Combined simulation and inversion of SP and resistivity logs for the estimation of connate-water resistivity and Archie’s cementation exponent

Jesús M. Salazar¹, Gong Li Wang¹, Carlos Torres-Verdín¹, and Hee Jae Lee¹

ABSTRACT

Knowledge of initial water saturation is necessary to estimate original hydrocarbon in place. A reliable assessment of this petrophysical property is possible when rock-core measurements of Archie’s parameters, such as saturation exponent $n$ and cementation exponent $m$, are available. In addition, chemical analysis of formation water is necessary to measure connate-water resistivity $R_w$. Such measurements are seldom available in most applications; if they are available, their reliability may be questionable. We describe a new inversion method to estimate $R_w$ and Archie’s cementation exponent from the combined use of borehole spontaneous-potential (SP) and raw array-induction resistivity measurements acquired in water-bearing depth intervals. Combined inversion of resistivity and SP measurements is performed assuming a piston-like invasion profile. In so doing, the reservoir is divided into petrophysical layers to account for vertical heterogeneities. Inversion products are values of invaded and virgin formation resistivity, radius of invasion, and static spontaneous potential SSP. Connate-water resistivity is calculated by assuming membrane and diffusion potentials as the main contributors to the SSP. Archie’s or dual-water equations enable the estimation of $m$. The new combined estimation method has been successfully applied to a data set acquired in a clastic formation. Data were acquired in a high permeability and moderately high-salt-concentration reservoir. Values of $R_w$ and $m$ yielded by the inversion are consistent with those obtained with a traditional interpretation method, thereby confirming the reliability of the estimation. The method is an efficient, rigorous alternative to conventional interpretation techniques for performing petrophysical analysis of exploratory and appraisal wells wherein rock-core measurements may not be available.

INTRODUCTION

Initial water saturation in hydrocarbon reservoirs has an enormous impact on calculating and producing original oil in place. When laboratory measurements (core, water analysis, etc.) are available, this variable is properly constrained. However, such measurements are not always available, and even if they are, their reliability may be questionable. Therefore, a strong need exists for alternative methods to estimate those parameters before calculating initial water saturation.

Two of the main parameters needed to calculate water saturation are connate-water resistivity $R_w$ and the cementation exponent $m$, which can be obtained from connate-water analysis and special core analysis, respectively. Core measurements are often expensive because they involve the cost of extracting the core sample and subsequent laboratory work. Moreover, measurements of water resistivity are difficult because of the need to acquire connate-water samples when wells are already in production and water injection/steam flooding has been applied to enhance production. Fluid samples taken by fluid-acquisition tools are often contaminated with mud filtrate and/or hydrocarbon.

In the early days of formation evaluation, spontaneous potential (SP) and resistivity were the only borehole measurements available for interpretation to log analysts or petrophysicists (Doll, 1949). One of the first physical models of SP was developed using a resistor network (Segesman, 1962), where dipole layers were simulated using voltage sources. Zhang and Wang (1997, 1999) developed a finite-element algorithm to simulate SP measurements using the vector and scalar potential theory. Their algorithm successfully reproduced the resistor model developed by Segesman.

In this paper, we simulate SP measurements using Zhang and Wang’s algorithm and resistivity measurements with the numerical-
mode matching (NMM) method (Chew et al., 1984; Zhang et al., 1999). We use raw borehole-corrected array-induction resistivity (AIT, a mark of Schlumberger) measurements and consider the effect of invasion and layer thickness on both resistivity and SP. Our objective is to calculate $R_e$ and $m$ for a field data set acquired in a water-bearing clastic formation based on the combined inversion of SP and raw array-induction measurements. The estimation method is based on quantitative simulations and interpretation of the measurements with realistic physical assumptions, not on qualitative interpretations. Additionally, to benchmark the technique, values of $R_e$ obtained with this method are compared to those obtained with Pickett plots (Pickett, 1966). Finally, we summarize the possible applications of the method.

**RESISTIVITY MODELING AND INVERSION**

The purpose of resistivity modeling (forward modeling) and inversion is to estimate the invaded-zone $R_{inv}$, virgin-zone $R_v$ resistivities, and the radius of invasion $r_{inv}$ from raw array-induction conductivity (inverse of resistivity) measurements. Figure 1 shows the subsurface model assumed in the simulations. Initially, we assume a single-layer model and a piston-like radial profile of invasion. The system is bounded at the top and bottom by shale beds (shoulders) with resistivities equal to $R_{sh, top}$ and $R_{sh, bot}$, respectively. Also, the model is radially bounded by a borehole with fluid resistivity equal to $R_m$ and the virgin zone, whose resistivity is one of the main inputs necessary to estimate water saturation.

**Forward model**

Induction tools measure formation conductivity $\sigma$ by inducing low-frequency electric currents into the formation surrounding the borehole. We adopt a 2D axial-symmetric model. In such a model, current loop sources reside at the center of the borehole, with a magnetic moment pointing upward; the conductivity is invariant in the azimuthal direction. Sedimentary rocks are generally not magnetic; hence, the magnetic permeability $\mu$ is assumed equal to the vacuum magnetic permeability $\mu_0$. As a result, the electric field comprises only the azimuthal component $E_r$ and varies only in the meridian plane.

Assuming there is only one source current loop, with radius $r_i$ and vertical position $z_s$, the governing equation in cylindrical coordinates $(r, z)$ for $E_r$ is given by

$$\left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial z^2} + k^2 \right] E_r(r, z, r_i, z_s) = -i\omega \mu_0 I \delta(r - r_i) \delta(z - z_s),$$  \hspace{0.5cm} (1)

where $I$ is the electric-current intensity, $\delta$ is Dirac’s delta function, $\omega$ is angular frequency, $k^2 = i\omega \mu_0 \sigma$, $i = \sqrt{-1}$, and the time convention $e^{-\omega t}$ is assumed, where $t$ is time. The boundary condition is $E_r(r, z)|_{z=0} = 0$, where $\Omega = \{(r, z)|0 < r < \infty, -\infty < z < \infty\}$. We assume a piecewise-constant spatial distribution of electrical conductivity:

$$\sigma(r, z) = \sigma_0(r) H(z_1 - z) + \sum_{m=1}^{M-1} \sigma_m(r)[H(z - z_m) - H(z - z_{m+1})],$$  \hspace{0.5cm} (2)

where $H(z)$ is the Heaviside function and $\sigma_0$ and $\sigma_m$ are the conductivities of the first and last layers, respectively. There is a total of $M + 1$ horizontal layers with boundaries $z_1, z_2, \ldots, z_M$; layer conductivities are given by

$$\sigma_m(r) = \sigma_{inv} H(r_{inv} - r) + \sigma_m H(r - r_{inv}),$$  \hspace{0.5cm} (3)

$$+ \sigma_{inv,m} H(r - r_{inv}) - H(r - r_{inv,m}),$$

where $r_{inv}$ is the radial length of invasion, $r_m$ is the borehole radius, $\sigma_{inv} = \sigma_m$ is the borehole conductivity, and $\sigma_m$ and $\sigma_{inv}$ are the invaded and virgin-zone conductivities, respectively.

We use the NMM method to solve the above 2D simulation problem (Chew et al., 1984; Zhang et al., 1999). This algorithm combines a 1D finite-element solution in the radial direction with an analytical solution in the vertical direction. When augmented with amplitude and slope basis functions (Zhang et al., 1999), the NMM is hundreds of times more computer efficient than either 2D finite-element or finite-difference method. (One function describes the amplitude, and the remaining function describes the slope of the electric field at nodal points.)

**Inversion**

Inversion of array-induction resistivity measurements is posed as the minimization of a quadratic cost function that comprises an additive term which weighs the data residuals and one additive term introduced to stabilize the inversion in the presence of noisy and sparsely sampled measurements. We adopt the quadratic cost function, given by

$$C(\bar{x}) = \frac{1}{2} \left\{ \| \bar{d}(\bar{x}) - \bar{d}^c \|^2 + \lambda^2 \| \bar{x} \|^2 \right\},$$  \hspace{0.5cm} (3)

where $\bar{x}$ includes the unknown model parameters (layer-by-layer values of $R_m$, $R_v$, and $r_{inv}$), $\lambda^2$ is the regularization (stabilization) parameter, and $\bar{d}$ is the vector of data residuals given by $\bar{d}(\bar{x}) = \bar{d} - \bar{d}^c$. In this expression, $\bar{d}(\bar{x})$ contains the indexed numerically simulated measurements and $\bar{d}^c$ contains the correspond-
ing indexed field measurements (raw array-induction conductivities). For the estimation of layer resistivities, we assume that layer boundaries are known a priori.

We approach the minimization of the quadratic cost function given by equation 3 using the distorted Born iterative method (DBIM) (Chew and Liu, 1994). The computation of sensitivities (entries of the Jacobian matrix) is crucial to solving the nonlinear minimization problem by iterative linear steps.

To calculate the sensitivities, we consider a generic single-transmitter, single-receiver induction system. Let \( r_T \) be the radius and \( z_T \) the vertical location of the transmitter. Likewise, let \( r_S \) and \( z_S \) be the radius and vertical location of the receiver, respectively. When applying a perturbation to the background conductivity \( \sigma_0 \), the corresponding perturbation of apparent conductivity \( \sigma_a \) is given by (Zhang, 1984)

\[
\Delta \sigma_a = \int_0^\infty \int_{-\infty}^\infty dr dz g(r, z) \Delta \sigma(r, z),
\]

where

\[
g(r, z) = \frac{2 \pi \omega^2 \mu_0^2 j}{K} \frac{r_G(r, z; r_T, z_T) G(r, z; r_S, z_S)}{G(r, z; r_T, z_T)},
\]

\( K \) is the tool constant, \( G(r, z; r_T, z_T) \) is the Green’s function that satisfies equation 1 with the right-hand side replaced with \(-\delta(r - r_T) \delta(z - z_T)\), and \( \Delta \sigma = \sigma - \sigma_0 \) is a perturbation of the background conductivity. Substituting equation 2 into equation 4 yields

\[
\frac{\partial \sigma_a(r, z)}{\partial \sigma_{xo,m}} = \int_{r_m}^{r_T} \int_z^{z_T + 1} dr dz g(r, z),
\]

\[
\frac{\partial \sigma_a(r, z)}{\partial \sigma_{t,m}} = \int_{r_m}^{r_T} \int_z^{z_T + 1} dr dz g(r, z),
\]

\[
\frac{\partial \sigma_a(r, z)}{\partial r_{inv,m}} = (\sigma_{xo,m} - \sigma_{t,m}) \int_0^{z_T} dz g(r_{inv,m}, z),
\]

where \( \sigma_{xo,m} \) and \( \sigma_{t,m} \) are invaded- and virgin-zone layer conductivities, respectively, and \( r_{inv,m} \) is the radius of invasion for each layer.

In the above equations, we assume the formation boundary \( z_m \) is known. Initially in the DBIM, the Jacobian matrix, whose entries are given by equation 5, is recalculated after updating the conductivity distribution (Chew and Liu, 1994). Because the induction problem is quasi-linear (Zhang, 1984), the Green’s function is approximated with that of a homogeneous medium penetrated by a borehole and fixed for all iteration steps. Consequently, at each step, the Jacobian matrix is reset by evaluating only the three integrals given by equation 5. This approach represents a highly cost-effective process because the repeated evaluation of \( g(r, z) \) is unnecessary. For the inversion, we assume that \( \sigma_{xo,m} \) and \( r_T \) are known and that the conductivity of the homogeneous medium is equal to 0.01 S/m. This strategy greatly reduces the computer cost required by calculating derivatives.

We invoke the multiplicative regularization technique described by Habashy and Abubakar (2004) to calculate the regularization parameter included in the quadratic cost function (equation 3) with the relationship

\[
\lambda^2 = \frac{||\mathbf{e}(\mathbf{x})||^2}{\beta},
\]

where \( \beta \) is a constant that can be determined with numerical experiments and, in our case, is equal to 2.0. In thick formations (over hundreds of feet long) containing many layers, the inversion process acts as a depth window sliding over the data set layer by layer. This is possible because induction is primarily a localized measurement, namely, apparent conductivity is mostly affected by layers close to the measurement point whereas the effect of layers far from the measurement point is comparatively small.

**SP MODEL**

The SP has four main components (Hallemberg, 1971): the diffusion potential, the membrane or Nernst potential (both diffusion and membrane are known as the electrochemical potentials), the electrokinetic or streaming potential, and the oxidation/reduction (redox) potential. Electrokinetic and redox potentials are negligible in borehole applications compared with the electrochemical potentials. Here, we assume the total potential measured by a borehole SP tool is solely the sum of the membrane and diffusion potentials (Wyllie and Southwick, 1954).

In a permeable zone at borehole conditions, the maximum differential potential (in absolute value) is known as the static SP (SSP). The SSP at borehole conditions is measured with respect to the shale baseline (Pirson, 1963) and, in millivolts, is given by

\[
\text{SSP} \equiv -70.7 \times \left( \frac{460 + T_F}{537} \right) \log \left( \frac{R_{mfe}}{R_{we}} \right),
\]

where \( R_{we} \) designates the equivalent water resistivity, \( R_{mfe} \) is the equivalent mud-filtrate resistivity, and \( T_F \) is the formation temperature in degrees Fahrenheit. We use equation 6 to estimate \( R_{we} \) via the combined inversion of SP and resistivity measurements.

**Modeling and inversion of SP**

The SSP is an electromotive force because of electric dipole layers distributed along the borehole wall, invasion fronts, and formation boundaries. In this study, vector potential theory (Zhang and Wang, 1997, 1999) is used to compute SP in a water-based, mud-filled borehole. Accordingly, electric dipole layers that could extend to infinity are replaced with magnetic current rings located at the intersection points of the borehole wall, invasion fronts, and formation boundaries.

In cylindrical coordinates \((r, z)\), the governing equation is given by

\[
\frac{1}{2\pi} \left[ \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\partial}{\partial z} \frac{1}{r} \frac{\partial V}{\partial r} \right]
\]

\[
= -e_{SSP} \delta(r - r_T) \delta(z - z_T),
\]

where \( V \) is the current potential (Zhang and Wang, 1997, 1999), \( e \) is the electrical conductivity of the formation, \( e_{SSP} \) is the total potential difference across dipole layers, and \( r_T \) and \( z_T \) are the radius and vertical locations of the transmitter and receiver, respectively.
Results from resistivity inversion are used directly in the inversion of SP measurements. In addition, we assume that mud resistivity, borehole radius, and layer boundary location are a priori information determined from other measurements. Therefore, in the SP inversion, we only invert for the magnitudes of SSP. From equation 7, we realize (1) the SP response is linear with respect to the source magnitude and (2) when there are many sources, the total SP response is the superposition of SP responses originating separately from all sources. Thus, one can write the equation for SP (SP) as

$$SP_n = \sum_{m=1}^{M} \varepsilon_{SSP,m}SP_{nm}, \quad n = 1,2,\ldots,N,$$

where \(N\) is the total number of SP measurements, \(M\) is the total number of SP sources, \(\varepsilon_{SSP,m}\) is the magnitude of the potential difference for the \(m\)th SP source, and \(SP_{nm}\) is the response to a unit SP source with radius \(r_{n,m}\) and vertical position \(z_{n,m}\), obtained by solving equation 7.

Equation 8 can be written in matrix notation as

$$\tilde{\mathbf{y}} = \mathbf{b},$$

where \(\tilde{\mathbf{y}} = (SP_{n1})_{N \times M}\), \(\mathbf{y} = (\varepsilon_{SSP,m})_{M \times 1}\), and \(\mathbf{b} = (SP_{n})_{N \times 1}\). Because the coefficient matrix \(\tilde{\mathbf{C}}\) is rectangular, we use the generalized singular-value-decomposition (SVD) method to solve equation 9. Accordingly, the solution of the eigenvalue problem is given by

$$\mathbf{y} = \sqrt{\mathbf{T}}\mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{b},$$

where \(\mathbf{U} \in R^{N \times M}\) and \(\mathbf{V} \in R^{M \times M}\) are two unitary matrices formed by eigenvectors, \(\mathbf{\Sigma} \in R^{N \times M}\) is a diagonal matrix that consists of singular values obtained from the SVD, and the superscript \(T\) designates the transpose of a matrix. We assume that SP sources are far from each other; therefore, no truncation of the spectrum of singular values is necessary, whereupon matrix \(\tilde{\mathbf{C}}\) is invertible.

Figure 2 shows a multilayer subsurface model (heterogeneous formation) that includes three invaded layers with different radii of invasion and resistivities for each cylindrical and vertical layer. The resistivity of each radial block (\(R_w\) and \(R_t\)), the radius of invasion, and the SSP at each layer boundary are the properties estimated with the combined inversion of SP and resistivity measurements.

**Comments on assumptions and limitations of the simulation of SP measurements**

The SP sources assumed in our algorithm are exactly the same ones used to construct the Schlumberger’s SP-3 chart (Schlumberger, 1991). That is to say, they are dipole layers that could extend to infinity from the borehole wall or from invasion fronts. Under the theory of current potential, these dipole layers are equivalent to many infinitesimal rings centered on the borehole axis. Physically, they are static magnetic-current rings that drive the flow of electric current in both the borehole and the formation. Such infinitesimal current rings and dipole layers are mathematically equivalent to each other. However, infinitesimal current rings are more amenable to numerical simulation.

The dipole-layer assumption for SP sources commonly has been used in log interpretation. Such an approach will continue to be used until a better assumption is introduced in the SP theory, which should be more physically complete and numerically easier to implement than the current one.

In modern induction tools, the reliability of SP measurements can be compromised as a result of the short distance that exists between the SP electrode and the grounded metal. However, the effect of that distance becomes appreciable only when formations are highly resistive (more than 100 \(\Omega\)m). The field case considered in this article corresponds to a very conductive formation, in which the resistivity is generally less than 20 \(\Omega\)m, whereby the effect of short distance between the SP electrode and the grounded metal is negligible. Considering that the favorable measurement condition for induction tools is generally low-resistivity formations, it is reasonable to assume that the effect of short distance between the SP electrode and the grounded metal is always negligible as long as this condition is satisfied.

**ESTIMATION OF \(R_w\) AND \(m\)**

Field SP and raw AIT measurements as well borehole and mud properties are used as input to the combined SP-resistivity inversion. Resistivity-inverted results are input to the SP inversion. To estimate connate-water resistivity, the inversion is carried out in a wet-sand interval. Once the SSP is calculated at each layer boundary from the inverted SP, the corresponding equivalent water resistivity is calculated with the maximum negative SSP using equation 6. Subsequently, an empirical correlation from log interpretation charts (Schlumberger, 1991; Bigelow, 1992) is used to estimate \(R_w\), namely,

$$R_w = \frac{R_{we} + 0.131 \cdot 10^{-\left(\frac{1}{\log(\tau_{w}/19.9)} - 2\right)}}{-0.5 \times R_{we} + 10^{0.0426/\log(\tau_{w}/50.8)}}.$$  

In our method, we assume that formation water is a solution of sodium chloride (NaCl). Therefore, equation 10 is not valid for all types of formation water or waters with very high salt concentration.
To estimate the cementation exponent, we initially assume a clean (shale-free) 100% water-saturated clastic rock. Archie’s equation (Archie, 1942) is used to compute the cementation exponent without specific adjustments for the presence of shale as follows:

\[ m = \frac{1}{\log(\phi)} \log \left( \frac{R_w}{R_i} \right), \]  

where \( \phi \) is the nonshale porosity taken as an average in the interval of analysis and the tortuosity factor \( \alpha \) (not shown in equation 11) is assumed equal to one. On the other hand, when the presence of clay is considered, we use the dual-water model for shaly sands (Clavier et al., 1984) because several of its governing parameters can be calculated from well logs (Dewan, 1983):

\[ m = \frac{1}{\log(\phi)} \log \left( \frac{R_w}{R_i} \left( 1 - S_b \left( 1 - \frac{R_w}{R_b} \right)^{-1} \right) \right), \]

where \( S_b \) and \( R_b \) are bound-water saturation and bound-water resistivity (a function of shale resistivity), respectively. Equation 12 is valid in wet sands and reduces to equation 11 in clean sands (where \( S_b = 0 \)). Figure 3 summarizes the steps used to estimate connate-water resistivity and cementation exponent via the combined inversion of resistivity and SP measurements.

**FIELD CASE STUDY**

We select a formation with a water-bearing depth zone to test the developed method on field measurements. The case under analysis corresponds to a north Louisiana (U. S.) fairly clean clastic formation. The reservoir is 100% water saturated with moderately high salt concentration (more than 100,000 ppm). Table 1 summarizes the thickness and average petrophysical properties of the formation. Figure 4 displays the well logs and estimated porosity and permeability for the formation under analysis. Well logs include gamma ray (GR), SP, and AIT resistivity (2-ft vertical resolution). Porosity is calculated from density-neutron logs via a nonlinear dual-mineral (shale-quartz) model, and permeability is calculated from porosity and irreducible water saturation using a modified Timur-Tixier equation (Balan et al., 1995). Water saturation is not shown here because the formation is completely water saturated (\( S_w = 100\% \)).

### Table 1. Summary of average petrophysical properties assumed for the case study.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>m</td>
<td>17.68</td>
</tr>
<tr>
<td>Interconnected porosity fraction</td>
<td></td>
<td>0.30</td>
</tr>
<tr>
<td>Water saturation fraction</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Volumetric shale concentration fraction</td>
<td></td>
<td>0.02-0.10</td>
</tr>
<tr>
<td>Permeability md (m²)</td>
<td></td>
<td>100-2000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\sim 10^{-14} - 2 \times 10^{-12})</td>
</tr>
</tbody>
</table>

Figure 3. Flow chart describing the algorithmic steps included in the combined simulation and inversion of resistivity and SP measurements to estimate \( R_w \) and \( m \) and to calculate water saturation in hydrocarbon-bearing zones.

Figure 4. Well logs and petrophysical properties for the case study. Separating AIT resistivity curves indicates invasion of mud filtrate into the water-bearing sand. The porosity track shows shale-corrected density and neutron porosity \( \phi_d^w \) and \( \phi_n^w \), effective initial-guess porosity \( \phi_{eo} \), and nonshale porosity \( \phi \) obtained with a nonlinear model that relates density with resistivity.
are layer boundaries, borehole radius and conductivity, and raw AIT conductivity data. Inversion results are values of $R_{xo}$, $R_t$, and $r_{inv}$ for each layer, which are additional inputs to the SP inversion. Once the inversion is finished, we obtain SSP values for each layer boundary.

Table 2 describes the inversion results for resistivity and radius of invasion. Figure 5 shows the results of resistivity inversion and compares field and simulated SP measurements at the conclusion of the inversion. Figure 6 compares field to simulated apparent array-induction resistivity measurements calculated with the inverted values of $r_{inv}$, $R_{xo}$, and $R_t$.

Assessment of $m$ and $R_w$

Equivalent water resistivity is obtained from the inverted SSP with equation 6. Subsequently, connate-water resistivity is computed at reservoir temperature using equation 10. Archie’s (equation 11 for clean sands) and dual-water (equation 12 for shaly sands) models yield $m$ using the updated values of $R_w$ and $R_t$ obtained from inversion and porosity from logs. Table 3 describes the inverted values of $R_w$ and $m$.

When we compare the values of $m$ obtained with Archie’s equation to the one obtained with the dual-water equation, we see a reduction of the cementation exponent. The resulting value of $m$, decreasing with increasing shaliness in the dual-water model, depends on the ratio between bound-water resistivity and true formation resistivity $R_b/R_t$. Clavier et al. (1984, their Figure 8) cite no clear tendency of the variations of $m$ as a function of shale content; measurements are scattered between $m = 1.5$ and $m = 2.5$ for a volumetric shale concentration between 0 and 0.50. Therefore, values of $m$ with behavior opposite to that observed in this paper are feasible.

Appraisal of $R_w$

To cross-validate the inverted value of connate-water resistivity, we constructed a traditional Pickett plot (Pickett, 1966) for log $R_t$ versus log(1/log($R_w$)), which yields a straight line whose slope is equal to $-m$. The ordinate at log(1/log($R_w$)) = 0 yields log($R_w$) (assuming $a = 1$).

Figure 7 displays a Pickett plot for the formation under analysis. The value of $R_w$ obtained from the Pickett plot is 0.037 $\Omega$m at 61 °C. Such a result represents a difference of 12% with respect to the value

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>$R_{xo}$ ((\Omega)m)</th>
<th>$R_t$ ((\Omega)m)</th>
<th>$r_{inv}$ (m)</th>
<th>SSP (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoulder</td>
<td>3.7586</td>
<td>3.7586</td>
<td>0</td>
<td>56.4128</td>
</tr>
<tr>
<td>3.66</td>
<td>1.8938</td>
<td>0.5425</td>
<td>0.35</td>
<td>−6.5244</td>
</tr>
<tr>
<td>5.79</td>
<td>1.1665</td>
<td>0.4849</td>
<td>0.38</td>
<td>−1.7396</td>
</tr>
<tr>
<td>8.23</td>
<td>1.0916</td>
<td>0.3387</td>
<td>0.54</td>
<td>−49.3705</td>
</tr>
<tr>
<td>Shoulder</td>
<td>4.2012</td>
<td>4.2012</td>
<td>0</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 3. Calculated values of connate-water resistivity and Archie’s cementation exponent.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_w$</td>
<td>(\Omega)m</td>
<td>0.042</td>
</tr>
<tr>
<td>Temperature</td>
<td>°C</td>
<td>61</td>
</tr>
<tr>
<td>NaCl</td>
<td>ppm</td>
<td>105,000</td>
</tr>
<tr>
<td>$m$ — Archie</td>
<td></td>
<td>1.74</td>
</tr>
<tr>
<td>$m$ — dual water</td>
<td></td>
<td>1.69</td>
</tr>
<tr>
<td>Volumetric shale concentra</td>
<td>Fraction</td>
<td>0.10</td>
</tr>
</tbody>
</table>
obtained from inversion. The relatively small difference between the Pickett plot calculation and the inversion result lends credence to the reliability of our combined inversion method.

Applications of inversion results

Ideally, in a reservoir penetrating a water zone, the objective is to use the results of inversion ($R_w$ and $m$) to estimate water saturation in the upper hydrocarbon zone within the same geologic formation. The method, parameters, and simulation approach used to compute water saturation, porosity, and initial guess of permeability are described by Salazar et al. (2006). Either resistivity matching or inversion (Salazar et al., 2005; Salazar et al., 2006) using the physics of mud-filtrate invasion (Alpak et al., 2003; Wu et al., 2005) is an alternative method for estimating permeability.

Values obtained from inversion provide a starting point to estimate water saturation and layer-by-layer permeability accurately. Correct assessment of water saturation leads to reliable estimation of in-place hydrocarbon saturation. Moreover, accurate estimation of permeability is necessary to select perforation intervals and layers for fluid injection and to forecast production.

CONCLUSIONS

The combined inversion of resistivity and SP measurements is a reliable method to estimate $R_w$ and $m$ in water-bearing formations. Results obtained from the inversion were consistent with those obtained from Pickett plots. The difference between the estimates obtained with the two methods was roughly 10%.

The combined inversion method is highly recommended in zones where NaCl is the most abundant solution component and where connate salt concentration does not change in short depth intervals. This method of inversion works very well in high-permeability, thick formations, and when connate water exhibits medium to high connate-water resistivity.

In depth zones where the most abundant salt components are different from NaCl (e.g., calcium chloride or potassium chloride), equation 6 is not valid. In addition, when the salt concentration of mud filtrate and connate water is similar, the deflection of the SP curve is marginal and, therefore, the inversion method is not recommended. In low-permeability formations ($k<5$ md), the electrokinetic components of the SP may become important for the total contribution to SSP and hence could be needed in the formulation before applying our inversion method.

In the absence of water zones, resistivity inversion can still be used to estimate $R_w$ and $R_t$ for the refined calculation of initial water saturation. This calculation method can be useful in the petrophysical assessment of exploratory and appraisal wells that are normally devoid of core samples and laboratory measurements.

The fact that service companies do not charge to acquire SP measurements contributes to the fact that neither service companies nor clients give SP the attention it deserves. Commonly, SP measurements consist of extremely smooth, low-quality curves that make interpretation more challenging and hence may have an erroneous impact on applying the inversion method described in this article.

ACKNOWLEDGMENTS

We are obliged to ConocoPhillips for permission to publish the field measurements described in this article. A note of special gratitude goes to Tom Barber, Barbara Anderson, and two anonymous reviewers, whose constructive comments and suggestions improved the original manuscript. The work reported in this paper was funded by the University of Texas at Austin’s Research Consortium on Formation Evaluation, jointly sponsored by Anadarko, Aramco, Baker Atlas, BHP Billiton, BP, British Gas, ConocoPhillips, Chevron, ENI E&P, ExxonMobil, Halliburton Energy Services, Hydro, Marathon Oil Corporation, Mexican Institute for Petroleum, Occidental Petroleum Corporation, Petrobras, Schlumberger, Shell International E&P, Statoil, TOTAL, and Weatherford.

REFERENCES


