Online stochastic control of dimensional quality in multistation manufacturing systems

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Abstract: In the current paper, a control law is devised for flexible tooling element adjustments in a multistation manufacturing system based on the in-process product measurements and the state-space model of dimensional error flow through this system. The control law is devised so that the dimensional product quality at the end of the line is as close to the nominal as possible in the least squares sense, taking into account the measurement and process noise, as well as the accuracy of actuation of flexible tooling elements. The condition of controllability of product quality is introduced and dimensional quality variations of the product with and without the newly introduced stochastic quality control law are compared. It is proven that the newly proposed control algorithm reduces variations of dimensional product errors and that the controllability condition is beneficial to both the expected product quality as well as for its variations. Capabilities of the newly proposed method for stochastic control of dimensional quality in multistation manufacturing systems are illustrated in an example of a machining system used for the manufacturing of an automotive cylinder head by a major automotive manufacturer.

Keywords: dimensional quality control, multistation manufacturing systems, stream of variation (SoV) methodology, stochastic control

1 INTRODUCTION

Dimensional quality problems due to process variation are among the most critical issues for multistation discrete-part manufacturing, such as auto-body assembly processes of machining processes for manufacturing of engine blocks, heads, or transmission components. Each manufacturing station in one such system introduces errors that propagate through the system and influence the final product quality. Therefore, the final part quality is a result of the complex interaction of errors caused by various manufacturing stations due to the propagation and accumulation of these errors. Understanding such processes is crucial for timely and accurate elimination of product quality errors and subsequent product quality improvement.

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Considerable efforts have been made to establish a mathematical connection between the root causes of dimensional quality and the measured part quality characteristics [1–4]. More recently, the so-called stream of variation (SoV) methodology [4–11] was introduced, which explicitly models the flow of dimensional errors from one station to another in the state-space form with the ordering index of the manufacturing station playing the role of the time index in the usual state-space models used in control theory. The special form of SoV models was utilized for:

1. Identification and description of process-level root causes of dimensional quality based on the distributed measurements of the product [12–16].
2. Formal and quantitative characterization of measurements taken in multistation systems based on the amount of information that measurements carry about the process-level root causes of quality problems [17–21].
3. Optimal selection of features that need to be measured and optimal allocation of sensors in order

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Considerable efforts have been made to establish a mathematical connection between the root causes of dimensional quality and the measured part quality characteristics [1–4]. More recently, the so-called stream of variation (SoV) methodology [4–11] was introduced, which explicitly models the flow of dimensional errors from one station to another in the state-space form with the ordering index of the manufacturing station playing the role of the time index in the usual state-space models used in control theory. The special form of SoV models was utilized for:

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3. Optimal selection of features that need to be measured and optimal allocation of sensors in order
to maximize the amount of information about the process reflected in the measurements [22, 23].

4. Evaluation of designs based on sensitivity of key quality features to perturbations of process-level factors [24, 25].


Nevertheless, the aforementioned work did not pay significant attention to the need to automatically adjust process parameters once process-level root causes of product quality have been identified. Lack of rapid, accurate measurements is one factor that prevents the acquiring of online information about part quality, which could subsequently be used to automatically correct the process and compensate manufacturing errors. Furthermore, lack of appropriate actuation devices that could autonomously correct process parameters, such as fixture locator positions or cutting tool path parameters, significantly reduced the practical significance of SoV-based automatic error compensation. Nevertheless, the use of recently developed laser-based [29] and laser holography-based [30] measurement devices for rapid and highly accurate inspection of products could yield highly detailed dimensional and non-dimensional measurements of every product that comes out of the manufacturing process (including machining), thus giving an immediate insight into the evolution of the health of the underlying manufacturing process. In addition, the recent development of flexible fixtures [31] signifies that automatic compensation of multistation manufacturing errors can soon become a reality and that systematic methods are needed to improve product quality through strategic utilization of rapid sensing and actuation capabilities that are increasingly becoming available.

In the present paper, a method is proposed for automatic control of dimensional product quality based on the SoV model of the process, using in-process measurements of the product and online adjustments of flexible tooling elements, such as recently developed flexible fixtures. The goal is to devise a control law for flexible tooling element adjustments based on the in-process product measurements so that the dimensional product quality at the end of the line is as close to nominal as possible, taking into account the measurement and process noise, as well as the accuracy of actuation of flexible tooling elements.

The remainder of the current paper is organized as follows. In section 2, there is a brief review of the previous work in explicit modelling of the flow of dimensional errors in multistation manufacturing systems, as well as various applications of those models for process diagnostics, measurement characterization, optimization of measurements and sensor locations, manufacturing system design, tolerance allocation, and automatic control of dimensional quality. Section 3 gives a succinct overview of the linear state-space modelling of the flow of dimensional errors in multistation manufacturing systems, yielding SoV models that explicitly connect process-level parameters with the measured product quality. Section 4 offers a novel method for SoV model-based stochastic control of dimensional quality in multistation manufacturing systems using in-process product measurements and online adjustments of process parameters. The effectiveness of the newly proposed dimensional quality control method is quantitatively evaluated and discussed in section 5. Section 6 offers a numerical illustration of the capabilities of the newly proposed stochastic quality control method in an automotive cylinder head machining process, while section 7 yields conclusions of the work presented in the present paper and offers guidelines for possible future work.

2 LITERATURE REVIEW

Explicit modelling of the flow of dimensional errors from one manufacturing station to another in order to formally predict, diagnose, and reduce variations in an automotive body assembly system was first suggested in reference [4], effectively yielding a model in the state-space form in terms of the manufacturing errors (having the role of system states) and assembly station index (having the role of the time index). The methodology therein was referred to as the SoV approach. Jin and Shi [5] developed an approach for linear state-space modelling of the flow of dimensional errors in a multistation automobile body assembly under the assumption of small process-level errors and rigid body workpieces, while Camello et al. [7] expanded this work by taking workpiece compliance into consideration. An SoV model was proposed in reference [8] to describe the dimensional deviations using the vectorial representation of workpiece features. In references [9] and [10], the model derived in reference [8] was linearized and a linear state-space model of the flow of dimensional errors in machining was obtained. Zhou et al. [11] also developed a linear state-space model by using differential motion vector representation of features.

The linear state-space form of the SoV models provides the foundation for further application of achievements in modern control theory and multivariate statistics to detection of sources of product quality problems. Ding et al. [14] and Huang et al. [15] presented fault identification methods in
multistation machining systems based on the SoV model of the corresponding process. Zhou et al. [17] used the linear state-space SoV model to formulate a linear mixed fault–quality model of the multistation manufacturing process to describe the relationship between process faults and product quality and to facilitate identification of process-level parameter faults using product measurements.

Diagnosability in multistation manufacturing processes can be defined as the capability of identifying the root causes of an observed workpiece quality problem. In reference [18] it was proposed to analyze diagnosability quantitatively by checking the rank of the regression matrix connecting the measured dimensional errors and corresponding root causes based on the SoV model. A thorough study on diagnosability of process faults in a multistation manufacturing process was provided in references [19] and [16]. In references [20] and [21], it was proposed to use properties of the variance/covariance matrix of root cause estimation error to quantify the amount of information contained in any set of measurements, yielding a measurement characterization criterion derived from the accuracy of root cause estimation based on those measurements.

In reference [22], a procedure for optimal selection of measurements was presented as a procedure for minimizing root cause estimation uncertainty associated with the selected measurement combination, or in other words, maximizing ‘informativeness’ of measurement combinations, where informativeness was expressed based on the measurement scheme analysis methods from references [20] and [21]. This tool offers a formal and systematic method for the design of the measurement schemes in the multistation manufacturing process.

In addition, the model of the flow of dimensional errors in multistation manufacturing systems can also be used to integrate the process and product information into the tolerance analysis and allocation [26–28], where manufacturing costs associated with tolerances of critical process requirements are minimized, subject to the constraint of satisfying product functionality.

With the recent development of inspection technology [29, 30] and flexible fixtures [31], much research has focused on the use of the SoV models of the flow of manufacturing errors in order to facilitate automatic control of product quality. Automatic control of process parameters based on in-process measurements of the product and the SoV model of the connection of those measurements with process-level parameters was addressed in reference [32]. In this paper, the authors address the problem of adjustment of process parameters between two adjacent jobs as well as the problem of in-process adjustments of downstream process parameters based on measurements of a semi-finished product in the middle of the process. Furthermore, the concept of compensability – which quantitatively depicts the capacity of error compensation in a specific system and is analogous to the concept of controllability in traditional control theory – is proposed, based on which compensable and non-compensable subspaces of dimensional errors are identified and quantitatively described. Nevertheless, this paper assumed that one has full control over all tooling elements of the process, which may not be true in reality, since flexible fixtures and machines most of the time need to be allocated only to selected locations in the system owing to their high costs. Furthermore, the error compensation method and corresponding characterization of the compensable and uncompensable subspaces introduced in reference [32] take into account only the deterministc effects, while the noise due to the linearization, unmodelled effects, process noise, and sensor imperfection was neglected.

In the current paper, a stochastic control law is proposed that takes into account distributed actuation capabilities as well as statistical characteristics of actuation accuracy and noise due to linearization, unmodelled effects, process noise, and sensor imperfection. The problem of characterizing this control law is also addressed using the residual product quality variations after the control law is applied.

3 SoV MODELLING METHODOLOGY

In references [5, 6, 9], and [10], propagation of dimensional errors in multistation manufacturing systems with in-process measurements of the product was described through SoV models in the linear state space form

\[ x(k) = A(k)x(k-1) + B(k)u(k) + W(k), x(0) = 0 \]
\[ y(k) = C(k)x(k) + D(k)u(k) + V(k), k = 1, 2, \ldots, N \]

where \( k \) denotes the manufacturing operation number, \( N \) denotes the total number of manufacturing operations, \( x(k) \) denotes the workpiece dimensional errors accumulated up to manufacturing operation \( k \), \( u(k) \) denotes errors in process parameters at operation \( k \) due to which new product errors are introduced at that operation, \( y(k) \) denotes measured dimensional errors after operation \( k \), \( W(k) \) is the ‘plant’ noise present due to the linearization errors and non-modelled effects, and \( V(k) \) is the noise term present due to the linearization effects and sensor noise.

The model of the linear state-space form, equations (1), describes how manufacturing errors are introduced, transformed, and accumulated as a
workpiece is being processed in a multistation manufacturing system. For each operation \( k \), matrix \( A(k) \) describes how errors accumulated up to and including operation \( k–1 \) are transformed and influence errors in operation \( k \), while matrix \( B(k) \) describes how new errors are introduced into the workpiece at operation \( k \). Matrix \( C(k) \) connects errors of the workpiece computer aided design (CAD) parameters \( x(k) \) to the measured errors \( y(k) \), while matrix \( D(k) \) describes how manufacturing errors introduced in operation \( k \) directly influence the measured errors \( y(k) \).

In the case of the linear state-space models derived in references [5] and [6] to describe the flow of dimensional errors in autobody assembly, the state vector \( x(k) \) in the state-space model was a vector of workpiece deviations after assembly at assembly station \( k \), consisting of deviations in position and orientation in each assembled part. The vector of inputs \( u(k) \) consisted of positional errors in fixture elements at station \( k \).

In the SoV models of machining processes presented in references [9] and [10], each workpiece feature was described by its position, orientation, and a set of scalar features – such as the cylinder diameter, hole diameter, and depth, and slot width and depth, etc. The feature positions and orientations were expressed in a coordinate system determined by some workpiece measurement datum features, and the state vector \( x(k) \) from the model in equations (1) comprised of errors in positions, orientations, and scalar parameters of each workpiece feature after machining operation \( k \). For each machining operation \( k \), the ‘input’ vector \( u(k) \) contains errors in fixture parameters and errors in parameters of the newly machined surfaces at that machining operation. Such errors in orientation, position, and scalar parameters (diameters of holes, slot depths, etc.) of the newly machined surfaces could occur owing to thermal effects, tool wear, or other error causes. Nevertheless, the existing SoV models of multistation machining are not able to represent the dependency of error parameters of the newly machined surfaces on other process parameters, such as tool wear or thermal errors. Further identification of tool-path-related errors would have to be accomplished through other methods, such as thermal error, or tool wear monitoring.

It should be noted that the model formulation reported in references [5] and [6] did not explicitly include direct propagation matrices \( D(k) \), which corresponds to the usual situation where the measurements after operation \( k \) contain only on-product measurements expressing characteristics of workpiece features relative to each other (distance between holes, parallelism between planes, orientation of a hole relative to a datum plane, etc).

However, in machining lines it is possible to also have on-machine measurements of a newly machined surface or machine fixtures, wherever an on-machine probe is available. Given the SoV modelling procedure for multistage machining systems described in references [9–11], this kind of measurement corresponds to direct measuring of elements of the vector \( u(k) \), which means that the vector of measurements contains within itself measurements of input components \( u(k) \). In that case, it is necessary to have a non-trivial (non-empty) matrix \( D(k) \) to express such measurements. This situation is thoroughly described in references [9] and [10], where a generic approach to modelling of the flow of dimensional errors in a multistage machining of prismatic parts is described.

Following model (1), manufacturing errors in a machining or assembly system represented using the linear state-space model (1) are introduced at each operation \( k \), \( k = 1, 2, \ldots, N \), through non-zero elements of vectors \( u(k) \). Furthermore, the existence of flexible fixturing and tooling elements in the manufacturing system means that certain components in the input vectors \( u(k) \) are under the operator’s control and can be adjusted to offset errors introduced through non-zero elements of vectors \( u(k) \), once those errors are observed through in-process measurements of the product.

In each manufacturing station \( k \), partition the vector of inputs \( u(k) \) from model (1) into vector \( u_C(k) \) consisting of components that can be automatically actuated by the operator, and vector \( u_E(k) \) consisting of components that cannot be automatically actuated by the operator. The vector \( u_C(k) \) would contain parameters of machine – tool axes at station \( k \) that can be controlled and fixture parameters of the flexible fixture at station \( k \), if such a fixture is installed in that station. The vector \( u_E(k) \) would consist of machine – tool axes at station \( k \) and fixture parameters that cannot be automatically actuated by the operator. In that case, the SoV model (1) becomes

\[
x(k) = A(k)x(k–1) + B_C(k)u_C(k) + B_E(k)u_E(k) + W(k), \quad x(0) = 0
\]

\[
y(k) = C(k)x(k) + D_C(k)u_C(k) + D_E(k)u_E(k) + V(k)
\]

where matrices \( B_C(k) \) and \( B_E(k) \) are obtained by appropriate partitioning of matrix \( B(k) \) from equations (1), and matrices \( D_C(k) \) and \( D_E(k) \) are obtained by appropriate partitioning of matrix \( D(k) \) in equations (1).

Assume that product has been processed in operations 1 to \( k \). Based on model (2) and in-process measurements of the product after operations 1 to \( k \), a control law will be postulated to minimize variations of product quality in the downstream manufacturing
stations using adjustments of controllable process parameters contained in vectors $u_C(k+1)$, $u_C(k+2), \ldots, u_C(N)$. Figure 1 illustrates the aforementioned problem that will be addressed in the current paper.

### 4 SoV – BASED STOCHASTIC CONTROL OF DIMENSIONAL QUALITY

In order to postulate the control law that would minimize variations of product quality after manufacturing operations and after product measurements are taken in stations 1 to $k$, the following assumptions will be made.

1. Vectors of uncontrollable process parameters $u_E(k)$ follow a Gaussian distribution with zero mean and a known variance–covariance matrix $K_E(k)$. Zero mean implies that the process has been calibrated, while matrix $K_E(k)$, which contains variances of process parameters at station $k$, can be obtained from a machine capability study. It is reasonable to assume that this matrix ($K_E(k)$) is a diagonal matrix with variances of process parameters at station $k$ on its main diagonal, where parameter variances can be estimated from a machine capability study.

2. Vectors of uncontrollable process parameters $u_E(k_1)$ and $u_E(k_2)$ from different manufacturing operations $k_1$ and $k_2$ are independent.

3. Vectors of controllable process parameters $u_C(k)$ can be described as

$$u_C(k) = \bar{u}_C(k) + \xi_C(k)$$

where $\bar{u}_C(k)$ is the desired vector of controllable parameters and $\xi_C(k)$ is a noise vector term following a Gaussian distribution with zero mean and a known variance–covariance matrix $K_C(k)$. Matrix $K_C(k)$ contains variances of controllable process parameters at station $k$ around their desired value and depicts the actuation accuracy provided by the flexible fixture and adjustable machine–tool axes in operation $k$. Nevertheless, the derivations presented in this paper are not constrained to only diagonal variance–covariance matrices.

4. Noise terms $\xi_C(k_1)$ and $\xi_C(k_2)$ associated with vectors of controllable process parameters $u_C(k_1)$ and $u_C(k_2)$ from different manufacturing operations $k_1$ and $k_2$, respectively, are independent.

5. Noise terms $\xi_C(k_1)$ associated with vectors of controllable process parameters $u_C(k_1)$ are independent of all uncontrollable process parameters $u_E(k_2)$ for any two operations $k_1$ and $k_2$.

6. Process noise vectors $W(k)$ and measurement noise vectors $V(k)$ are independent Gaussian processes with zero mean and known variance–covariance matrices $K_W(k)$ and $K_V(k)$ respectively.

7. Process noise vectors $W(k_1)$ and $W(k_2)$ associated with different manufacturing operations $k_1$ and $k_2$, respectively, are independent. In addition, measurement noise vectors $V(k_1)$ and $V(k_2)$ associated with different manufacturing operations $k_1$ and $k_2$, respectively, are independent.

If the product has been manufactured in stations 1 to $k$, with in-process measurements of quality errors

![Fig. 1 Optimal quality control problem addressed in this paper](image)
being \(y(1), y(2), \ldots, y(k)\), the relation of the measured product quality with controllable and uncontrollable process parameters \(u_{C}(1), u_{C}(2), \ldots, u_{C}(k)\) and \(u_{E}(1), u_{E}(2), \ldots, u_{E}(k)\) can be described as

\[
Y_{1-k} = T_{1-k} U_{1-k} + M_{1-k} E_{1-k} + \eta_{1-k}
\]

where

\[
Y_{1-k} = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(k) \end{bmatrix}, \quad U_{1-k} = \begin{bmatrix} u_{C}(1) \\ u_{C}(2) \\ \vdots \\ u_{C}(k) \end{bmatrix}, \quad E_{1-k} = \begin{bmatrix} u_{E}(1) \\ u_{E}(2) \\ \vdots \\ u_{E}(k) \end{bmatrix}
\]

\[
T_{1-k} = \begin{bmatrix} T_{1,1} & 0 & \cdots & 0 \\ T_{2,1} & T_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ T_{k,1} & T_{k,2} & \cdots & T_{k,k} \end{bmatrix}, \quad M_{1-k} = \begin{bmatrix} M_{1,1} & 0 & \cdots & 0 \\ M_{2,1} & M_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ M_{k,1} & M_{k,2} & \cdots & M_{k,k} \end{bmatrix}, \quad s_{1-k} = \begin{bmatrix} Q_{1,1} & 0 & \cdots & 0 \\ Q_{2,1} & Q_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ Q_{k,1} & Q_{k,2} & \cdots & Q_{k,k} \end{bmatrix}, \quad Q_{i,j} = C(i) \Phi(i,j), \quad i > j
\]

and matrices \(\Phi(i,j)\) are the well-known discrete-time state transition matrices defined as

\[
\Phi(k_1,k_2) = \begin{cases} A(k_2+1)A(k_2+2)\cdots A(k_1) & \text{for } k_1 > k_2 \\ I & \text{for } k_1 = k_2 \end{cases}
\]

It can be observed that the matrix \(T_{1-k}\) describes the way controllable process parameters in operations 1 to \(k\) (‘upstream operations’) influence the in-process measurements from operations 1 to \(k\), while matrix \(M_{1-k}\) describes the way uncontrollable process parameters in operations 1 to \(k\) influence the in-process measurements from operations 1 to \(k\).

Since controllable parameters \(u_{C}(k)\) in each operation can be described as \(u_{C}(k) = u_{C}(k) + \xi_{C}(k)\), where \(u_{C}(k)\) is the desired vector of controllable parameters and \(\xi_{C}(k)\) is a noise vector term following a Gaussian distribution with zero mean and a known variance-covariance matrix \(K_{C}(k)\), model (3) can be rewritten as

\[
Y_{1-k} = T_{1-k} U_{1-k} + M_{1-k} E_{1-k} + \eta_{1-k}
\]

where

\[
U_{1-k} = \begin{bmatrix} u_{C}(1) \\ u_{C}(2) \\ \vdots \\ u_{C}(k) \end{bmatrix}, \quad \eta_{1-k} = \epsilon_{1-k} + T_{1-k} \xi_{C}(1) + \xi_{C}(k)
\]

The noise term \(\eta_{1-k}\) is a Gaussian random vector with zero mean and variance – covariance matrix \(K_{\eta}(k)\) that can be easily expressed based on variance – covariance matrices of noise processes \(\xi_{C}(k), W(k)\), and \(V(k)\). It is independent of the vector of uncontrollable process parameters \(E_{1-k}\).

Continuation of manufacturing through stations \(k + 1, k + 2, \ldots, N\) would result in measured quality errors \(y(k+1), y(k+2), \ldots, y(N)\) that can be expressed using SoV model (2) as

\[
Y_{k+1-N} = T_{k+1-N} U_{k+1-N} + M_{k+1-N} E_{k+1-N} + \epsilon_{k+1-N}
\]

where

\[
Y_{k+1-N} = \begin{bmatrix} y(k+1) \\ y(k+2) \\ \vdots \\ y(N) \end{bmatrix}, \quad U_{k+1-N} = \begin{bmatrix} u_{C}(k+1) \\ u_{C}(k+2) \\ \vdots \\ u_{C}(N) \end{bmatrix}, \quad E_{k+1-N} = \begin{bmatrix} E_{k+1-N} \end{bmatrix}
\]

\[
T_{k+1-N} = \begin{bmatrix} T_{k+1,1} & T_{k+1,2} & \cdots & T_{k+1,k} \\ T_{k+2,1} & T_{k+2,2} & \cdots & T_{k+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ T_{N,k+1} & T_{N,k+2} & \cdots & T_{N,N} \end{bmatrix}, \quad M_{k+1-N} = \begin{bmatrix} M_{k+1,1} & M_{k+1,2} & \cdots & M_{k+1,k} \\ M_{k+2,1} & M_{k+2,2} & \cdots & M_{k+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ M_{N,k+1} & M_{N,k+2} & \cdots & M_{N,N} \end{bmatrix}
\]
process parameters in operations can be seen as the matrix that describes the way in-process measurements from operations \( k+1 \) to \( N \) influence the in-process measurements from operations \( k+1 \) to \( N \). Similarly, matrix \( T_{k+1-N} \) describes the way controllable process parameters in operations \( 1 \) to \( k \) influence the in-process measurements from operations \( k+1 \) to \( N \) (‘upstream operations’), while matrix \( M_{k+1-N} \) describes the way uncontrollable process parameters in operations \( 1 \) to \( k \) (‘upstream operations’) influence the in-process measurements from operations \( k+1 \) to \( N \) (‘downstream operations’).

Since controllable parameters \( u_c(k) \) in each operation can be described as \( u_c(k) = u_c(k) + \xi_c(k) \), where \( u_c(k) \) is the desired vector of controllable parameters and \( \xi_c(k) \) is a noise vector following a Gaussian distribution with zero mean and a known covariance matrix \( K_c(k) \), model (5) can be rewritten as

\[
Y_{k+1-N} = T_{k+1-N} \cdot U_{k-1} + T_{k+1-N} \cdot \xi_c(k) + \epsilon_{k+1-N}
\]

where

\[
\epsilon_{k+1-N} = [u_c(k+1) \, \xi_c(k+1) \, \cdots \, \xi_c(k+1) \, \cdots \, u_c(N) \, \xi_c(N)]
\]

\[

\nu_{k+1-N} = \epsilon_{k+1-N} + \epsilon_{k+1-N}
\]

The noise term \( \nu_{k+1-N} \) is a Gaussian random vector with zero mean and variance-covariance matrix \( K_{\nu_{k+1-N}} \) that can be easily expressed based on variance-covariance matrices of noise processes \( \xi_c(k), E(k), W(k), \) and \( V(k) \). It is independent of the vector of uncontrollable process parameters \( E_{1-k} \).

Based on measured product quality \( Y_{1-k} \) in operations \( 1 \) to \( k \), controllable parameters \( U_{k+1-N} \) in downstream operations \( k+1 \) to \( N \) can be adjusted in such a way that the variations of quality in those operations are minimized. It is assumed that adjustments of the downstream controllable parameters are linear in terms of the measured part quality \( Y_{1-k} \) in operations up to operation \( k \) and in controllable parameters \( U_{1-k} \) in stations \( 1 \) to \( k \); in other words, the control law will be of the shape

\[
U_{k+1-N} = -G_u U_{k-1} - G_y Y_{1-k}
\]

where gain matrices \( G_u \) and \( G_y \) will be tuned to minimize variance of the downstream manufacturing quality \( Y_{k+1-N} \).

In order to facilitate easier derivation of the control law, control law (7) will be represented as

\[
U_{k+1-N} = -G_u U_{k-1} - G_y \hat{E}_{1-k}
\]

where \( \hat{E}_{1-k} \) is itself a linear combination of vectors \( Y_{1-k} \) and \( U_{1-k} \), estimating the uncontrollable parameters \( E_{1-k} \) based on vectors \( Y_{1-k} \) and \( U_{1-k} \).

Substituting equation (8) into equation (6) yields

\[
Y_{k+1-N} = [T_{k+1-N} - T_{k+1-N} G_u] \cdot U_{k-1} + [M_{k+1-N} \cdot T_{k+1-N} - T_{k+1-N} G_u] \cdot E_{1-k} + \epsilon_{k+1-N} + \epsilon_{k+1-N}
\]

The term \( \nu_{k+1-N} \) is the noise term that cannot be predicted by measured part quality \( Y_{1-k} \) and commands \( U_{1-k} \) up to operation \( k \), which is why it cannot be mitigated by any control law based on the observations made up to operation \( k \).

Based on equation (9), in order to minimize the effects of commands \( U_{1-k} \) and uncontrollable parameters \( E_{1-k} \) on the product quality variables \( Y_{k+1-N} \) it would be desirable to make the norm of matrices \( T_{k+1-N} - T_{k+1-N} G_u \) and \( M_{k+1-N} \cdot T_{k+1-N} - T_{k+1-N} G_u \) as small as possible (preferably zero). This can be done by selecting

\[
G_u = T_{k+1-N}^+ M_{k+1-N} \quad G_y = T_{k+1-N}^+ M_{k+1-N} \quad H_{1-k}
\]

where \( H^+ \) denotes the Moore–Penrose inverse of a matrix \( H \) [35]. Substituting this into equation (9) yields

\[
Y_{k+1-N} = T_{k+1-N} \cdot U_{k-1} + T_{k+1-N} \cdot \xi_c(k) + \epsilon_{k+1-N}
\]

\[
U_{k+1-N} = T_{k+1-N}^+ \cdot T_{k+1-N} \cdot U_{k-1} - T_{k+1-N}^+ \cdot T_{k+1-N} \cdot \xi_c(k) + \epsilon_{k+1-N}
\]

\[
\nu_{k+1-N} = \epsilon_{k+1-N} + \epsilon_{k+1-N}
\]

\[
\nu_{k+1-N} = \epsilon_{k+1-N} + \epsilon_{k+1-N}
\]
Since $I - T_{k+1-N} T_{k+1-N}^T$ is the projection operator on the null space of matrix $T_{k+1-N}$ [33], then if the rank of the matrix $T_{k+1-N}$ is equal to the number of its rows, i.e. if the rank of $T_{k+1-N}$ is equal to the number of measured product quality errors in manufacturing operation $k + 1$ to $N$, the null space of matrix $T_{k+1-N}^T$ is trivial (zero), meaning that

$$[T_{k+1-N} 1 \cdot k - T_{k+1-N} G_U] = 0 \quad [M_{k+1-N} 1 \cdot k - T_{k+1-N} G_E] = 0$$

In other words, when the rank of matrix $T_{k+1-N}$ is equal to the number of its rows, control gains can be found in control law (7) to adjust the downstream control parameters $U_{k+1-N}$ so that the variation of the downstream product quality is only due to estimation error $[E_{1-k} - E_{1-k}]$ of the uncontrollable parameters $E_{1-k}$ in the upstream manufacturing operations 1 to $k$, and due to the term noise $v_{k+1-N}$, consisting of variations in process noise, sensor noise, accuracy of controllable parameters, and variations of downstream uncontrollable parameters. The noise term $v_{k+1-N}$ cannot be predicted based on commands and measurements in upstream operations 1 to $k$. The aforementioned condition on the rank of matrix $T_{k+1-N}$ physically means that controllable parameters in downstream manufacturing operations can be used to offset any deterministic influence from upstream operations 1 to $k$. In analogy with traditional control theory, this condition corresponds to controllability between operations $k + 1$ and $N$.

Inspection of equation (11) shows that minimizing in some sense the estimation error $[E_{1-k} - E_{1-k}]$ of the uncontrollable parameters $E_{1-k}$ in the upstream manufacturing operations 1 to $k$, using observed commands $U_{1-k}$ and product measurements $Y_{1-k}$ up to operation $k$ would result in improved quality $Y_{k+1-N}$ measured in downstream operations $k + 1$ to $N$. Given linear model (3) and Gaussianity assumptions, the problem of minimizing the estimation error in the least squares sense reduces to obtaining a linear least squares estimator (LLSE) of uncontrollable parameters $E_{1-k}$ using commands $U_{1-k}$ and product measurements $Y_{1-k}$.

Given linear model (4) and Bayesian assumptions listed at the beginning of this section, this can be accomplished as in reference [34]

$$\hat{E}_{1-k} = K_{k+1-N}^T \cdot M_{k+1-N}^T k  \cdot [M k+1-N 1 \cdot k \cdot K_{k+1-N}^T k + K_{k+1-N}^T k]^{-1} [Y_{1-k} - T_{1-k} U_{1-k}]$$

where $K_{k+1-N}^T k$ denotes the variance–covariance matrix of uncontrollable parameters $E_{1-k}$ in operations 1 to $k$. The variance–covariance $K_{k+1-N}^T k$ matrix of estimation error $[E_{1-k} - E_{1-k}]$ is [34]

$$K_{k+1-N}^T k = K_{k+1-N}^T k - K_{k+1-N}^T k M_{k+1-N}^T k \times [M k+1-N 1 \cdot k K_{k+1-N}^T k + K_{k+1-N}^T k]^{-1} [Y_{1-k} - T_{1-k} U_{1-k}]$$

and its trace is minimized through the LLSE estimation.

Thus, the control law minimizing the variance of product quality in operations $k + 1$ to $N$ based on measurements and commands from operations 1 to $k$ can be summarized as

$$\hat{U}_{k+1-N} = - T_{k+1-N} T_{k+1-N} 1 \cdot k \hat{Y}_{1-k}$$

$$\hat{Y}_{k+1-N} = - T_{k+1-N} M_{k+1-N} 1 \cdot k K_{k+1-N}^T k M_{k+1-N}^T k \times [M k+1-N 1 \cdot k K_{k+1-N}^T k + K_{k+1-N}^T k]^{-1} [Y_{1-k} - T_{1-k} \hat{U}_{1-k}]$$

The gain matrices $G_U$ and $G_Y$ in the linear control law (7) that minimize the variance of the downstream manufacturing quality $Y_{k+1-N}$ are

$$G_U = T_{k+1-N} M_{k+1-N} 1 \cdot k \times [I - K_{k+1-N}^T k M_{k+1-N}^T k K_{k+1-N}^T k M_{k+1-N}^T k + K_{k+1-N}^T k]^{-1} Y_{1-k}$$

$$G_Y = T_{k+1-N} M_{k+1-N} 1 \cdot k K_{k+1-N}^T k M_{k+1-N}^T k [M k+1-N 1 \cdot k K_{k+1-N}^T k M_{k+1-N}^T k + K_{k+1-N}^T k]^{-1}$$

5 DISCUSSION ON PROPERTIES OF SOV-BASED STOCHASTIC DIMENSIONAL QUALITY CONTROL

This section discusses the effects of the control law on the mean and variance–covariance matrix of product quality characteristics output in operations $k + 1$ to $N$.

5.1 Properties of the mean vector of downstream product quality features

Based on equation (9) expressing the part quality in operations $k + 1$ to $N$, the expected value of part quality output by those operations is

$$E(Y_{k+1-N}) = T_{k+1-N} 1 \cdot k 1 \cdot k I - T_{k+1-N} T_{k+1-N}^T 1 \cdot k U_{1-k} = T_{k+1-N} 1 \cdot k \Pi_Y(T_{k+1-N}^T 1 \cdot k) U_{1-k}$$

where $\Pi_Y(T_{k+1-N}^T 1 \cdot k)$ denotes the projection operator onto the null space of matrix $T_{k+1-N}^T 1 \cdot k$. Since vector $Y_{k+1-N}$ denotes measured quality errors in downstream operations $k + 1$ to $N$ that should ideally be zero, it is obvious from equation (12) that introduction of control signals can cause the expected quality in operations $k + 1$ to $N$ to be off nominal, since $E(Y_{k+1-N})$ obviously does not have to be zero. The reason for this is that commands $U_{1-k}$ undertaken in operations 1 to $k$ also affect
part quality in downstream stations \( k + 1 \) to \( N \). In cases where the system is not controllable between operations \( k + 1 \) and \( N \), i.e. when \( \Pi_N(T_{k+1\to N}^T) \neq 0 \), it is not possible to guarantee that downstream control commands in operations \( k + 1 \) to \( N \) can compensate for this effect, thus causing \( E[Y_{k+1\to N}] \) expressed in equation (12) to be not equal to zero.

In case controllability between operations \( k + 1 \) and \( N \) cannot be guaranteed, the expected quality in operations \( k + 1 \) to \( N \) can be kept at nominal by selecting upstream control commands \( U_{k-1} \) to be in the range space of \( T_{k+1\to N}^T \), in which case \( \Pi_N(T_{k+1\to N}) \cdot U_{k-1} = 0 \) and thus \( E[Y_{k+1\to N}] = 0 \). Nevertheless, in order to ensure that control commands do not have any adverse effects on the expected quality in downstream stations, it is necessary to arrange flexible fixtures and corresponding controllable parameters in such a way that controllability is ensured between two successive operations in the system where adjustments of controllable parameters can be made. Thus, the control methodology introduced in the current paper also offers guidelines for appropriate design and allocation of flexible fixtures in the multistation manufacturing system.

5.2 Properties of the variance–covariance matrix of downstream product quality features

The variations of product quality in operations \( k + 1 \) to \( N \) can be analysed by observing the variance–covariance matrix of product quality in operations \( k + 1 \) to \( N \) with and without the control law that utilizes observations of product measurements and control commands in operations 1 to \( k \), as introduced in the previous section.

Using equation (6), it is possible to find the variations away from nominal in the product quality output by operations \( k + 1 \) to \( N \) without any control of the controllable parameters

\[
\begin{align*}
[K_{Y_{k+1\to N}}]_{\text{No control}} &= T_{k+1\to N}[k]K_E^{k+1}T_{k+1\to N}^T[k] \nonumber \\
&+ M_{k+1\to N}[k]K_{E}^{k+1}M_{k+1\to N}^T[k] \\
&+ T_{k+1\to N}[k]K_{E}^{k+1}T_{k+1\to N}^T[k] \\
&+ M_{k+1\to N}[k]K_{E}^{k+1}M_{k+1\to N}^T[k] + K_{p}^{k+1\to N} \\
&= T_{k+1\to N}[k]K_{E}^{k+1}M_{k+1\to N}^T[k] + K_{p}^{k+1\to N}
\end{align*}
\]

(13)

where

\[
K_{E}^{k+1\to N} = T_{k+1\to N}[k]K_{E}^{k+1}T_{k+1\to N}^T[k] \\
+ T_{k+1\to N}[k]K_{E}^{k+1}T_{k+1\to N}^T[k] \\
+ M_{k+1\to N}[k]K_{E}^{k+1}M_{k+1\to N}^T[k] + K_{p}^{k+1\to N}
\]

Using equation (10), it is possible to find the variance–covariance matrix \([K_{Y_{k+1\to N}}]_{\text{With control}}\) with control of the product quality output by operations \( k + 1 \) to \( N \) in the case when controllable parameters \( U_{k+1\to N} \) in operations \( k + 1 \) to \( N \) are adjusted based on measured product quality \( Y_{k+1\to k} \) and commands \( U_{k-1} \) in operations 1 to \( k \), using the control law derived in the previous section. Basic matrix algebra yields

\[
[K_{Y_{k+1\to N}}]_{\text{With control}} = [I - \Pi_R(T_{k+1\to N})][M_{k+1\to N}[k] - 1] \\
- \cdot [K_E^{k+1}\cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})]^T \\
+ \Pi_R(T_{k+1\to N})M_{k+1\to N}[k] - k \cdot K_{E}^{k+1} \\
- \cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})] + K_{p}^{k+1\to N}
\]

(14)

Substituting equation (11) into equation (14) gives

\[
[K_{Y_{k+1\to N}}]_{\text{With control}} = [I - \Pi_R(T_{k+1\to N})][M_{k+1\to N}[k] - 1] \\
- \cdot [K_E^{k} \cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})] \\
+ \Pi_R(T_{k+1\to N})M_{k+1\to N}[k] - K_{E}^{k} \\
- \cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})] + K_{p}^{k+1\to N} \\
- \cdot \Pi_R(T_{k+1\to N})[\Pi_R(T_{k+1\to N})] \\
\]

(15)

Please note that

\[
[I - \Pi_R(T_{k+1\to N})]^T = \Pi_R(T_{k+1\to N}) \text{ and } [I - \Pi_R(T_{k+1\to N})]^T = I - \Pi_R(T_{k+1\to N})
\]

where

\[
\Sigma = M_{k+1\to N}[k]\cdot K_E^{k}\cdot M_{k+1\to N}[k]^T\cdot [M_{k+1\to N}[k] - 1]^{-1}
\]

and

\[
\Pi_R(T_{k+1\to N}) = I - \Pi_N(T_{k+1\to N})
\]

is the projection operator onto the range space of matrix \( T_{k+1\to N} \).

The variance–covariance matrix \([K_{Y_{k+1\to N}}]_{\text{With control}}\) from equation (15) can be written as

\[
[K_{Y_{k+1\to N}}]_{\text{With control}} = [I - \Pi_R(T_{k+1\to N})][M_{k+1\to N}[k] - 1] \\
- \cdot [K_E^{k}\cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})] \\
+ \Pi_R(T_{k+1\to N})M_{k+1\to N}[k] - K_{E}^{k} \\
- \cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})] + K_{p}^{k+1\to N} \\
- \cdot \Pi_R(T_{k+1\to N})[\Pi_R(T_{k+1\to N})]
\]

(16)

Therefore, comparing equations (13) and (16) yields

\[
[K_{Y_{k+1\to N}}]_{\text{With control}} = [K_{Y_{k+1\to N}}]_{\text{No control}} - \Psi
\]

(17)

where

\[
\Psi = \Pi_R(T_{k+1\to N})[\Pi_R(T_{k+1\to N}) \\
+ \Pi_R(T_{k+1\to N})M_{k+1\to N}[k] - 1] \\
- \cdot [K_E^{k}\cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})] \\
+ [I - \Pi_R(T_{k+1\to N})]M_{k+1\to N}[k] - 1] \\
- \cdot K_E^{k}\cdot M_{k+1\to N}[k][I - \Pi_R(T_{k+1\to N})]
\]

(18)
Since $\Psi$ is a positive semi-definite matrix, analysis of equation (17) leads to the conclusion that the control law introduced in section 3 results in a reduction of the variance–covariance matrix of product quality output by operations $k+1$ to $N$ in the least squares sense. Furthermore, based on the assumptions listed in section 3, the aforementioned control law is optimal in the least squares sense.

Further inspection of equation (16) leads to the conclusion that more significant reduction of quality variations in operations $k+1$ to $N$ can be achieved if the dimension of the range space of matrix $T_{k+1\rightarrow N}$ is equal to the number of columns in $T_{k+1\rightarrow N}$, or equivalently if the null space of matrix $T_{k+1\rightarrow N}$ is zero (trivial null space). This condition is equivalent to matrix $T_{k+1\rightarrow N}$ having its rank equal to the number of its rows, i.e. to the controllability condition introduced in the previous section. In that case

$$\Pi_k(T_{k+1\rightarrow N}) = I - \Pi_N(T_{k+1\rightarrow N}) = I$$

and therefore the variance–covariance matrix $[K_{Y}^{k+1\rightarrow N}]_{\text{With control}}$ satisfies

$$[K_{Y}^{k+1\rightarrow N}]_{\text{With control}} = [K_{Y}^{k+1\rightarrow N}]_{\text{No control}} - M_{k+1\rightarrow N} \cdot k \cdot \sum_{i=1}^{N} M_{i\rightarrow k}^T \cdot k \cdot \sum_{i=1}^{N} M_{i\rightarrow k}^{-1} \cdot M_{i\rightarrow k} \cdot k$$

In addition, good estimation of uncontrollable parameters $E_1\rightarrow k$ mirrored in the small trace of the matrix

$$K_{E_1\rightarrow k} = K_{E_1\rightarrow k}$$

implies a large trace of

$$\Sigma = M_{k+1\rightarrow N} \cdot k \cdot [K_{E_1\rightarrow k} - M_{E_1\rightarrow k} \cdot k]$$

and thus a reduction of the trace of the variance–covariance matrix $[K_{Y}^{k+1\rightarrow N}]_{\text{With control}}$. In other words, measurements with a smaller root cause estimation uncertainty, as discussed in references [20] and [21], enable a more effective reduction of the quality variations using the newly proposed stochastic control law. This condition corresponds to the need to pursue measurements that convey the largest amount of information about the uncontrollable parameters in the manufacturing system.

Inspection of equations (13) and (17) proves that variation of product quality output by operations $k+1$ to $N$ can be reduced by strategic adjustments of controllable process parameters in those operations, based on measurements of the product made in upstream operations, up to operation $k$. Essentially, this reduction is made because of the least squares estimation of uncontrollable parameters in operations 1 to $k$, whose influence is then mitigated using controllable parameters in operations $k+1$ to $N$.

Furthermore, it can be noted that pursuit of the controllability condition of matrix $T_{k+1\rightarrow N}$ having its rank equal to the number of its rows yields beneficial results in terms of both expected part quality and part quality variations in operations $k+1$ to $N$. Therefore, achieving controllability through allocation of flexible fixtures and controllable parameters across a manufacturing system should be an important goal in the design of multistation manufacturing systems.

### 6 Numerical Illustration

This section presents a numerical illustration of the potential implementation of the SoV-based online stochastic quality control method proposed in section 4 and discussed in section 5. The purpose of this section is to illustrate the potential impact of the newly proposed methods in a real industrial process. One should, however, note that the control law described in section 4 will display the mean and variance–covariance properties in section 5 as long as the multistation process is modelled in the linear state-space form (2), i.e. irrespective of what the numerical values of the state-space model matrices are.

The impact of the newly proposed method will be demonstrated in the machining process used to produce an automotive cylinder head, as shown in Fig. 2. The process plan and locating datum features are identified in Table 1 and correspond to the process plan used by a major US automotive manufacturer.

Each plane was described by a unit vector defining the plane orientation and one point in the plane defining its position. Cylindrical features were described by a unit vector defining the cylinder axis, one point on the cylinder axis defining its position, and by the cylinder diameter. The resulting state-space model for $N=5$ operations had the state vector of dimension 28 (for details on SoV modelling of dimensional machining errors in this process see references [9] and [10]). It is assumed that all fixture and tool movement parameters in operations 1 to 4 are uncontrollable, while a flexible fixture exists in the last station performing the fifth machining operation (machining of the slot), so that fixture adjustments in that last operation can be made based on measurements made after operation 4 in
order to minimize, in the least squares sense, the variations in the product quality after the last machining operation.

After operation 4, measurements of the cover face and rough datum points X1, X2, X3, Y1, Y2, and Z1 are performed and are expressed in the coordinate system defined by the machined joint face (primary measurement datum feature), datum hole B (secondary measurement datum feature), and datum hole C (tertiary measurement datum feature); this part coordinate system is also indicated in Fig. 1. After the last (fifth) machining operation, measurements of the orientation and distance of the slot with respect to the plane defined by rough datum points X1, X2, and X3 are performed. Such measurements could easily be taken using a coordinate measurement machine (CMM).

Statistical properties of the quality characteristics measured after operation 5 have been evaluated using equations (17) and (18) for the cases with and without the newly proposed method for SoV-based stochastic quality control respectively. The numerical values necessary for the evaluation of matrices \(K_{5}^{Y}/C_{1}^{1}/C_{1}^{1}/C_{1}^{1}/Y_{C_{2}^{2}/C_{3}^{3}}\) No control and \(K_{5}^{Y}/C_{1}^{1}/C_{1}^{1}/C_{1}^{1}/Y_{C_{2}^{2}/C_{3}^{3}}\) With control are shown in the Appendix. It should be noted that the variance-covariance matrices for the measurement noise and process noise that are reported in the Appendix are somewhat distorted so that the proprietary process capability indices are not disclosed. It should also be noted that it is assumed that the flexible fixture used in the last machining operation is equally as accurate as the fixtures used in operations 1 to 4, which also does not reflect reality since flexible fixtures display a higher positioning variation than rigid fixtures.

For the numerical values reported in the Appendix, the standard deviation of the orientation of the slot with respect to the rough datum surface defined by points X1, X2, and X3 reduces from \(1.8937 \times 10^{-4}\) radians for the case without implementation of the newly proposed control algorithm, to \(1.4882 \times 10^{-4}\) radians when the quality control algorithm is employed. The standard deviation of the position of the slot with respect to the rough datum surface defined by points X1, X2, and X3 reduces from 25.4 \(\mu\)m without implementation of the newly

\(\text{Table 1} \quad\text{Process plan for machining of the cylinder head shown in Fig. 1}\)

<table>
<thead>
<tr>
<th>Locating surfaces</th>
<th>Machining operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setup 1</td>
<td>Primary: rough datum points X1, X2, and X3&lt;br&gt;Secondary: rough datum points Y1 and Y2&lt;br&gt;Tertiary: rough datum point Z1</td>
</tr>
<tr>
<td>Setup 2</td>
<td>Primary: machined cover face (CF)&lt;br&gt;Secondary: rough datum points Y1 and Y2&lt;br&gt;Tertiary: rough datum point Z1</td>
</tr>
<tr>
<td>Setup 3</td>
<td>Primary: machined joint face (J)&lt;br&gt;Secondary: machined hole B&lt;br&gt;Tertiary: machined hole C</td>
</tr>
</tbody>
</table>
proposed control algorithm to 18.1 μm when the quality control algorithm is employed.

The reason why such significant improvements could be made is because the rank matrix quality control algorithm is employed. Proc. IMechE Vol. 221 Part B: J. Engineering Manufacture JEM458/C211

The solution of this problem will involve formulation of a metric involving the variations of all quality characteristics given the control law (with possibly multiple decision points) to serve as the objective function in the subsequent optimization of where the measurement points should be (after which station measurements should be taken), where the flexible fixtures, should be (actuation points), and finally where decisions should be made. Based on this metric, an optimization procedure should be made to solve the allocation of measurements, flexible fixtures, and decision points. However, this problem is also outside the scope of the current paper and will be addressed in future work.

7 CONCLUSIONS AND FUTURE WORK

A stochastic control law that is optimal in the least squares sense is derived in the present paper to facilitate control of dimensional quality in multistation manufacturing systems using the SoV model of the system. In-process measurements taken in the middle of the process are used to devise a control law that takes into account the influence of actuation accuracy of flexible tooling elements whose parameters can be controlled, variations of uncontrolled process parameters, process noise, and measurement accuracy, in order to minimize in the least squares sense the variations of product quality output by the stations that are downstream from the one where measurements are taken. Condition of error controllability is mathematically defined as a condition that ensures achieving on-target expected product quality output by the downstream stations. It is proven that the newly proposed control law reduces variations of the downstream product quality and that the controllability condition has beneficial effects on both the expected product quality output by the operations downstream from the in-process measurements, and its variations.

The control algorithm devised in this paper attempts to reduce variations in all downstream operations once in-process measurements of the product are made. This approach is optimal in the least squares sense if process parameter adjustments based on in-process measurements are made just at one point in the manufacturing system. If such adjustments are made in several places in the system, additional information obtained through additional in-process measurements of product quality can and should be taken into account. An optimal solution to this problem is one possible topic for future research.

Furthermore, reduction of variations of product quality using in-process measurements and online adjustments of controllable process parameters as well as the controllability condition can be used to optimally allocate measurement and actuation stations in a multistation manufacturing system to ensure controllability conditions and the maximum possible reduction of product quality variations. ACKNOWLEDGEMENT

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REFERENCES


APPENDIX

Numerical values used in section 6

The numerical values of the process model gave the following transformation matrices necessary for evaluation of matrices \( [K_T^2]_5 \) and \( [K_T^3]_5 \) with control are enclosed below.

- Matrix \( M_{5.5} \) describes the way uncontrollable process parameters in operation 5 (in this case it is the ‘downstream operation’) influence the in-process measurements in operation 5

\[
M_{5.5} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

- Matrix \( T_{5.5} \) describes the way controllable process parameters in operation 5 influence the in-process measurements from operation 5

\[
T_{5.5} = \begin{bmatrix}
-0.00525 & 0.00525 & 0 & 0 & 0 & 0 \\
0.00525 & 0.00262 & -0.00787 & 0 & 0 & 0 \\
-0.66667 & 0.66667 & -1 & 0 & 0 & 0
\end{bmatrix}
\]

- Matrix \( M_{5.5[5]} \) describes the way uncontrollable process parameters in operations 1 to 4 (in this case these are the ‘upstream operations’)
influence the in-process measurements from operation 5 (in this case this is the ‘downstream operation’)

\[
M_{5:4|1-4} = \begin{bmatrix}
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-153.7 & -153 & 1 & 153 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

- Matrix \(T_{5:4|1-4}\) describes the way controllable process parameters in operations 1 to 4 influence the in-process measurements from operation 5

\[
T_{5:4|1-4} = \begin{bmatrix}
-0.00453 & 0.00453 & 0 & 0 & 0 & -0.00525 & 0.00525 & 0 & 0 & 0 & 0 \\
0.00263 & 0.00379 & -0.006413 & 0 & 0 & -0.00350 & -0.00175 & 0.00525 & 0 & 0 & 0 \\
-0.85368 & 0.6806 & -0.82692 & 0 & 0 & -1.3211 & 0.54418 & -0.2231 & 0 & 0 & 0 \\
\end{bmatrix}
\]

- Matrix \(M_{1:4}\) describes the way uncontrollable process parameters in operations 1 to 4 (in this case these are the ‘downstream operations’) influence the in-process measurements from operations 1 to 4

\[
M_{1:4} = \begin{bmatrix}
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-115.45 & -22.3 & 1 & 114.75 & -130.7 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.00327 & 0 & 0 & 0 & 0 & -0.00327 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 0 & 0.86111 & 0 & 0 & 0 & 0.13899 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00327 & 0 & 0 & 0 & 0.00327 \\
-1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 & 0 & 0 & 0.4297 & 0 & 0 & 0 & -0.4297 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

- Matrix \(T_{1:4}\) describes the way controllable process parameters in operations 1 to 4 influence the in-process measurements from operations 1 to 4
Variance–covariance matrix $K^{5-5}$ describes measurement variations for features machined in operation 5

$$K^{5-5} = \begin{bmatrix} 1.6 \times 10^{-9} & 0 & 0 \\ 0 & 1.6 \times 10^{-9} & 0 \\ 0 & 0 & 4 \times 10^{-6} \end{bmatrix}$$

Variance–covariance matrix $K^{1-4}$ describing measurement variations for features machined in operations 1 to 4, is a diagonal matrix with diagonal elements

- $[K^{1-4}_{1,1}] = [K^{1-4}_{2,2}] = [K^{1-4}_{3,3}] = [K^{1-4}_{4,4}] = [K^{1-4}_{5,5}] = [K^{1-4}_{6,6}] = [K^{1-4}_{7,7}] = [K^{1-4}_{8,8}] = 1.6 \times 10^{-9}$

- $[K^{1-4}_{10,10}] = [K^{1-4}_{11,11}] = [K^{1-4}_{12,12}] = [K^{1-4}_{13,13}] = [K^{1-4}_{14,14}] = [K^{1-4}_{15,15}] = [K^{1-4}_{16,16}] = 4 \times 10^{-6}$

Variance–covariance matrix $K^{5-5}$, describes variations of uncontrollable parameters in operation 5

$$K^{5-5} = \begin{bmatrix} 4.9 \times 10^{-9} & 0 & 0 \\ 0 & 4.9 \times 10^{-9} & 0 \\ 0 & 0 & 4.9 \times 10^{-5} \end{bmatrix}$$

Variance–covariance matrix $K^{1-4}$, describing variations of controllable parameters in operations 1 to 4, is a diagonal matrix with the following diagonal elements

- $[K^{1-4}_{1,1}] = [K^{1-4}_{2,2}] = [K^{1-4}_{3,3}] = [K^{1-4}_{4,4}] = [K^{1-4}_{5,5}] = [K^{1-4}_{6,6}] = [K^{1-4}_{7,7}] = [K^{1-4}_{8,8}] = 1.6 \times 10^{-9}$

- $[K^{1-4}_{10,10}] = [K^{1-4}_{11,11}] = [K^{1-4}_{12,12}] = [K^{1-4}_{13,13}] = [K^{1-4}_{14,14}] = [K^{1-4}_{15,15}] = [K^{1-4}_{16,16}] = 4 \times 10^{-6}$
• Noise terms $W(k)$ due to unmodelled effects and nonlinearities are neglected in this numerical study.

The control strategy described in section 4 and discussed in section 5, yields the following matrices, $[K_{Y}^{5,5}]_{\text{No control}}$ and $[K_{Y}^{5,5}]_{\text{With control}}$, for the numerical values enclosed in this appendix.

Since components in positions (1,1) and (2,2) denote variances of small angular errors of a planar feature in two orthogonal directions, the total standard deviation of that error is estimated as a square root of the variances in the two orthogonal directions. In the case without control the standard deviation of the angular error is $1.8937 \times 10^{-4}$ radians, and in the case with control the standard deviation of the angular error is $1.4882 \times 10^{-4}$ radians.

\[
[K_{Y}^{5,5}]_{\text{No control}} = \begin{bmatrix}
1.7707 \times 10^{-8} & 7.0732 \times 10^{-10} & 2.3228 \times 10^{-6} \\
7.0732 \times 10^{-10} & 1.8154 \times 10^{-8} & 1.14842 \times 10^{-6} \\
2.3228 \times 10^{-6} & 1.14842 \times 10^{-6} & 6.4555 \times 10^{-4}
\end{bmatrix}
\]

\[
[K_{Y}^{5,5}]_{\text{With control}} = \begin{bmatrix}
1.0824 \times 10^{-8} & 8.2055 \times 10^{-10} & 1.2585 \times 10^{-6} \\
8.2055 \times 10^{-10} & 1.1323 \times 10^{-8} & 0.6569 \times 10^{-6} \\
1.2585 \times 10^{-6} & 0.6569 \times 10^{-6} & 3.2615 \times 10^{-4}
\end{bmatrix}
\]