PORE-SCALE QUANTIFICATION OF APPARENT PERMEABILITY AND ELECTRICAL RESISTIVITY OF HYDROCARBON-BEARING SHALE IN THE PRESENCE OF GAS DESORPTION

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ABSTRACT

We introduce a new pore-scale method that includes the effects of no-slip and slip flow, Knudsen diffusion, and Langmuir desorption to model gas flow in porous media. A weighting factor combines the effects of Knudsen diffusion, no-slip, and slip flows in the pore space. Boundary conditions model Langmuir desorption at the surface of grains and organic matter while finite pressures are enforced at the inlet and outlet. Spatial distributions of pressure and fluid velocity in the interstitial domain are calculated using a generalized Laplace equation. Subsequently, apparent permeability is estimated from fluid velocity distributions. Our model is applicable to both conventional and unconventional reservoirs such as carbonates, shales, gas, tight gas and coal-bed methane. Furthermore, the pore-scale simulation method calculates electrical conductivity of gas-bearing shale in the presence of organic matter, clay bound water, and conductive grains.

Previously, we suggested that slip flow and Knudsen diffusion become increasingly dominant flow mechanisms in the case of nanometer pore sizes. In addition, Langmuir desorption from organic matter surfaces, and to a less extent, from grain surfaces, becomes important in the calculation of stored gas in shale-gas formations. This paper focuses on shale-gas strata where all of the aforementioned mechanisms are present. Modeling results indicate that contributions from slip-flow and Knudsen diffusion become dominant in these formations, whereas Langmuir desorption has no appreciable impact on apparent permeability when rock morphology is not altered because of gas desorption. Nevertheless, desorption is crucial to continuously supply gas while maintaining reservoir pressure over long periods of time.

Connate water as well as surface conductivity due to clay minerals are important components of the pore-scale electrical-conductivity model of gas-bearing shale. Organic matter is gas adsorbent, whereby its presence alters the morphology of water at the pore-scale; consequently, it blocks electrically conductive paths and decreases the conductivity of the formation. Our pore-scale model allows a critical study of the \( \Delta \log R \) evaluation method in gas-bearing shale from a physical point of view. Additionally, we examine two Archie's saturation exponents to model rocks which contain electrically conductive grains.

INTRODUCTION

In the past few years, there has been renewed interest in source rocks as reservoirs (Gault and Stotts 2007; Passey et al. 2010). This trend is driven by the higher prices of petroleum and the depletion of conventional reservoirs. However, there is neither common consensus regarding how to calculate petrophysical parameters, nor a "standard" model to quantify gas production in these formations (Passey et al. 2010; Freeman et al. 2010; Sondergeld, et al. 2010b; Ambrose et al. 2010; Shabro et al. 2011). It is evident that the oil industry requires more technically focused attention to hydrocarbon-bearing shale as increasingly complex technologies are needed to produce them in an optimal manner.

Recent pore-scale characterizations of gas-bearing shale have advanced the physical understanding of shale morphologies and their transport mechanisms. Scanning Electron Microscopy (SEM), Focused Ion Beam-Scanning Electron Microscopy (FIB-SEM), and Atomic Force Microscopy (AFM) methods are examples of pore-scale characterization methods (Loucks et al. 2009; Javadpour 2009; Schieber 2010; Ambrose et al. 2010; Sondergeld et al. 2010b). Nevertheless, the expensive and time consuming nature of these techniques combined with their extremely small field of view significantly limits their practical applicability in determining petrophysical parameters. A comprehensive pore-scale study based on current physical understanding of the pore network and transport mechanisms in shale strata is crucial to identify and evaluate possible methods to characterize these formations and understand their gas production.

Permeability of gas-bearing shales has been the main focus of many research activities (Javadpour et al.
It was shown that high organic matter content is an essential petrophysical characteristic of gas-producing shale (Passey et al. 2010). Several groups have attempted to correlate total organic carbon (TOC) with conventional logs (Fertl and Chilingar 1988; Passey et al. 1990). Passey et al. (1990) introduced the DLogR method to estimate TOC from resistivity and sonic-porosity logs. Clay content is also important in the modeling of electrical resistivity of shale formations (Passey et al. 2010). Furthermore, the validity of Archie’s equation has been challenged for several formations (Montaron 2008; Montaron and Han 2009; Haro 2010), particularly for gas-bearing shale. A comprehensive pore-scale model is needed to accurately and reliably quantify the electrical conductivity of rocks in the presence of organic matter, clay minerals, and conductive grains.

In this paper, the combined effects of Knudsen diffusion, slip flow and Langmuir desorption are taken into account in a finite-difference pore-scale model to quantify fluid flow in gas-bearing shale. In addition, we model electrical resistivity to account for the effects of organic matter, adsorbed gas, clay minerals, and conductive grains in hydrocarbon-bearing shale.

**SHALE-GAS MORPHOLOGIES**

It is accepted that gas-bearing shales exhibit pore-throat structures in the nanometer range. Pore-scale SEM images of shale formations have indicated the existence of pores, clay minerals, and organic matter (Loucks et al. 2009; Schieber 2010; Passey et al. 2010). Existence of connected pore-throat structures to transport gas was confirmed by FIB-SEM images (Sonderegger et al. 2010b). Pore-scale images acquired with AFM also confirmed the existence of nano-pores in these formations (Javadpour 2009). Figure 1 shows pores in the range of tens to hundreds of nanometers in four cross-sectional images of gas-bearing shale. AFM images confirm that the observed pore structures in SEM images are not artifacts from high-energy electron beams impacting the surface of shale formations. Figure 2 shows a schematic of gas-bearing shale. We make the following assumptions in our pore-scale model: (1) grains are covered by clay minerals; (2) clay minerals are saturated with immobile water, referred to as clay bound water (CBW); (3) organic matter occurs between grains; (4) grains and clays are water wet, and organic matter is gas adsorbent; (5) free water and free gas exist in the pore structure; (6) organic matter may connect adsorbed gas with free gas, hence partially block the free passage of water.

![Figure 1: AFM images of gas-bearing shale. Dark areas represent pores and throats in these formations and their sizes are commonly in the nanometer range.](image1)

![Figure 2: Two dimensional schematic of pore and throat structure of gas-bearing shale.](image2)
We use three-dimensional (3-D) grid-blocks in Cartesian coordinates to implement the above pore-scale assumptions to quantify petrophysical properties of gas-bearing shale. Each grid may represent either grain, clay bound water, free water, free gas, organic matter, or adsorbed gas. First, a grain pack is constructed and compacted (Silir et al. 2003) to a grain radius distribution between 0.25 µm and 1.5 µm as described in Table 1. Subsequently, the grain pack is digitized using a 10 nm grid size to yield a 3-D grid-block. Grid size is changed to 3 nm, 30 nm, and 100 nm to obtain 3-D grid-blocks with various porethroat sizes. The effect of size on the contribution of each transport mechanism is examined by modeling these 3-D grid-blocks. Next, the 3-D grid-blocks are uniformly cemented (Silir et al. 2003) to coat the grains with clay minerals and achieve lower pack porosities (12% and 8.5%).

To model organic matter and adsorbed gas at the pore-scale, organic matter is added to the surface of clay minerals and grains at random locations. Next, organic matter is expanded toward clay minerals and grains until we achieve a predefined value of TOC. Then, free-water or free-gas grids neighboring organic matter are changed to adsorbed gas. Figure 3 describes the resulting pore-scale image of shale gas with 12% porosity, 4% organic matter content, and 1% adsorbed gas. Some of the clay minerals are in contact with gas in these 3-D grid-blocks which stand in contrast to our initial assumption that the formation is water-wet. Such a condition usually occurs when rendering two-phase pore-scale models from digital images (Shabro et al. 2010). A grid in these images can only assume one of the phases. To achieve high non-wetting phase saturations, it is impossible to place all non-wetting phase grids away from grains. Higher-resolution images and grid refinement can mitigate this problem. However, image size is limited by the computer resources available to quantify the ensuing digital porous medium. Previously, we discussed the trade-off between image resolution, grid-refinement, and computer resources (Shabro et al. 2010). Throughout this paper, we substitute each grid in 3-D grid blocks with 8 grids of the same constituent to increase the corresponding accuracy of modeling results.

Porosity ($\phi$) is the ratio of free pore volume to total volume; free water saturation ($S_{wf}$), free gas saturation ($S_{gf}$), and adsorbed gas volumetric ratio ($S_{ag}$) are the ratio of free water, free gas, and adsorbed gas to free pore volume, respectively. TOC and CBW are calculated from the volumetric ratio of organic matter and clay mineral to total volume, respectively. Total water saturation ($S_{tw}$) is calculated as the volumetric ratio of free water and clay bound water to total fluid volume in porous media.

<table>
<thead>
<tr>
<th>Grain radius [µm]</th>
<th>Volumetric concentration [%]</th>
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<tr>
<td>0.25</td>
<td>3</td>
</tr>
<tr>
<td>0.5</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>46</td>
</tr>
<tr>
<td>1.5</td>
<td>40</td>
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Figure 3: Pore-scale schematic of grains, clay-bound water, free water, free gas, organic matter and adsorbed gas included in a gas-bearing shale. $S_{gf}$=25%, $S_{gf}$=67%, $S_{ag}$=8%, $\phi$=12%, CBW=22%, $S_{tw}$=75%, TOC=4%.
METHOD

We previously developed a systematic pore-scale approach to model gas flow in mudrocks (shales and siltstone) with pore-throat sizes in the nanometer range (Shabro et al. 2009). This model included both Knudsen diffusion and advection transport at various Knudsen regimes while neglecting inertia and gas compressibility effects. Geometrical and physical parameters were combined with a space-dependent weighting factor (d) and interstitial pressure was calculated with a generalized Laplace equation. The method enforced Neumann’s boundary condition at the grain-pore surface along with the adoption of finite pressures as boundary conditions at both inlet and outlet. Appendix A provides a brief mathematical description of this modeling strategy. The following paragraphs describe the addition of gas desorption to our existing pore-scale flow modeling method.

In the presence of gas desorption from surfaces of organic matter and clay minerals, mass balance requires that gas generation influences volumetric gas fluxes. Mass balance is expressed as

\[ \nabla \cdot J = \frac{\text{Surface}}{\text{Volume}} (J_{\text{des}} - J_{\text{ads}}), \]

where \( J \) is gas volume flux in the interstitial space, \( J_{\text{des}} \) is desorption volume flux and \( J_{\text{ads}} \) is adsorption volume flux (Ruthven 1984). Desorption volume flux is given by

\[ J_{\text{des}} = K_{\text{des}} \theta, \]

where \( K_{\text{des}} \) is the desorption coefficient and \( \theta \) is fractional coverage. Fractional coverage is the fraction of total surface sites for gas molecules that are occupied by adsorbed gas molecules. Desorption flux depends on both material properties and fractional coverage. Adsorption mass flux is defined by

\[ J_{\text{ads}} = K_{\text{ads}} (1 - \theta) P, \]

where \( K_{\text{ads}} \) is adsorption coefficient, and \( (1 - \theta) \) is fractional vacancy (the fraction of unfilled surface sites for gas adsorption). Adsorption flux depends on material properties, fractional vacancy, and pressure. Both adsorption and desorption mechanisms are continually present at the pore-scale. When the contribution of desorption exceeds the contribution from adsorption, the medium is effectively desorbing, whereas in the reverse condition, the medium is effectively adsorbing. At equilibrium, these two mechanisms are equal to each other, and the fractional coverage is given by

\[ \theta = \frac{P}{K_{\text{des}} + P}. \]

Equilibrium fractional coverage depends on pressure and \( K_{\text{ads}}/K_{\text{des}} \). Adsorption flux decreases when pressure decreases in the porous medium. Desorption flux surpasses adsorption flux until a lower-equilibrium fractional coverage becomes effective.

We add desorption and adsorption fluxes from Eqs. 1, 2, and 3 to the generalized Laplace equation for gas flow (Eq. A7 developed in Appendix A) to obtain

\[ \nabla \cdot (\theta \nabla P) = \frac{k_{\text{eq}} \theta - k_{\text{eq}} (1 - \theta) P}{\text{Gridsize}}. \]

The volume to surface ratio in Eq. 1 transforms to the inverse of grid size for each grid. Fractional coverage for all grids is set equal to equilibrium fractional coverage at inlet pressure. Equation 5 is solved via the conjugate gradient method (Shewchuk 1994) to yield the spatial distribution of pressure. Then, apparent permeability is calculated from fluid velocities in the interstitial space. The difference between calculated apparent permeability at the inlet and outlet (\( \Delta k \)) represents the effects of desorption on apparent permeability.

Similarly, we solve the electrical conductivity equation at the pore-scale via

\[ \nabla \cdot (\sigma \nabla V) = 0, \]

where \( \sigma \) is electrical conductivity of the medium, and \( V \) is voltage. Pertinent electrical conductivities are assigned to each grid according to their constituent. Finite voltages are enforced at the inlet and outlet. Equation 6 is solved via the conjugate gradient method (Shewchuk 1994), and electrical conductivity is calculated from the spatial distribution of electrical current.

RESULTS AND DISCUSSION

First, we investigate the effect of slip flow and Knudsen diffusion on apparent permeability. The 3-D grid-block with porosity of 8.5% is modeled with the corresponding parameters listed in Table 2. Grid size determines the smallest throat size and it is varied between 3 nm to 100 nm to model various porous media. Figure 4 shows that apparent permeability increases due to slip flow and Knudsen diffusion. Conventional permeability for grid sizes equal to 3 nm, 10 nm, 30 nm, and 100 nm are 0.017 μm², 0.19 μm², 1.7 μm², and 19 μm², respectively. Contributions due to slip flow and Knudsen diffusion on fluid flow are negligible when the smallest throat is 100 nm; however, apparent permeability is significantly higher if the smallest throat size is 3 nm or 10 nm. Additionally, apparent permeability increases when pressure decreases. Figure 4 shows calculated variations of apparent permeability changes when pressure is varied from 175 to 2800 psi. We have shown that this effect is partially responsible for the higher-than-expected gas...
production rates observed in gas-bearing shale (Javadpour 2009; Shabro et al. 2011).

<table>
<thead>
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<th>Table 2: Summary of modeling parameters assumed in the calculations reported in this paper.</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>μ [Pa.s]</td>
</tr>
<tr>
<td>ρ_{init} [kg.m^{-3}]</td>
</tr>
<tr>
<td>K_{ads} [m.Pa^{-1}.s^{-1}]</td>
</tr>
<tr>
<td>K_{des} [m.s^{-1}]</td>
</tr>
<tr>
<td>α</td>
</tr>
<tr>
<td>M [kg.kmol^{-1}]</td>
</tr>
<tr>
<td>T [K]</td>
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We find that apparent permeability calculated from velocity fields will vary when gas desorption is taken into account. However, the effect is negligible when porous media are in equilibrium at initial conditions. Figure 5 shows the relative change in the calculated apparent permeability from velocity fields at the inlet and outlet of porous media. Table 3 describes modeling parameters for Cases Nos. 1 to 5. The effect of gas desorption on calculated apparent permeability is higher in porous media with smaller grid sizes. Changes in apparent permeability are below 0.2% in Case No. 1. If the gas desorption coefficient is increased 1000 times, (Case No. 2), changes in apparent permeability are below 1%. On the other hand, when we increase desorption coefficient 100 times and decrease the pressure gradient 100 times, (Case No. 3), changes in apparent permeability are still below 3%. When porous media are in adsorption-desorption equilibrium during gas production, changes in apparent permeability due to desorption become negligible. Cases Nos. 4 and 5 assume that equilibrium fractional coverage is calculated at pressures 0.0001% and 0.001% higher than inlet pressure. The non-equilibrium condition results in higher effective gas desorption, thereby affecting the calculated apparent permeability by as much as 1000%. Previously, we showed that gas production in gas-bearing shale follows adsorption-desorption equilibrium (Shabro et al. 2011). Therefore, we conclude that gas desorption does not significantly affect the apparent permeability of gas-bearing shale.

![Figure 4: Permeability variation vs. pressure calculated for a 2-D grid block with 8.6% porosity and with the smallest throat diameters equal to 3 nm, 10 nm, 30 nm, and 100 nm. Reported values are apparent permeability divided by conventional permeability to identify the effects of slip flow and Knudsen diffusion.](image)

![Figure 5: Permeability variations due to desorption vs. grid size calculated from pore-scale modeling of a 3-D grid block with 12% porosity. Parameters for Cases Nos. 1 through 5 are described in Table 3.](image)

<table>
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<tr>
<th>Table 3: Summary of modeling parameters assumed in the calculation of apparent permeability variations due to gas desorption.</th>
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<tr>
<td>Case</td>
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<tr>
<td>P_{init} [Pa \times 10^5]</td>
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<tr>
<td>P_{in} [Pa \times 10^6]</td>
</tr>
<tr>
<td>ΔP_{ads} [kPa.m^{-1}]</td>
</tr>
<tr>
<td>K_{ads} [m.Pa^{-1}.s^{-1} \times 10^{-19}]</td>
</tr>
<tr>
<td>K_{des} [m.s^{-1} \times 10^{-13}]</td>
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</table>

Formation conductivity is calculated via pore-scale modeling of electrical conductivity for the 3-D grid-block with 1.2% porosity, 4% TLC, water resistivity of 0.035 ohm-m, and clay bound water resistivity of 0.012 ohm-m. These resistivity values are equivalent to salinities of 50,000 ppm and 200,000 ppm, respectively. Figure 6 describes the calculated values of formation electrical conductivity vs. water saturation. Total water saturation varies between 64% and 93% due to
abundance of CBM. The high electrical conductivity of CBM results in a relatively low Archie’s saturation exponent, between 1.0 and 1.5.

\[ \sigma_n \] vs. \( S_w \)

**Figure 6:** Formation conductivity vs. water saturation calculated for the grain-pack shown in Figure 3.

Next, we vary TOC from 1.5% to 10.8% to quantify its effect on electrical conductivity. As predicted by the so-called $A\log R$ method, resistivity increases almost linearly with respect to TOC when there is more organic matter in the formation. **Figure 7** describes the calculated values of electrical resistivity of the 3-D grid-block with 12% porosity at irreducible water saturation and residual gas saturation vs. TOC.

\[ \rho_n \] vs. TOC

**Figure 7:** Calculated relative formation resistivity to water at irreducible water saturation ($S_w=64\%$) and residual gas saturation ($S_w=83\%$) vs. TOC. Electrical resistivity increases with TOC in the formation.

Finally, we model the 3-D grid-block of **Figure 3** with connate water resistivity equal to 0.012 ohm.m and grain resistivity equal to 1.2 ohm.m, and assume that clay minerals are also grains with resistivity equal to 1.2 ohm.m. **Figure 8** shows that two saturation regimes arise with different values of Archie’s saturation exponents in these examples. At low values of water saturations, matrix electrical conductivity becomes important and Archie’s saturation exponent approaches an abnormally low value (0.4). At higher values of water saturation, connate water conductivity becomes dominant and Archie’s saturation exponent falls within the common range (here equal to 1.7).

\[ \sigma_n \] vs. \( S_w \)

**Figure 8:** Formation conductivity vs. water saturation calculated when the grain matrix is electrically conductive.

**CONCLUSIONS**

We developed a pore-scale gas flow model that includes Knudsen diffusion and advection (slip and no-slip flows) as well as Langmuir desorption-adsorption. The method solves a generalized Laplace equation and enforces either Neumann’s boundary or desorption-adsorption flux boundary condition at the grain-pore surface along with the adoption of finite pressures at both inlet and outlet as boundary conditions. When modeling apparent permeability in gas-bearing shale, neglecting slip flow and Knudsen diffusion may result in quantification errors as large as 2000%. Langmuir desorption does not change apparent permeability significantly if adsorption-desorption equilibrium is maintained during gas production and the rock morphology is not altered because of gas desorption.

Electrical resistivity in gas-bearing shale was modeled in the presence of organic matter and adsorbed gas. Organic matter, as a gas adsorbent constituent, alters the morphology of water at the pore-scale and blocks conductive paths, thereby increasing the rock’s electrical resistivity. Our modeling results are consistent with the so-called $A\log R$ method. Additionally, we observed two Archie’s saturation exponents at low and high values of water saturations when the matrix of porous media is conductive.
ACKNOWLEDGEMENTS

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LIST OF ACRONYMS

3-D Three-dimensional
AFM Atomic Force Microscopy
FIB SEM Focused Ion Beam Scanning Electron Microscopy
SEM Scanning Electron Microscopy

NOMENCLATURE

c(d_{min}) Calibration function, [ ]
CBW Clay bound water, [ ]
d Weighting factor, [m²·Pa⁻¹·s⁻¹]
d_{min} Digital equivalent of r_{min}, [ ]
d_{grid} Distance to the closest wall, [ ]
f Slip flow coefficient, [ ]
J Volume flux, [m²·s⁻¹]
J_{ads}, J_{des} Adsorption and desorption volume
J_{ads}, J_{des} flux, [m²·s⁻¹]
k Permeability, [m²], [d = Darcy = 9.869 × 10⁻¹³ m²]
A_k Apparent permeability difference at
A_k the inlet and outlet
K_{ads} Adsorption coefficient, [m·Pa⁻¹·s⁻¹]
K_{des} Desorption coefficient, [m·Pa⁻¹·s⁻¹]
L Porous media length, [m]
M Molar mass, [kg·kmol⁻¹]
P, P₁, P₂ Pressure, [Pa]
P_{ave} Average pressure, [Pa]
P_{in} Inlet pressure, [Pa]
P_{init} Initial pressure, [Pa]
ΔP_{in-out} Inlet pressure minus outlet pressure,
ΔP_{in-out} [Pa]
R Resistivity [ohm-m]
R_g Universal gas constant, 8.314 × 10³
R_g Pa·m³·kmol⁻¹·K⁻¹
R_w Water resistivity [ohm-m]
r Tube radius, [m]

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APPENDIX A: PORE-SCALE MODELING OF SLIP FLOW AND KNUDSEN DIFFUSION

The overall equation for gas flow in a cylindrical tube is given by (Javadpour 2009)

\[ J = -\frac{2r}{3r_T \rho_{avg}} \left( \frac{8 r_T}{\pi M} \right)^{1/2} \left( \frac{P_1 - P_2}{\mu L} \right), \]  

(A-1)

where \( J \) is volume flux, \( r \) is the tube radius, \( R \) is the universal gas constant, \( T \) is temperature, \( M \) is gas molar mass, \( \rho_{avg} \) is gas average density at \( P_{avg} = (P_1 + P_2)/2 \), \( \mu \) is gas viscosity at atmospheric pressure, \( P_1 \) and \( P_2 \) are pressures at the inlet and outlet of the tube, respectively, and \( L \) is the length of the tube. The first and second terms in the bracket represent diffusive and
no-slip/slip gas flow due to pressure forces, respectively. Coefficient $F$ defines the contribution of slip flow and is given by

$$F = 1 + \left( \frac{8\pi RT}{M} \right)^{0.5} \left( \frac{\mu}{p_{avg}} \right)^{(2/\alpha - 1)}$$  \hspace{1cm} (A-2)

where, $p_{avg}$ is the average pressure of the system and $\alpha$ is the tangential momentum accommodation coefficient. Coefficient $F$ approaches 1 for conventional formations, which suggests a no-slip boundary condition on the surface of grains. Accordingly, Eq. A-1 simplifies to Hagen-Poiseuille’s equation for conventional rock formations.

We substitute Eq. A-2 into Eq. A-1 and rearrange terms to obtain

$$J = -\left( Ar + Br^2 \right) \nabla P,$$  \hspace{1cm} (A-3)

where

$$A = \left( \frac{8RT}{\pi M} \right)^{0.5} \left( \frac{2}{3p_{avg}} \right), \quad \frac{2}{3RTP_{avg}} \right)$$

and

$$B = 1/8\mu.$$  \hspace{1cm} (A-4)

At steady-state conditions, assuming no sink or source, mass conservation in the system enforces

$$\nabla \cdot J = 0.$$  \hspace{1cm} (A-6)

By combining Eqs. A-3 and A-6, we obtain a generalized Laplace equation of the form

$$\nabla \cdot d \nabla P = 0,$$  \hspace{1cm} (A-7)

where $d$ is a weighting factor. The weighting factor is calculated for all grids along each of the three Cartesian directions as

$$d = \left( \frac{Ar_{max} + Br_{max}^2}{2Ar_{max} + Br_{max}^2} \right) \times \frac{2d_{max}d_{grid}^2 - d_{grid}^2}{c(r_{max})},$$

where $r_{max}$ is the radius of the largest circle that can be inscribed in a specific pore in the spatial domain perpendicular to the direction of flow which incluses the grid of interest. Likewise, in Eq. A-8 $d_{max}$ is the digitized version of $r_{max}$, $d_{grid}$ is the distance of the grid from the closest grain wall, and $c(r_{max})$ is a calibration function included to match the flow in the digitized system with flow in an equivalent tube.

Gas density is assumed constant in our model for small pressure gradients. The effect of inertia is neglected because creeping flow is expected to dominate in porous media, especially in unconventional reservoirs where pore-throat sizes are in the nanometer range.