. However, values of water saturations estimated from both inversion and standard log analysis methods are lower than those of core data. We believe that the mismatch is due to the coring process applied on the unconsolidated sands, which modified the original water saturation in the core samples, thereby no longer representing in-situ conditions.

Most of the differences encountered between core and inverted values of water saturation take place within the upper (xx12-xx28 m) and lower (xx65-xx75 m) shaly intervals where gamma-ray readings fluctuate between the corresponding values for clean sand and pure shale. For these 2 zones, the values of water saturation calculated from conventional well-log analysis exhibit a constant positive bias of more than 50% while the values of water saturation estimated from the inversion method exhibit strong fluctuations due to the laminations of shale and sand. The inversion method shows that among these laminated zones, 50%-70% are pay zones and the rest are shale layers with clay-bound water.

Both inversion and standard well-log interpretation methods properly identified the main reservoir from xx28 to xx63 m. In addition, results from the inversion method indicate a potential pay zone at xx10-xx28 m. If we assume that the reservoir is at capillary equilibrium, this depth segment should be a hydrocarbon zone and be able to produce some free water. The estimation from well-log interpretation shows an oil/water contact at approximately xx63 m while inversion results together with the gamma-ray log indicate that the oil/water contact is actually at approximately xx75 m. Unfortunately, there are no pressure-gradient data available in this well to cross-validate the estimation of the water/oil contact locations.
Table 3- Summary of dual-water parameters and rock and fluid properties used to estimate water saturation and porosity in the example of inversion with field data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
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<tbody>
<tr>
<td>Tortuosity factor (a)</td>
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<td>1.00</td>
</tr>
<tr>
<td>Cementation exponent (m)</td>
<td>-</td>
<td>1.9</td>
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<tr>
<td>Saturation exponent (n)</td>
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<tr>
<td>Clay bound water resistivity</td>
<td>Ohm-m</td>
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<td>Sand matrix density</td>
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</tr>
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<td>Hydrocarbon density (mix.)</td>
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<td>Shale Resistivity</td>
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<td>Shale Porosity</td>
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Bayesian Inversion Method for Thin Beds: In beds thinner than 1 foot, we use a statistical estimation method in order to avoid selecting bed boundaries, which is often difficult. The Bayesian inversion method (Tarantola, 2005) is used to estimate global petrophysical properties and their uncertainty in a thin-bed interval.

We assume a model consisting of a sequence of equal-thickness thin beds which can be either sand or shale. The thickness of each bed is 0.25 ft. For each bed, if it is a shale layer, its properties such as density and resistivity, porosity, and water saturation are fixed to be a predefined value, while if it is a sand bed, then we use Archie’s equation

\[
\sigma = \frac{\phi^n S_w^m}{a R_w},
\]  

and the density compositional equation

\[
\rho = \phi[(1-S_w)\rho_b + S_w\rho_s] + (1-\phi)\rho_{sand},
\]  

to establish a relationship between measurable properties, i.e. conductivity \(\sigma\), density \(\rho\) and its petrophysical properties, i.e. porosity \(\phi\) and water saturation \(S_w\).

The cost function implemented for Bayesian inversion is given by

\[
C(L, \phi, S_w) = \| \sigma_{log} - \sigma \|^2 + \alpha \| \rho_{log} - \rho \|^2,
\]  

where the vector \(L\) defines the model lithology.

The uncertainty of model parameters, including lithology \(L\), porosity \(\phi\), and saturation \(S_w\) is estimated using the Bayesian theorem (Tarantola, 2005), given by

\[
f_{post}(L, \phi, S_w | \sigma_{log}, \rho_{log}) = \gamma L(\sigma_{log}, \rho_{log} | L, \phi, S_w) f_{pri}(L, \phi, S_w),
\]  

where \(f_{post}(L, \phi, S_w)\) is the prior probability density function (PDF) with respect to lithology, porosity, and water saturation, and \(L(\sigma_{log}, \rho_{log} | L, \phi, S_w)\) is the data likelihood function of a given realization of the model. The function \(f_{post}(L, \phi, S_w | \sigma_{log}, \rho_{log})\) designates the posterior probability density function, and \(\gamma\) is a constant normalization factor.

Porosity and water saturation are assumed to be dependent on lithology but independent to each other. The prior PDF of model parameters is given by (Besag et al., 1995)

\[
f_{pri}(L, \phi, S_w) = f(\phi, S_w | L) \cdot f(L),
\]  

where

\[
f(\phi, S_w | L) = f(\phi | L) f(S_w | L).
\]
In the above formulation, the prior PDF of lithology takes a binary distribution of 50% sand and 50% shale, whereas the prior PDF of porosity $\phi$ is a truncated Gaussian distribution between 0 and 1, with a mean of 0.175 and a standard deviation of 0.08. Finally, the prior PDF of water saturation, $S_w$, is also a truncated Gaussian distribution between 0 and 1, with a mean of 0.3 and a standard deviation of 0.15.

We consider a data likelihood function $L(\sigma_{\log}, \rho_{\log} | L, \phi, S_w)$ in the form of a Gaussian distribution which can be calculated from the quadratic cost function given by eq. (11), namely,

$$L(\sigma_{\log}, \rho_{\log} | L, \phi, S_w) = e^{-\frac{1}{2\sigma^2}C(\phi, L)},$$

where $\sigma$ is the standard deviation of the input measurements due to presence of noise (we assume that the measurements are independent and uncorrelated). Sampling of the model space to calculate the posterior PDF given by eq. (12) is performed with the Metropolis method (Metropolis et al., 1953, Kirkpatrick et al., 1983).

Figure 12 shows the synthetic model constructed to benchmark and validate the Bayesian inversion method described above. The model consists of 50 layers with a uniform thickness equal to 0.25 ft. Sand and shale layers constitute 60% and 40% of the total, respectively. Table 4 describes the petrophysical, fluid, and Archie’s parameters assumed in the calculation of layer densities and electrical resistivities. The model was design to exhibit a linear capillary transition zone for the distribution of water saturation in sand layers. Petrophysical properties of shale layers were assumed constant and not subject to inversion. Figure 13 shows the density and raw AIT conductivity logs simulated for the synthetic model and contaminated with zero-mean 5% additive Gaussian random noise. For clarity, only 2

### Table 4

<table>
<thead>
<tr>
<th>Variable</th>
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<th>Value</th>
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<td>Connate water resistivity</td>
<td>Ohm-m</td>
<td>0.02</td>
</tr>
<tr>
<td>Sand matrix density</td>
<td>g/cm$^3$</td>
<td>2.65</td>
</tr>
<tr>
<td>Shale density</td>
<td>g/cm$^3$</td>
<td>2.75</td>
</tr>
<tr>
<td>Water density</td>
<td>g/cm$^3$</td>
<td>1.00</td>
</tr>
<tr>
<td>Hydrocarbon density (mix.)</td>
<td>g/cm$^3$</td>
<td>0.8</td>
</tr>
<tr>
<td>Shale Resistivity</td>
<td>Ohm-m</td>
<td>1.0</td>
</tr>
<tr>
<td>Shale Porosity</td>
<td>fraction</td>
<td>0.10</td>
</tr>
</tbody>
</table>
(AIBC1 and AIBC7) out of 8 raw electrical conductivity logs are shown in the figure. The sampling rates of the simulated density and conductivity logs are 6 inches and 3 inches, respectively.

Figure 14 shows the estimated PDFs for net-to-gross ratio, global sand porosity, hydrocarbon reserves, and global sand water saturation. The same figure shows the most probable values for each property, along with their uncertainty: the shorter the spread of the PDF, the lower the uncertainty of the estimation. We note that the estimated global properties are consistent with the corresponding distributions of the true model, thereby lending credence to the Bayesian inversion approach. Figure 15 describes the layer-by-layer probability for each thin bed to be a sand bed. The vertical distribution of sand-show probabilities is consistent with the true sand-shale distributions shown in Figure 12 (Track 2).

For comparison, we calculated petrophysical properties using the AIT deep conductivity log, $\sigma_{\log}$, and the bulk density log, $\rho_{\log}$. Table 5 summarizes the comparison between global property estimations obtained with the Bayesian inversion method and standard well-log interpretation together with true values. The comparison indicates that the standard method of well-log interpretation underestimates hydrocarbon reserves by 48%, whereas the Bayesian inversion method overestimates hydrocarbon reserves by 26%.

![Figure 14: Probability density functions (PDFs) of net-to-gross ratio (upper left panel), sand porosity (lower left panel), hydrocarbon reserves (upper left panel), and sand water saturation (lower right panel) obtained from Bayesian inversion.](image)

![Figure 15: Estimated probability for each thin bed to be a sand unit.](image)

<table>
<thead>
<tr>
<th></th>
<th>Bayesian</th>
<th>Log</th>
<th>True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average sand porosity</td>
<td>0.195</td>
<td>0.1066</td>
<td>0.168</td>
</tr>
<tr>
<td>Average sand water saturation</td>
<td>0.475</td>
<td>0.7084</td>
<td>0.5621</td>
</tr>
<tr>
<td>Hydrocarbon reserves (STB/acre)</td>
<td>1700</td>
<td>696</td>
<td>1340</td>
</tr>
<tr>
<td>Net-to-gross ratio</td>
<td>0.375</td>
<td>0.16</td>
<td>0.32</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

This study of density and induction log simulation and inversion in thin beds in clastic formations shows that the joint inversion method has the ability to improve the detection and evaluation of thin-bed petrophysical properties.

We developed deterministic and statistical inversion methods to approach the quantitative joint interpretation

---

5 Assuming pure sand, and using Archie’s equation.
6 Assuming pure sand, and using Archie’s equation.
7 Using the same parameter shown in Table 2
of density and resistivity logs. With the joint inversion method proposed in this paper, thin bed pay-zones can be more accurately identified and quantified than using the standard well-log interpretation procedures. This new method can make a substantial difference in the estimation of pay per unit area and net-to-gross ratio in thinly-bedded formations. In most cases, joint-inversion processing helps to accurately identify more potential hydrocarbon-bearing beds compared to standard log processing and interpretation. Inversion requires only the real part of AIT conductivity raw measurements and mono-sensor density measurements which are available for many wells. The joint inversion method can potentially be applied to many existing logs to improve thin-bed identification and petrophysical assessment.

Tests performed on synthetic and field data indicate that the joint inversion method is numerically more stable when compared to the sequential inversion method applied separately on density and resistivity logs. Moreover, the joint inversion method can substantially reduce non-uniqueness in the estimation and hence increase the accuracy of the estimated values of density and resistivity. Finally, the joint inversion method directly estimates petrophysical properties such as porosity and water saturation which ensures a consistent petrophysical interpretation. The structure of the joint inversion method is open to incorporate other logs, e.g. gamma-ray and sonic logs, when there is a fast forward method for numerical simulation.

ACKNOWLEDGEMENTS

We especially thank the management of BP for suggesting this challenging research project. Our deepest gratitude goes to Kerr-McGee for providing the field data used to test the joint inversion method. The work reported in this paper was funded by the University of Texas at Austin's Research Consortium on Formation Evaluation, jointly sponsored by Anadarko, Aramco, Baker Atlas, BP, British Gas, ConocoPhillips, Chevron, ENI E&P, ExxonMobil, Halliburton Energy Services, Hydro, Marathon Oil Corporation, Mexican Institute for Petroleum, Occidental Petroleum Corporation, Petrobras, Schlumberger, Shell International E&P, Statoil, TOTAL, and Weatherford.

REFERENCES


APPENDIX A: Calculation of the Entries of the Jacobian Matrix.

Evaluation of the Jacobian matrix is an essential step in most nonlinear inversion problems. Due to the lack of an analytical solution, the evaluation of the Jacobian matrix is computationally expensive if it is calculated with finite differences. In order to improve the performance of the inversion process while maintaining its accuracy, we approximate the entries of the Jacobian matrix in our inversion algorithm. The derivatives of log measurements with respect to model parameters are calculated using chain rules, as follows:

\[
\frac{\partial \rho_c}{\partial \phi} = \frac{\partial \rho_c}{\partial \rho} \cdot \frac{\partial \rho}{\partial \phi},
\]

(A-1)

\[
\frac{\partial \sigma_c}{\partial \phi} = \frac{\partial \sigma_c}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial \phi},
\]

(A-2)

\[
\frac{\partial \sigma_c}{\partial \omega} = \frac{\partial \sigma_c}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial \omega} + \frac{\partial \sigma}{\partial \phi} \cdot \frac{\partial \phi}{\partial \omega},
\]

(A-3)

\[
\frac{\partial \sigma_t}{\partial \omega} = \frac{\partial \sigma_t}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial \omega},
\]

(A-4)

and

\[
\frac{\partial \sigma_t}{\partial \omega} = \frac{\partial \sigma_t}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial \omega},
\]

(A-5)

where \( \rho_c \) are the mono-sensor density log curves, \( \rho_t \) is the model density, \( \sigma_c \) are the AIT conductivity measurement curves, and \( \sigma_t \) and \( \sigma_w \) are the model conductivity in the flushed and virgin zones, respectively.

To reduce the cost of computations, the derivative

\[
\frac{\partial \rho_c}{\partial \sigma_c} \cdot \frac{\partial \sigma_c}{\partial \sigma_w} \cdot \frac{\partial \sigma_w}{\partial \phi}
\]

is approximated with the response of a homogenous and isotropic formation and thus can be computed and stored prior to performing the inversion (Wang et al. 2007). On the other hand, the calculation of the derivatives

\[
\frac{\partial \rho_c}{\partial \phi} \cdot \frac{\partial \phi}{\partial \sigma_c} \cdot \frac{\partial \sigma_c}{\partial \sigma_t} \cdot \frac{\partial \sigma_t}{\partial \omega},
\]

and

\[
\frac{\partial \sigma_t}{\partial \omega}
\]

can be performed analytically using the dual-water equation and the density composition equation at each...
iteration. Derivatives computed with the dual-water Eq. (1) and their related equations are given by

\[ \frac{\partial \rho_{ns}}{\partial \phi_{ad}} = S_{ns} \rho_{ns} \left( 1 - S_{ns} \right) \rho_{ns} - \rho_{ns}, \]  

(A-15)

and

\[ \frac{\partial \rho_{nf}}{\partial \phi_{ad}} = \phi_{ad} \left( \rho_{ns} - \rho_{ns} \right). \]  

(A-16)

**APPENDIX B: Rapid Simulation of Density Logs.**

The forward simulation used in the joint inversion method is based on approximate spatial flux-scattering functions calculated for specific source-sensor configurations (Mendoza et al., 2007). For completeness, we begin the description of this method of approximation with the simulation of density logs using particle-transport techniques based on the Monte Carlo solution of Boltzmann’s equation.

We simulate density measurements with the particle-transport simulation code MCNP maintained by Los Alamos National Laboratory (X-5 Monte Carlo Team, 2003). The assumed source-sensor configuration corresponds to the Longhorn Nuclear Logging Tool (LNLT) introduced by Mendoza et al. (2007). The simulated particle counts (flux) are processed using spine-and-rib corrections. We consider a synthetic model to describe the relative accuracy of Mendoza et al.’s (2007) approximate simulation with respect to density-log simulations performed with MCNP. The synthetic formation is constructed based on field logs across a clastic formation consisting of alternations of sand and shale. Shale is assumed consisting of illite, namely,

\[(K,H_{3}O)(Al,Mg,Fe)_{2}(Si,Al)_{4}O_{10}(OH)_{2},(H_{2}O),\]

and the sand content is assumed quartz (SiO₂). Formation fluid and borehole fluid are both water.

Simulated far- and near-detector counts are entered into spine-and-rib corrections to produce the corrected density reading and the correction applied to the far density log, \( \Delta \rho \). Counts from the far and near detectors are also used to calculate the corresponding density readings. Figure B-1 shows one example of simulation result. We note that the particle-transport simulation with MCNP is computationally demanding. For instance, the simulation of the density log for this example (about 111.5 ft of log at a sampling rate of 6 ft) required 218.8 hours of CPU time on a 3.2GHz, 4G RAM PC.

Figure B-2 shows the spatial sensitivity of the assumed density tool (LNLT) for both far and near detectors.
The sensitivity map describes the relative probability that a photon generated at a given point in the formation will make its way to the detector. The rapid simulation method proposed by Mendoza et al. (2007) is based on the application of these sensitivity functions as linear and nonlinear functions that weigh the spatial distribution of formation density to calculate the corresponding particle counts. Subsequently, mono-sensor densities are calculated from single-sensor counts from a spine cross-plot. Two counts, from long spacing (LS) and short spacing (SS), are processed to produce the formation density log.

Figure B-3 compares the density logs simulated with both the particle-transport code (MCNP) and Mendoza et al.’s (2007) sensitivity weighing procedure. We observe that the sensitivity-based simulation is in general within 5% of the MCNP simulation, except across the interface between the 2 beds that exhibit the largest density contrast. Compared to hundreds of hours of CPU time required to calculate the density log with MCNP, the sensitivity-based density simulation is performed in just a few seconds of CPU time.

Figure B-1: Example of MCNP simulation of density logs in a thinly-bedded formation. Far and near detector counts are converted to density readings. Both counts are corrected using spine and rib plots to produce the corrected density and the correction made to the far density reading. Track 1: true formation density (thin blue line), and spine-rib corrected density (thick blue line). Track 2: density correction $\Delta \rho$. Track 3: true formation density (thin blue line), near sensor density (black line), far sensor density (red line), and spine-and-rib corrected density (thick blue line).

Figure B-2: Spatial sensitivity function for the long- (upper panel) and short-spaced detectors (lower panel of the Longhorn Nuclear Logging Tool (LNLT)).

Figure B-3: Track 1: comparison between sensitivity-based density simulation (black line) and MCNP-based density simulation (blue line) and true formation density (red line). Track: relative difference between the two simulations.
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