Simulation of triaxial induction measurements in dipping, invaded, and anisotropic formations using a Fourier series expansion in a nonorthogonal system of coordinates and a self-adaptive hp finite-element method

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ABSTRACT

Borehole triaxial induction instruments were designed to diagnose and measure rock electrical conductivity parallel and perpendicular to the bedding plane. Experience has shown that the interpretation of triaxial induction measurements often requires numerical modeling for a proper diagnosis of rock electrical conductivity anisotropy in the presence of geometric effects such as dipping wells, layer boundaries, and invasion. We introduce a new algorithm to simulate triaxial induction measurements that combines a Fourier series expansion in a nonorthogonal system of coordinates with a 2D goal-oriented, self-adaptive, high-order hp finite-element method. This procedure enables the accurate and reliable simulation of triaxial induction measurements across reservoir rock formations with extreme contrasts of electrical conductivity while reducing the 3D computational complexity associated with deviated wells. Numerical results indicate that borehole dip effects on triaxial induction measurements are larger than on standard coaxial induction measurements.

INTRODUCTION

The oil and gas exploration industry has traditionally resorted to induction tools to measure the electrical conductivity of reservoir rocks. Electrical conductivity of hydrocarbon-bearing rocks is related to the interconnected pore volume occupied by connate water, hence, to hydrocarbon reserves. Across thinly bedded sand-shale rock sequences (wherein the thickness of the layers is shorter than the vertical resolution of borehole-induction measurements), hydrocarbon pay zones exhibit lower values of resistivity parallel to the bedding plane (horizontal resistivity) than perpendicular to it (vertical resistivity). In such transversely isotropic (TI) rock formations, vertical conductivity is governed by the resistive hydrocarbon-bearing sand formations whereas horizontal conductivity is usually dominated by low-resistivity shale layers (Luling et al., 1994). Thus, measurements of vertical conductivity are a better hydrocarbon indicator than those of horizontal conductivity (Anderson et al., 2002).

In addition to measurements of vertical and horizontal conductivity, quantifying the hydrocarbon saturation of sand layers included in thinly bedded sand-shale sequences requires knowledge of intrinsic vertical and horizontal shale conductivities and the volumetric proportion of shale layers (Hagiwara, 1997). Therefore, the ability to measure horizontal and vertical resistivities is critical for accurate petrophysical interpretation of hydrocarbon pay zones in thinly bedded siliciclastic sequences.

Standard induction logging devices include several transmitter and receiver coils (each of which is coaxial with the borehole); therefore, they are only sensitive to the horizontal resistivity of rock formations penetrated by vertical wells (Barber et al., 2004; Abubakar et al., 2006). Consequently, conventional borehole induction devices usually measure abnormally low values of electrical resistivity in thinly laminated sand-shale sequences, which leads to pessimistic estimates of hydrocarbon pore volume.

Multicomponent triaxial induction tools were designed to diagnose and measure horizontal and vertical conductivity of rock formations (Krieghauser et al., 2000; Zhdanov et al., 2001; Rosthal et al., 2003). These tools include three mutually orthogonal transmitter coils located at the same position on the tool axis and three colocated...
mutually orthogonal receiver coils. Such a coil configuration enables the measurements of nine components of magnetic fields via nine coupling combinations between each transmitter and each of the available receivers.

Deviated wells are commonly used nowadays to access and produce hydrocarbon reservoirs, including the case of thinly bedded formations; they penetrate longer distances within hydrocarbon-bearing layers. Consequently, accurate evaluation of hydrocarbon pay zones in deviated wells has become increasingly important. Geometric and physical constraints such as well deviation angle, thin layers, the presence of invaded layers, and the presence of electrically anisotropic layers render the interpretation of triaxial induction measurements difficult without numerical simulation. The simulation of triaxial induction measurements acquired in deviated wells is still challenging due to the associated 3D nature of the induced electromagnetic (EM) fields (see, for instance, Druskin et al., 1999; Newman and Alumbaugh, 2002; Davvydycheva et al., 2003; Hou et al., 2006; Mallan and Torres-Verdín, 2007; Zhong et al., 2008). In addition, the complexity of arbitrary 3D geometries is taxing on computational requirements whereby general-purpose 3D simulation algorithms often fail to calculate accurate solutions in a limited amount of CPU time.

Recently, a new method was introduced to simulate EM measurements acquired in deviated wells that penetrate 2D (multilayer) formation models. The method uses a particular nonorthogonal system of coordinates \( \xi \) \((\xi_x, \xi_y, \xi_z)\) wherein material properties become invariant with respect to the quasi-azimuthal direction \( \xi_z \) (Pardo et al., 2008). Any function in the new coordinate system is periodic and thus can be expressed in terms of a Fourier series expansion with respect to \( \xi_z \). Invoking a Fourier series expansion in the nonorthogonal system of coordinates reduces the 3D formulation to a sequence of 2D problems wherein each 2D problem is coupled only with a maximum of five other 2D problems. Thus, the corresponding stiffness matrix associated with the numerical solution becomes pentadiagonal. Such a degree of matrix sparsity becomes a major advantage over general 3D formulations (which consist of a sequence of fully coupled 2D problems).

The proposed method is efficient and accurate for the simulation of triaxial induction measurements acquired in a deviated well. In the simulation, we consider the presence of the wellbore, the tool mandrel, TI formations, and large contrasts of electrical conductivity. We also consider the presence of invasion and other 2D effects in the formation. However, the method also has some limitations. First, it is not suitable for the simulation of borehole-centric tool measurements in deviated wells (although the method is appropriate for simulating measurements with tool eccentricity in vertical wells using a particular system of coordinates; e.g., Nam et al. [2010]). Second, the method breaks down as the dip angle approaches 90° (horizontal well) because the change of coordinates becomes singular.

To simulate 3D current source \( M_x, M_y, M_z \), we numerically approximate them using Dirac delta functions in the azimuthal direction. For all types of EM sources, we consider a finite cross-section in the \( \rho-z \) plane. When considering vertical wells, our new nonorthogonal system of coordinates is identical to the cylindrical system of coordinates, whereby the 3D formulation reduces to a so-called 2.5D problem.

To solve the resulting 3D variational formulation of EM fields, we use a goal-oriented, self-adaptive, high-order \( hp \) finite-element (FE) method (Pardo et al., 2008, where \( h \) denotes element size and \( p \) the polynomial order of approximation within each element). This numerical algorithm automatically constructs an optimal grid for a problem under consideration through local-mesh refinements. By making proper combinations of \( h \) and \( p \) refinements across the grid, the algorithm performs highly accurate simulations even in the presence of high contrasts of material properties in limited CPU time. In addition, the self-adaptivity of the method enables an explicit assessment of solution accuracy.

The remainder of this paper describes the triaxial induction tool assumed in the study and outlines the simulation method of triaxial induction measurements acquired in deviated wells with 3D source-receiver configurations. We subsequently validate the simulation algorithm using semi-analytical solutions. Finally, we describe numerical simulations of triaxial induction measurements for cases of deviated wells penetrating a multilayer formation model that exhibits large contrasts of electrical resistivity, transverse isotropy in electrical conductivity, and invasion.

TRIAXIAL INDUCTION TOOL

A triaxial induction tool measures nine orthogonal magnetic field components, i.e., three orthogonal magnetic fields \((H_x, H_y, H_z)\) separately excited by each of three orthogonal impressed magnetic currents \((M_x, M_y, M_z)\), leading to the so-called \(H_{xx}, H_{xy}, H_{xz}, H_{yy}, H_{yz}, H_{zz}, H_{xy}, H_{yz}, \) and \(H_{zz}\) components (where \(H_j\) denotes \(H\) excited by \(M_j\)). For this purpose, such tools include three colocated orthogonal transmitters and three colocated orthogonal receivers (Figure 1a). In this paper, we assume a tool wherein the receiver coils and the transmitter coils are 1.016 m (40 in) apart.

Description of the triaxial tool

In the simulation of triaxial induction measurements, we assume a model of a realistic triaxial induction tool with a resistive mandrel whose dimensions are 9 cm in diameter and 3.016 m in length (Figure 1). The resistivity of the mandrel is 10^3 \(\Omega\) m. Source and receiver antennas are assumed to have finite-sized cross sections whose dimensions are 0.01 × 0.01 m on the \(\rho-z\) plane in cylindrical coordinates \((\rho, \varphi, z)\). The center of the cross section in the \(\rho-z\) plane of all antennas is located at 0.05 m from the center of the mandrel. Transmitters and receivers are located 1.016 m apart in the \(z\)-direction. The midpoint between both antennas in the \(z\)-direction is located at the vertical center of the mandrel.

SIMULATION METHOD

We simulate triaxial induction measurements acquired in a deviated well that penetrates a 1D subsurface model. In a cylindrical system of coordinates \((\rho, \varphi, z)\), whose vertical axis \(z\) coincides with the center of the borehole (and with the center of the tool in the absence of tool eccentricity), the resulting geometry of the deviated well is fully 3D. By using a nonorthogonal coordinate system \(\xi = (\xi_x, \xi_y, \xi_z)\) (Figure 2) described in Pardo et al. (2008), material properties become invariant with respect to the quasi-azimuthal direction \(\xi_z\). In the new coordinate system, any function becomes peri-
odic and thus can be expressed in terms of its Fourier series expansion with respect to $\xi_2$. Even though the geometry of deviated wells is reduced in complexity when using the new system of coordinates we still need to numerically represent 3D sources and receivers. In this section, we first describe the nonorthogonal system of coordinates and our numerical implementation of 3D sources and receivers. Subsequently, we derive the Fourier FE variational formulation for the simulation of triaxial measurements in deviated and vertical wells.

**Nonorthogonal system of coordinates**

The 3D geometry of a deviated well (with a dip angle $\theta$) penetrating a 1D earth (Figure 2a) is reduced to a quasi-2D one, with material properties that are invariant with respect to the quasi-azimuthal direction $\xi_2$ when invoking a nonorthogonal coordinate system $\xi = (\xi_1, \xi_2, \xi_3)$ (Figure 2b; Pardo et al., 2008). The new system is defined in terms of a Cartesian system of coordinates $x = (x_1, x_2, x_3)$ (with $x_3$ positive downward along the axis of the borehole direction as shown in Figure 2b) as

$$\begin{align*}
x_1 &= \xi_1 \cos \xi_2 \\
x_2 &= \xi_1 \sin \xi_2 \\
x_3 &= \xi_3 + \theta_0 f_1(\xi_1) \cos \xi_2,
\end{align*}$$

where $\theta_0 = \tan \theta$ and $f_1$ is defined for given $\rho_1$ and $\rho_2$ as

$$f_1(\xi_1) = \begin{cases} 0 & \xi_1 < \rho_1 \\ \rho_2(\xi_1 - \rho_1)/\rho_2 - \rho_1 & \rho_1 \leq \xi_1 \leq \rho_2 \\ \xi_1 & \xi_1 > \rho_2 \end{cases},$$

where $\rho_1$ is the interface between subdomains 1 and 2, and $\rho_2$ is the interface between subdomains 2 and 3, as shown in Figure 2b. The resulting system of coordinates is globally continuous, bijective, and exhibits a positive Jacobian (Pardo et al., 2008). The nonorthogonal system of coordinates is globally continuous, bijective, and

![Figure 2](image)

Figure 2. (a) Cross-sectional view of a deviated well penetrating a layered formation with a dip angle $\theta$. The $x_3$-direction (in a Cartesian system of coordinates) and the $\xi_3$-direction (in a nonorthogonal system of coordinates) correspond to the axis of the borehole (with positive values pointing downward). Circles in (a) indicate the “quasi-azimuthal” direction $\xi_2$ in the nonorthogonal system of coordinates, which comprises three domains with different systems of coordinates. (b) Describes the cross section corresponding to $\xi_2 = 0$: Subdomain 1 is part of the borehole, which includes the logging instrument; subdomain 3 corresponds to the formation. Subdomain 2 is the remainder part of the borehole that is not contained in subdomain 1 and “glues” subdomain 1 with subdomain 3 so that the resulting nonorthogonal system of coordinates is globally continuous, bijective, and with a positive definite Jacobian.
coordinate system breaks down as dip angle $\theta$ approaches 90° (horizontal well) because $\theta_0 \approx \theta$ tends to infinity when $\theta$ approaches 90°. We note that the nonorthogonal system of coordinates reduces to a cylindrical system of coordinates in subdomain 1. Because $\xi_2$ is defined in a bounded domain, namely $[0, 2\pi]$, any function $G$ in the new coordinate system is periodic with a period of $2\pi$. Therefore, one can express $G$ in terms of its Fourier series expansion with respect to $\xi_2$, $G = \sum_{n=-\infty}^{\infty} f_n(G) e^{in\xi_2}$, where $e^{in\xi_2}$ are the modes and $f_n(G) = \frac{1}{2\pi} \int_{0}^{2\pi} G e^{-in\xi_2} d\xi_2$ are the modal coefficients, which are invariant with respect to $\xi_2$.

Modeling sources and receivers

We define subdomain 1 in such a way that it contains the domain occupied by the triaxial tool. Thus, sources and receivers are defined in the cylindrical system of coordinates whose vertical axis $z$ coincides with the center of the borehole, i.e., with the center of the tool. In the cylindrical system of coordinates, $M_1$ can be approximated with a small solenoidal coil ($I_z$) thereby becoming a 2D source whereas $M_2$ and $M_3$ are represented as 3D sources. For these 3D sources, we employ a Dirac delta function in the $\xi_2$-direction, i.e., $\delta_I(\xi_2 - \xi_{2B})$, where $\xi_{2B}$ is 0° and 90° for $M_2$ and $M_3$, respectively. The same considerations are applied for the simulation of $H_1$, $H_2$, and $H_3$.

FFE variational formulation for triaxial induction measurements in deviated wells

Assuming a time dependence of the form $e^{i\omega t}$, Maxwell’s equations in a Cartesian system of coordinates in a domain $\Omega$ are given by

$$\nabla \times \mathbf{E} = -j\omega \mu \mathbf{H} - \mathbf{M}^{imp}, \quad \text{(Faraday’s law)}$$

$$\nabla \times \mathbf{H} = (\sigma + j\omega\varepsilon)\mathbf{E} + \mathbf{J}^{imp}, \quad \text{(Ampere’s law)}.$$  \hspace{1cm} (3)

where $\mathbf{H}$ and $\mathbf{E}$ denote the magnetic and electric fields, respectively, $\mathbf{J}^{imp}$ and $\mathbf{M}^{imp}$ are impressed electric and magnetic current densities, respectively, and $\varepsilon$, $\mu$, and $\sigma$ denote dielectric permittivity, magnetic permeability, and electrical conductivity tensors, respectively. In the formulation, $\Gamma = \partial \Omega$ is the outer boundary where we assume $\mathbf{n} \times \mathbf{E} = 0$ for a large enough computational domain. Triaxial induction tools measure $H_1$, $H_2$, and $H_3$ excited by $M_1$, $M_2$, and $M_3$ in a Cartesian coordinate system $\mathbf{x} = (x_1, x_2, x_3)$ with $x_3$ in the borehole direction (Figure 1a).

By multiplying Faraday’s law (equation 3) by $\mu^{-1}$, premultiplying the resulting equation by $\nabla \times \mathbf{F}$ and integrating over domain $\Omega$ by parts, applying Ampere’s law, we arrive at the corresponding variational formulation given by

$$\langle \nabla \times \mathbf{F}, \mu^{-1} \nabla \times \mathbf{E} \rangle_{L^2(\Omega)} - \langle \mathbf{F}, k^2 \mathbf{E} \rangle_{L^2(\Omega)} = -j\omega (\mathbf{F}, \mathbf{J}^{imp})_{L^2(\Omega)} - \langle \nabla \times \mathbf{F}, \mu^{-1} \mathbf{M}^{imp} \rangle_{L^2(\Omega)}$$

$\forall \mathbf{F} \in H^1(\text{curl}; \Omega), \hspace{1cm} (4)$

where $k^2 = \omega^2 \varepsilon - j\omega \sigma$ and we denote the $L^2$-inner product of two complex functions $f_1$ and $f_2$ as $\langle f_1, f_2 \rangle_{L^2(\Omega)} = \int_{\Omega} \overline{f_1} f_2 d\Omega$ (the symbol “$\overline{\cdot}$” denotes the complex conjugate).

To derive the FFE variational formulation, we first apply the change of coordinates (i.e., $f = f(\varphi)$, where $\varphi = \phi(\xi)$) to the original variational formulation 4 and construct a Fourier series expansion for $\mathbf{E}, \mathbf{D}, \mathbf{H}_{new}, \mathbf{J}_{new}, \mathbf{M}_{new}$ of the resulting formulation, which are newly defined material coefficients and load tensors in $(\xi_1, \xi_2, \xi_3)$ (they include the Jacobian determinant of the change of coordinates, e.g., $\mathbf{M}_{new} = J^{-1} M^{imp} J^{-1}$). By multiplying the resulting variational formulation of Fourier modes by a monomodal test function $\mathbf{F} = f_2 e^{i\phi}$ and by invoking the orthogonality of Fourier modes in $L^2([0, 2\pi])$, the FFE formulation reduces to (refer to Pardo et al. [2008] for a detailed derivation)

$$\langle f_1(\nabla \times \mathbf{F}), f_2 e^{i\phi} \rangle_{L^2(\Omega_{2D})} - \langle f_1(\mathbf{F}), f_2(\mathbf{J}^{imp}) \rangle_{L^2(\Omega_{2D})}$$

$$- j\omega \langle f_1(\mathbf{F}), f_2(\mathbf{J}^{imp}) \rangle_{L^2(\Omega_{2D})}$$

$$= -j\omega f_1(\mathbf{F}) f_2(\mathbf{J}^{imp})_{L^2(\Omega_{2D})}$$

$\forall f_1(\mathbf{F}) \in V_f(\Omega_{2D})$, \hspace{1cm} (5)

where we employ the Einstein summation convention with $- \infty \leq \ell \leq \infty$ and assume that $\Gamma$ is independent of $\xi_2$, and $V_f(\Omega_{2D}) = f_1(\Gamma_f(\text{curl}; \Omega))$.

Vertical well

When considering a vertical well (whose dip angle is 0°), our new nonorthogonal system of coordinates is identical to a system of cylindrical coordinates. Therefore, the FFE variational formulation 5 reduces to the so-called 2.5D problem, i.e.,

$$\langle f_1(\nabla \times \mathbf{F}), \mathbf{\mu}^{-1} f_2(\nabla \times \mathbf{E}) \rangle_{L^2(\Omega_{2D})}$$

$$= -j\omega \langle f_1(\mathbf{F}), f_2(\mathbf{J}^{imp}) \rangle_{L^2(\Omega_{2D})}$$

$$= -j\omega f_1(\mathbf{F}) f_2(\mathbf{J}^{imp})_{L^2(\Omega_{2D})}$$

$$\forall f_1(\mathbf{F}) \in V_f(\Omega_{2D})$$. \hspace{1cm} (6)

hp FE method

For each 2D Fourier mode resulting from the above formulation, we employ an hp FE discretization as our solution method, where $h$ denotes the element size and $p$ denotes the polynomial order of approximation. Figure 3 shows an example of a self-adaptive, goal-oriented hp grid, where we observe rapid spatial variations of $h$ and $p$ throughout the computational domain. Intuitively, small elements with a low order of approximation are optimal to approximate field singularities or solutions with high gradients whereas large elements...
with large orders of approximation are intended to approximate the
smooth part of the solution. The resulting system of 2D problems is
solved with a direct solver of linear equations.

NUMERICAL RESULTS
Verification of the algorithm
To validate our 2.5D and 3D formulations, we consider the 1D
earth model without the borehole shown in Figure 4. For this valida-
tion, we ignore the resistivity of the tool so that magnetic fields for
the resulting models can also be calculated with a semi-analytical
method for benchmarking results. Notwithstanding, we still enforce
the geometry of the triaxial tool described in Figure 1b.

2.5D formulation
To validate the 2.5D formulation, we consider the three-layer
earth model without the borehole shown in Figure 4. The layers are
0.02, 10000, and 1 /H9024 m in resistivity from top to bottom, with a
thickness of 1.4 m for the second layer. Relative depth is set to zero
at the boundary between the first and second layers. We compute tri-
axial measurements of Hzz, Hxz, Hzx, Hz, Hxy, and Hyx for the model under consideration (Figure 1b) and com-
pare them to semi-analytical solutions calculated with em1d, which
is a general-purpose FORTRAN code developed by K. H. Lee (per-
sonal communication, 1984) to simulate EM fields induced in 1D
vertical subsurface models by point-source magnetic transmitters.

Given that Hzz is measured with 2D sources and receivers, exact
solutions are obtained for Hzz (Figure 5a) using only one (central)
Fourier mode. Because only the central Fourier mode of Hzz is sensi-
tive to the 2D source M, numerical simulation of Hzz (Figure 5b), us-
ing one central mode yields exactly the same result even though we
measure the 3D field Hz. For the case of 3D sources and receivers
used for measuring Hz, Hz, and Hxy, converged solutions are obtained
when using three and five Fourier modes for Hz, Hz, and Hxy, respectiv-
ely. Note that because the center of each source (and receiver) is located
0.05 m from the center of the mandrel (Figure 1b), we might have
nonzero values for Hz, Hz, and Hxy due to the geometry of the triaxial
tool; their values become null when the source and receiver are
aligned along any line parallel to the z-axis.

3D formulation for a deviated well
The 3D formulation for deviated wells with 2D sources has al-
ready been validated; interested readers are referred to Pardo et al.

Figure 3. Example of a final hp grid automatically generated with
our self-adaptive goal-oriented refinement strategy for a triaxial tool
operating in a borehole environment. Different shades indicate dif-
ferent polynomial orders of approximation from 1 (light gray) to 8
(dark gray). The transmitter is located at z = 1.25 m and the receiver
at z = 0 m.

Figure 4. Three-layer formation model without borehole. Magnetic
fields are simulated with a triaxial tool but without the presence of a
mandrel for comparison against 1D semi-analytical solutions calcu-
lated with em1d.
(2008) for additional technical details. We validate the 3D formulation by considering an example of one 3D source and one receiver. Specifically, we measure $H_{xx}$ in a 60° deviated well penetrating a 1 Ωm homogeneous formation (Figure 6b). The borehole is assumed to have the same resistivity as that of the formation, resulting in a homogeneous medium for which we compute semi-analytical solutions with em1d. Furthermore, the source is fixed while the receiver moves along the borehole.

Because $H_{xx}$ for the deviated well is the same as that for the vertical well, we compare $H_{xx}$ for the deviated well obtained using our $hp$ algorithm against semi-analytical calculations with em1d for the vertical well. Figure 6a indicates convergence of real and imaginary parts of $H_{xx}$ to the semi-analytical results as we increase the number of Fourier modes from three to nine (Figure 6a).

Triaxial measurements acquired in a borehole penetrating a formation with extreme resistivity contrasts

We simulate triaxial induction measurements using the tool described in Figure 2 for 0° (vertical), 30°, and 60° deviated wells in a formation composed of five layers (Figure 7). Concomitantly, we test the performance of our algorithm under extreme contrasts in electrical resistivity even though layer resistivities are too high or low for actual formations in practice. Each layer has a resistivity of 100, 0.05, 10,000, 1, and 20 Ωm (from top to bottom, respectively) whereas the thicknesses of the second, third, and fourth layers are 1.5, 3, and 3 m, respectively. The borehole is 0.2 m in diameter and 100 Ωm in resistivity.

![Figure 5. Comparison of (a) $H_{zz}$, (b) $H_{xz}$, (c) $H_{xx}$, and (d) $H_{xy}$ simulated for the 1D earth model shown in Figure 4 at 20 kHz, using the $hp$ Fourier FE method ($hp$-FFEM) against semi-analytical solutions calculated with em1d.](image-url)
We include invasion in the third and fourth layers and possibly TI electrical resistivity in the second and fourth layers. Invasion zones in the third and fourth layers are included with a radial distance equal to 0.1 m and resistivities equal to 500 and 10 1Ωm, respectively. When considering formation anisotropy, vertical resistivities in the second and fourth layers are 0.5 and 10 1Ωm, respectively.

In the following, we quantify the influence of invasion, TI electrical conductivity, dip angle, shoulder-bed effects, and borehole effects on triaxial induction measurements. Because most of the information about electrical formation properties is contained in the imaginary part of the measured magnetic fields, we concentrate the description to the imaginary parts of triaxial measurements. Moreover, even though it is customary to transform imaginary parts of $H_z$ into apparent conductivities, we will not perform such a transformation to ease the comparison of standard induction coupling $H_z$ (which is usually measured by standard induction tools) against triaxial induction couplings ($H_{xx}$ and $H_{yy}$).

**Convergence history of $H_{xx}$ for vertical and 60° deviated wells**

Before analyzing simulated triaxial induction measurements, Figure 8 shows the convergence history of real and imaginary parts of $H_{xx}$ for vertical and 60° deviated wells. For $H_{xx}$ in the case of a vertical well, converged solutions are obtained using five Fourier modes.

Figure 6. (a) Convergence history of simulated triaxial induction measurements of $H_{xx}$ (real and imaginary parts) as a function of the number of Fourier modes (3, 7, and 9 modes) at 20 kHz for a 60° deviated well penetrating a homogeneous formation (resistivity = 1 Ωm). (b) Borehole and tool have the same resistivity as that of the formation. Simulations obtained with our self-adaptive FE algorithm are compared to semi-analytical solutions calculated with em1d.

Figure 7. Formation model consisting of five horizontal layers of resistivities of 100, 0.05, 10,000, 1, and 20 1Ωm (from top to bottom, respectively), constructed for the analysis of numerical simulations. The thickness of the second, third, and fourth layers is 1.5, 3, and 2 m from top to bottom, respectively. Wellbore dip angle is 0° (vertical), 30°, or 60°. Borehole radius = 0.1 m and borehole resistivity = 100 1Ωm. The model includes layers with either invasion or TI electrical resistivity. Invasion is included in the third and fourth layers with a radial distance ($R$) of 0.1 m and resistivities of 500 and 10 1Ωm in the third and fourth layers, respectively. Transversely isotropic (TI) electrical resistivity is assumed in the second and fourth layers with vertical resistivities equal to 0.5 and 10 1Ωm in the second and fourth layers, respectively.
whereas for the case of a $60^\circ$ deviated well $H_x$ converges with nine-Fourier modes. We note that the most conductive layer ($0.05 \, \Omega\,m$) gives rise to the most dominant effect in the simulated response, whereby we scarcely observe the effects of the fourth (conductive) layer on the real part of $H_x$ for vertical and $60^\circ$ deviated wells. This behavior is partially due to our description of results with a linear scale (a logarithmic scale is not applicable here because of the presence of negative values).

**Effects of dip angle on triaxial induction measurements**

Dip effects on triaxial induction couplings $H_{xx}$ and $H_{yy}$ are more pronounced than those on conventional standard measurements ($H_{zz}$) across the most conductive (second) layer and increase with increasing dip angle (Figure 9). Moreover, dip effects on $H_{xx}$ and $H_{yy}$ across the fourth conductive layer are also prominent whereas those on $H_{zz}$ are negligible when described with the linear scale of

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**Figure 8.** Convergence history of real (top) and imaginary components (bottom) of $H_{xx}$ simulated at 20 kHz for a well penetrating the five-layer formation model described in Figure 7. Panels (a) and (b) describe the convergence history for a vertical and a $60^\circ$ deviated well, respectively.
Figure 9. The magnitude of imaginary parts across conductive layers decreases with an increase in dip angle. We also observe that the horizontal distance between the transmitter and receiver increases with increasing dip angle whereas the vertical distance decreases.

Effects of invasion on triaxial induction measurements in deviated wells

To investigate the sensitivity of triaxial measurements in the presence of invasion, we include a shallow radial length (0.1 m) of invasion in the third and fourth layers. Triaxial measurements $H_{xx}$ and $H_{yy}$ for vertical and 60° deviated wells are sensitive to shallow invasion whereas conventional induction measurements $H_{zz}$ (for both wells) remain almost insensitive (Figure 10).

Effects of anisotropy on triaxial induction measurements in deviated wells

Figure 11a confirms that the standard induction coupling $H_{zz}$ in a vertical well is insensitive to TI electrical resistivity (the anisotropy ratio is 10 as described in Figure 7); it only becomes sensitive to TI electrical resistivity in deviated wells. The effects of TI electrical resistivity on $H_{zz}$ increase with increasing dip angle. This behavior is attributed to the fact that electrical currents induced by $M_z$ in a vertical well flow only in the radial direction and are not affected by vertical resistivity whereas in deviated wells more current flows in the vertical direction with increasing dip angle. On the other hand, triaxial couplings $H_{xx}$ and $H_{yy}$ are sensitive to anisotropy in vertical and deviated wells (Figure 11b and c, respectively). Even with the linear scale of Figure 11, $H_{xx}$ and $H_{yy}$ are sensitive to electrical anisotropy in the fourth layer whereas $H_{zz}$ measurements remain insensitive. Furthermore, effects of electrical anisotropy on $H_{xx}$ and $H_{yy}$ decrease with increasing dip angle.

Frequency effects on triaxial induction measurements

To investigate frequency effects on triaxial measurements, we simulate $H_{xx}$ for the case of a vertical well penetrating the five-layered formation at 2 MHz and 20 kHz in Figure 12. We observe that $H_{xx}$ at 20 kHz exhibits smaller variations (in real and imaginary parts) than at 2 MHz.

The computational time and memory requirements are highly dependent on the specific problem and the desired numerical accuracy. For most problems, a few minutes (between one and five) are enough to secure simulations with very high numerical accuracy. For the most challenging problems, possibly involving casing and/or very difficult 3D effects, the computational time rapidly increases to one hour (and sometimes even more) due to the need for a finer spatial...
Figure 10. Comparison of the imaginary parts of (a) $H_{zz}$, (b) $H_{xx}$, and (c) $H_{yy}$ simulated at 20 kHz for vertical (top), and 60° (bottom) deviated wells penetrating the formation model described in Figure 7 without (left-facing triangles with solid lines) and with the presence of invasion (right-facing triangles with solid lines), respectively.
Figure 11. Comparison of the imaginary parts of (a) $H_{zz}$, (b) $H_{xx}$, and (c) $H_{yy}$ simulated at 20 kHz for vertical (top), 30° (middle), and 60° (bottom) deviated wells penetrating the formation model described in Figure 7 with isotropic (left-facing triangles with solid lines) and TI (right-facing triangles with solid lines) layers, respectively.
grid to obtain accurate results. All cases considered in the paper were solved on a sequential machine; usage of parallel computations was not necessary.

CONCLUSIONS

We introduced and successfully benchmarked a new algorithm to simulate 3D triaxial induction measurements (including the presence of the logging tool) in deviated wells by combining the use of a Fourier series expansion in a nonorthogonal system of coordinates with a goal-oriented, high-order, self-adaptive hp FE method. Our algorithm method is suitable neither for a 90° deviated well (a horizontal well) nor for borehole-eccentered tool measurements acquired in a deviated well. Validations of the algorithm against semianalytical solutions for 2.5D and 3D formulations confirmed the accuracy and reliability of the simulation method. Numerical experiments for a model with high contrasts of formation resistivity indicated that dip effects on triaxial induction measurements ($H_{xx}$ and $H_{yy}$) are larger than those of standard borehole induction measurements ($H_{src}$), which are usually measured with coaxial induction tools. Triaxial couplings $H_{xx}$ and $H_{yy}$ remained sensitive to the shallow-invasion model considered in our study even though $H_{src}$ was not sensitive to invasion. Transversely isotropic electrical resistivity effects on 3D measurements of $H_{src}$ and $H_{xy}$ are largest in vertical wells; they diminish with decreasing dip angles in deviated wells. Even though standard borehole-induction measurements $H_{src}$ are not sensitive to TI electrical resistivity in vertical wells, effects of TI electrical resistivity on $H_{src}$ in-cresce with increasing borehole dip angle. In vertical wells, $H_{src}$ at 20 kHz exhibits smaller variations than at 2 MHz.

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