Tutorial: Scheduling Multiclass Queueing Networks via Fluid Models

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Fluid Optimization Problems. Several papers have studied only the optimization problem which arises from formulating the fluid model equivalent of a multiclass queueing network (or more general networks). Probably the best introduction is Weiss [11] which discusses a number of fluid optimization problems and objective functions. Similar papers which appeared about the same time and also make good introductory reading are Weiss [12] and Avram, et al. [1].

A fluid model problem in which the objective function represents linear holding costs can be formulated as a *separated continuous linear program* (SCLP). The papers mentioned above solved some simple SCLPs. Luo and Bertsimas [5] developed an efficient algorithm to solve a class of problems which include the SCLP. Weiss [10], however, provided a finite simplex-like algorithm which solves a general SCLP.

Fluid Translations. In the last few years, many researchers in the queueing community have contributed to the literature on scheduling networks via fluid models. Many of these use the fluid model to gain scheduling insight in one regime, using other approximations (most notably diffusion approximations) to gain further insight. A good place to start reading about fluid translations is in a set of papers by Bertsimas, Gamarnik, and Sethuraman [2, 3], who basically examine finite (non-dynamic) scheduling problems. Dai and Weiss [4] use a different approach to fluid translation and obtain probabilistic bounds on the translation error. Maglaras [6] uses a so-called discrete review policy to obtain asymptotically optimal fluid translations. Magalaras' paper also contains a nice example showing the difficulty of converting fluid solutions to optimal policies. Meyn's recent papers [7, 8] outline a program for using the fluid model to do scheduling and routing for a more general class of networks. Veatch [9] has also looked at fluid scheduling and input control.

References

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