Quantitative interpretation of pulsed neutron capture logs: Part 1 — Fast numerical simulation

Jordan G. Mimoun¹, Carlos Torres-Verdín², and William E. Preeg³

ABSTRACT

Pulsed neutron capture (PNC) logs are commonly used for formation evaluation behind casing and to assess time-lapse variations of hydrocarbon pore volume. Because conventional interpretation methods for R logs assume homogeneous formations, errors may arise, especially in thinly bedded formations, when appraising petrophysical properties of hydrocarbon-bearing beds. There exist no quantitative interpretation methods to account for shoulder-bed effects on R logs acquired in sand-shale laminated reservoirs. Because of diffusion effects between dissimilar beds, R logs acquired in such formations do not obey mixing laws between the responses of pure-sand and pure-shale end members of the sedimentary sequence. We have developed a new numerical method to simulate PNC rapidly and accurately. The method makes use of late-time, thermal-neutron flux sensitivity functions (FSFs) to describe the contribution of multilayer formations toward the measured capture cross section. It includes a correction procedure based on 1D neutron diffusion theory that adapts the transport-equation-derived, base-case FSF of a homogeneous formation to simulate the response of vertically heterogeneous formations. Benchmarking exercises indicate that our simulation method yields average differences smaller than two capture units within seconds of computer central processing unit time with respect to PNC logs simulated with rigorous Monte Carlo methods for a wide range of geometrical, petrophysical, and fluid properties.

INTRODUCTION

Since the early days of Neutron Lifetime (trademark of Lane-Wells Co., now Baker Hughes) and Thermal Neutron Decay Time (TDT) (trademark of Schlumberger) logging instruments (Youmans et al., 1964; Wahl et al., 1970), pulsed neutron capture (PNC) tools have become indispensable for cased-hole saturation monitoring.

PNC logs are used to discriminate hydrocarbon-bearing from saltwater-bearing reservoir units because of the sizable difference of thermal-neutron decay rates in saline water and hydrocarbons. The difference in physical behavior stems from the presence in water of chlorine, which is a strong absorber of neutrons in common subsurface formations. In addition to identifying pay zones among water-saturated layers, PNC measurements are acquired throughout the life of hydrocarbon-producing wells to monitor variations of saturation with time. Time-lapse monitoring is a widely used application of R logs in the petroleum industry and has received much attention (Clavier et al., 1971b; Nutt and Watfa, 1989; Kimminau and Plasek, 1992; Cowan and Wright, 1999). The extension of time-lapse logging in waterflooded reservoirs is the estimation of waterflood residual oil saturation via the log-inject-log procedure (Richardson et al., 1973; Reedy, 1984). Also, equipping PNC tools with dual-detector systems improves the detection of gas, reduces borehole environmental effects, and provides the interpreter with an apparent porosity from the ratio of both detectors’ counts (Dewan et al., 1973; Serpas et al., 1977).

Despite recent breakthroughs in resistivity measurements behind casing (Bartenhagen et al., 2001), the need remains for resistivity-independent saturation evaluation in both open- and cased-hole logging operations. Low-contrast, low-resistivity pays (Simpson and Menke, 2010), as well as non-Archie formations (e.g., carbonates and shaly sands), may limit the reliance on resistivity measurements, hence the need for an alternative method for saturation appraisal and cross-validation of interpretations. This is why the
PNC time-lapse technique is routinely used in hydrocarbon-producing wells: changes can be directly associated with fluid-saturation changes.

To date, simulating PNC measurements with rigorous Monte Carlo methods remains the standard method used to quantify PNC tool responses (Butler, 1987). Previous attempts were made to develop interpretation methods based on Monte Carlo simulators, such as that described by Peeters et al. (1994). Nevertheless, the long computer central processing unit (CPU) time associated with Monte Carlo techniques renders them impractical for use in implementing inversion methods in either the assessment of complex lithologies or the joint quantitative petrophysical interpretation with other open- or cased-hole borehole measurements.

There also exist finite-difference numerical simulators of PNC measurements based on diffusion theory. Even though the underlying physics is governed by the Boltzmann transport equation, in some instances, it may be approximated by neutron diffusion. In that context, it is worth noting that Jennings and Weber (1995) took a remarkable first step toward the fast quantitative interpretation of PNC logs. However, borehole environmental effects may cause significant simulation errors: Hamzah (1996) reports errors on the simulated counts above 30% in the cases of 15.2- and 25.4-cm-diameter boreholes filled with salt water. This is why the departure curves for TDT determined by Locke and Smith (1975) with a diffusion-based numerical simulator were eventually substituted with Monte Carlo-based numerical simulation techniques (Peerg and Scott, 1986).

In fashion similar to Mendoza et al.’s (2010) approach to numerically simulate neutron and density borehole measurements, in this paper we develop a new method for the fast simulation of PNC measurements using Monte Carlo-derived flux sensitivity functions (FSFs). However, unlike neutron and density measurements, PNC measurements can be substantially affected by shoulder beds to the extent that it is not possible to precompute such FSFs for all multilayer cases. Instead, simulations are initialized with base-case FSFs for homogeneous formations, to subsequently account for vertical heterogeneities by adapting the FSFs to the presence of layer boundaries using 1D diffusion theory.

To benchmark our FSF-correction method, we generate synthetic PNC measurements using the Monte Carlo N-Particle (MCNP) (Trademark of Los Alamos National Laboratory) code (X-5 Monte Carlo Team, 2005). We also calculate the corresponding multilayer FSFs with MCNP, which we compare to diffusion-corrected FSFs. The excellent agreement with the calculated FSFs for the two methods validates our FSF-correction strategy, thereby enabling the rapid numerical simulation of PNC logs with no further need of Monte Carlo methods.

**TOOL CONFIGURATION**

We invoke a generic Longhorn PNC tool that exhibits the traditional configuration of logging tools. It includes a 14-MeV accelerator source, which emits fast neutrons through both casing and cement into surrounding rock formations. Since the earliest days of PNC logging it has been recognized that detecting neutron-capture gamma rays is preferable to detecting neutrons (Youmans et al., 1964; Mills et al., 1965; Wahl et al., 1970). This is why we equip the Longhorn tool with two scintillation crystals, which are located 35 and 58 cm away from the source, respectively. Owing to its better vertical resolution, henceforth we exclusively focus our attention on the photon counting rate at the short-spaced detector.

The hypothetical 4.30-cm-diameter tool exhibits a measurement response similar to that of commercial tools. Table 1 reports comparisons against experimental results reported by Peerg and Scott (1986), which indicate less than 1.25% error for the Longhorn PNC tool in terms of MCNP-simulated measurements of formation thermal decay time \( \tau \) at the near detector. In terms of \( \Sigma \), this represents an error of less than 0.20 capture unit (c.u.) where 1 c.u. = 10\(^{-3}\) cm\(^{-1}\).

**MONTE CARLO-SIMULATED MEASUREMENTS**

Throughout this study, we generate PNC logs using both Monte Carlo simulations and the fast, approximate numerical method described in a subsequent section of this paper. Monte Carlo-derived synthetic PNC logs will serve as the benchmark in the study.

Because the Boltzmann equation, which describes the transport of neutrons, cannot be solved analytically in a borehole-and-formation geometry (Steinman et al., 1988), we instead resort to numerical solutions. To that end, we use the Monte Carlo code MCNP that reproduces the transport of neutrons and photons, from the emission of fast neutrons and their slowing down through numerous collisions with atomic nuclei, to their capture and subsequent emission of neutron-induced gamma rays that are counted at the detector. Unlike deterministic methods, Monte Carlo methods need no averaging approximations in space, energy, or time, and they can easily include arbitrary distributions of material properties. This flexibility makes MCNP a commonly used technique for simulating such borehole nuclear measurements.

From MCNP results, we obtain gamma-ray counts at the near detector that are similar to those acquired in practice but without any background counts. Even though some of the recently commercialized PNC tools resort to a dynamic parameterization technique based on a database of laboratory measurements (Plasek et al., 1995), Morris et al. (2005) reports that the traditional signal processing technique based on fitting the data with two

<table>
<thead>
<tr>
<th>( C_{w} ) (kppm)</th>
<th>( C_{BH} ) (kppm)</th>
<th>( \tau_{sim} ) (s)</th>
<th>( \tau_{exp} ) (s)</th>
<th>( \Delta \tau ) (%)</th>
<th>( \Delta \Sigma ) (c.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>36.9</td>
<td>0</td>
<td>277.4</td>
<td>274</td>
<td>1.25</td>
<td>−0.20</td>
</tr>
<tr>
<td>36.9</td>
<td>36.9</td>
<td>271.4</td>
<td>270</td>
<td>0.53</td>
<td>−0.09</td>
</tr>
<tr>
<td>36.9</td>
<td>250</td>
<td>270</td>
<td>270</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>73.7</td>
<td>0</td>
<td>213.7</td>
<td>214</td>
<td>−0.13</td>
<td>0.03</td>
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<tr>
<td>73.7</td>
<td>73.7</td>
<td>213.8</td>
<td>214</td>
<td>−0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>73.7</td>
<td>250</td>
<td>213.3</td>
<td>212</td>
<td>0.61</td>
<td>−0.13</td>
</tr>
</tbody>
</table>
exponential decays is reliable to separate borehole and formation responses. Thus, we express the photon counts as a function of time $M(t)$ as

$$M(t) = A_{BH} \exp\left(-\frac{t}{\tau_{BH}}\right) + A_F \exp\left(-\frac{t}{\tau_F}\right),$$

(1)

where $t$ is time; $\tau_{BH}$ and $\tau_F$ are the thermal decay times of the borehole and formation signals, respectively; and $A_{BH}$ and $A_F$ are the corresponding amplitudes. Although magnetic resonance-like inversion techniques have been recently introduced to extract $\tau_F$ from $M(t)$ (Flaum et al., 2008), such linear regressions may not correctly discriminate closely spaced signals, such as in a high-$\Sigma$ formation and an average saline borehole. Instead, we approach the problem with a nonlinear, two-exponential-decay fit, based on the Levenberg-Marquardt algorithm (Marquardt, 1963). This enables the dynamic fit of both borehole and formation responses. All the synthetic Longhorn-PNC-tool measurements reported throughout this paper originate from the application of that technique.

The purpose of this study is twofold: (a) to rapidly simulate PNC logs, and (b) to use the forward modeling method to correct for shoulder-bed effects in thinly bedded formations. Therefore, we restrict the data processing step to discriminating formation from borehole signals; we do not attempt to correct for borehole diffusion and other residual borehole effects. A number of papers (Olesen et al., 1987; Murdoch et al., 1990) provide detailed insight to such complementary corrections.

Because MCNP-simulated measurements stem from a random sampling method, it is crucial to ascertain their reliability. Besides increasing the number of particle histories, which entails longer CPU times, we improve the simulation statistics via several variance reduction techniques. To that end, we first improved the photon sampling in the vicinity of the detector using a next-event estimator that deterministically biases the angle of collisions. We also defined time importance to improve the neutron photon sampling as a function of time. For each Monte Carlo-simulated measurement, the corresponding thermal-neutron FSF is obtained as a by-product of the simulation.

**LIMITATIONS OF $\Sigma$ MIXING LAWS**

When all the components (rock matrix, shale, and fluids) are homogeneously distributed in space, $\Sigma$ obeys a linear mixing law, where each component is weighted by its relative volumetric concentration (Clavier et al., 1971a). For instance, one can write the total capture cross section in a clay-free formation as

$$\Sigma_{total} = (1 - \phi) \Sigma_{ma} + \phi (1 - S_w) \Sigma_{sh} + \phi S_w \Sigma_{sw},$$

(2)

where $\phi$ is porosity; $S_w$ is water saturation; and $\Sigma_{ma}$, $\Sigma_{sh}$, and $\Sigma_{sw}$ are the capture cross sections of the rock matrix, hydrocarbon, and water, respectively. In shaly formations, $\Sigma$ obeys a linear mixing law similar to equation 2 as long as the shale is uniformly distributed in the rock such that the formation remains a homogeneous mixture. This situation occurs in the cases of structural and dispersed shale, for which one can write

$$\Sigma_{total} = C_{sh} \Sigma_{sh} + (1 - C_{sh}) \Sigma_{t},$$

(3)

where $C_{sh}$ is volumetric concentration of shale and $\Sigma_{sh}$ and $\Sigma_{t}$ are the capture cross sections of the shale and nonshale components, respectively.

In heterogeneous formations, neutrons may be captured more promptly in high-$\Sigma$ layers that act as sinks of neutrons, whereas they preferentially last longer in lower-$\Sigma$ layers, as pointed out by Allen et al. (1965) in terms of invaded and virgin zones. This behavior indicates that neutrons have a specific spatial variation in layered formations that does not solely depend on the relative volume of each component. Stieber (1970) proposes to compensate for shale effects using the formula

$$C_{Stieber} = \frac{C_{sh}}{3 - 2C_{sh}}$$

(4)
in lieu of the linear shale index $C_{sh}$ included in equation 3. The latter formula was determined by trial and error over hundreds of data sets.

Figure 1 describes two cases of studies that emphasize the shortcomings of conventional mixing laws in heterogeneous formations. For the particular tool position with respect to the bed boundary between shale and hydrocarbon-bearing sand for cases 1 and 2, Table 2 shows the discrepancies associated with the assumption of the linear and the Stieber nonlinear mixing laws between source and detector. Comparison against MCNP
simulations of the Longhorn PNC tool indicates an error of more than 68% for the former, which stems from its inability to account for the spatial variations of neutron flux in heterogeneous formations. For instance, the relative contribution of shale layers to the measurement is lower than expected based on their thickness in both cases. The discrepancy is especially notorious with dissimilar beds, as illustrated by case 2. The Stieber model yields a reduced error compared with the linear model; nevertheless, because it is an empirical model, it may not be reliable for all formation types. Core data would be required to modify accordingly the tuning constants included in equation 4.

However, when beds become thin enough in a region immediately surrounding the detector with respect to the diffusion length $L_d$, neutrons flow across them as though they were homogeneously mixed. Consequently, there exists a threshold above which heterogeneities begin to affect the measurement, thereby causing deviations from the linear model. For example, Haley (1995) reports that beds thicker than 10 cm are likely to cause measurement biases in a 34% porosity, 100-kppm salinity sand-shale laminated formation. Thus, the effective $\Sigma$ in a layered formation becomes a weighted average of the $\Sigma$ of all the layers that interact with the neutrons. These weights, which represent the importance of each layer toward the effective $\Sigma$, are no longer equal to the relative volume of each bed.

**FORMULATION**

By definition, the capture cross section $\Sigma$ quantifies the ability of a material to capture neutrons. The lower the neutron energy, the more likely capture phenomena will take place due to the general inverse-of-energy cross section dependence, whereby neutrons at thermal energies are the most likely to be absorbed. Consequently, monitoring the population of thermal neutrons at thermal energies are the most likely to be absorbed. The more likely capture phenomena will take place due to the slow down of energetic neutrons from the source on their way to the detector does not exhibit a symmetric probability distribution function: the closer to the detector a neutron is absorbed, the more likely it will make a count at the detector, which is given by the thermal-neutron flux sensitivity function. We approximate this physical interpretation of $\Sigma$ with the following Fredholm integral equation of the first kind:

$$\Sigma(r) \approx \int dr_0 FSF[r, r_0, \Sigma(r_0)]\Sigma(r_0), \quad (5)$$

where $r_0$ is an arbitrary position vector that spans the whole space; $\Sigma(r_0)$ is the capture cross section at position $r_0$; and $FSF[r, r_0]$ is the thermal-neutron flux sensitivity function, which describes the relative contribution of all the $\Sigma(r_0)$ at position $r$. Equation 5 is independent of energy in our problem because we assume that all absorbed neutrons are at their thermal energy state. Moreover, for the case of an infinite, homogeneous formation, $\Sigma(r_0)$ is constant and independent of $r_0$: the FSF must obey the following normalization:

$$\int dr_0 FSF[r, r_0, \Sigma(r_0)] = 1. \quad (6)$$

We determine the late-time importance of thermal neutrons to ensure cancellation of undesired early-time borehole effects. Furthermore, we assume that fluid reequilibration takes place shortly upon setting casing, hence making it possible to neglect invasion effects in the simulation. Also, we assume non-dipping beds. These three assumptions allow us to remove the dependence of the solution of equation 5 on the radial and azimuthal directions; the problem is now describable with the vertical direction only. We finally express the total capture cross-section $\Sigma$ at depth $z$ as

$$\Sigma(z) = \int dz_0 FSF_{LT}[z, z_0, \Sigma(z_0)]\Sigma(z_0), \quad (7)$$

where $z_0$ is an arbitrary location that spans the vertical direction, $\Sigma(z_0)$ is the capture cross section at depth $z_0$, and $FSF_{LT}[z, z_0, \Sigma(z_0)]$ is the normalized, radially averaged, late-time, thermal-neutron flux sensitivity function, which quantifies the relative contribution from all the $\Sigma(z_0)$ at depth $z$.

The above-described formulation exclusively concentrates on the transport of neutrons, disregarding the subsequent transport of gamma rays. This approximation is justified by the physics governing the phenomenon of pulsed neutron capture, which is independent of the detection system, i.e., thermal neutron or capture gamma rays (Allen et al., 1965). Both neutron and photon count rates yield similar values of $\Sigma$ (Wahl et al., 1970); neutron and photon fluxes are proportional, and there is no delay between neutron capture and gamma ray emission because photons travel at the speed of light (Locke and Smith, 1975). Therefore, even though detecting neutron-capture gamma rays is practically preferable, we may expect similar results by focusing only on thermal neutrons, which simplifies the problem for analysis.

**DEPENDENCE OF FSFS ON FORMATION PROPERTIES**

**Homogeneous formations**

Although $\Sigma$ is uniform everywhere in a homogeneous formation, the slowdown of energetic neutrons from the source on their way to the detector does not exhibit a symmetric probability distribution function: the closer to the detector a neutron is absorbed, the more likely it will make a count at the detector, hence the fairly skewed bell-shaped function described in Figure 2. The FSF is primarily influenced by the slowing-down length $L_s$, which is correlated to the hydrogen index $H_I$. Such a behavior causes potential problems in low-porosity and gas-bearing formations. Figure 3 shows the Monte Carlo-derived, thermal-neutron FSFs for three homogeneous formations that span the

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**Table 2.** Formation capture cross section calculated with MCNP simulations of the Longhorn PNC tool $\Sigma_{MCNP}$, compared against estimations from the linear mixing law $\Sigma_{linear}$ as well as from the nonlinear Stieber model $\Sigma_{Stieber}$ for cases 1 and 2, whose configurations are described in Figure 1. $\Delta \Sigma_{linear}$ and $\Delta \Sigma_{Stieber}$ are the relative errors with respect to $\Sigma_{MCNP}$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Sigma_{MCNP}$ (c.u.)</th>
<th>$\Sigma_{linear}$ (c.u.)</th>
<th>$\Sigma_{Stieber}$ (c.u.)</th>
<th>$\Delta \Sigma_{linear}$ (%)</th>
<th>$\Delta \Sigma_{Stieber}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.8</td>
<td>36.3</td>
<td>27.2</td>
<td>52.5</td>
<td>14.3</td>
</tr>
<tr>
<td>2</td>
<td>22.3</td>
<td>37.6</td>
<td>27.1</td>
<td>68.6</td>
<td>21.5</td>
</tr>
</tbody>
</table>
range of $L_s$ from 13.21 to 37.86 cm; Table 3 reports volumetric compositions and other petrophysical properties of interest for each case of study. Calculated FSFs in low-$HI$ formations are shifted toward the detector because neutrons scatter longer before thermalization and absorption.

Changes of $\Sigma$ cause negligible effects on FSFs for a similar value of $L_s$, Figure 4 compares Monte Carlo-calculated, thermal-neutron FSFs for homogeneous sandstone formations saturated with either fresh or salt water for four different values of porosity. For each of these four sets of cases, both $L_s$ and the diffusion coefficient $D$ show comparable values, whereas $\Sigma$ exhibits significant variations (Table 4). However, FSFs exhibit negligible changes, although $\Sigma$ may vary from 10.73 to 37.93 c.u. As noted by Randall et al. (1978), neutron absorbers, such as chlorine, have a small effect on the slowdown process, as well as on the diffusion of neutrons. In other words, thermal-neutron FSFs depend marginally on $\Sigma$ when absorption is uniform within the volume of investigation.

**Impact of heterogeneities**

Figure 5 compares the thermal-neutron FSFs in homogeneous and heterogeneous formations for the same cases displayed in Figure 1. The transition between two distinct materials causes distortions in the FSF. Because absorption is no longer uniform in such layered formations, thermal neutrons are more likely to last longer in low-$\Sigma$ materials, whereas they are absorbed more promptly in

<table>
<thead>
<tr>
<th>Formation</th>
<th>Volumetric composition</th>
<th>$\phi$ (%)</th>
<th>$\rho_b$ (g/cm$^3$)</th>
<th>$HI$</th>
<th>$L_s$ (cm)</th>
<th>$\Sigma_{int}$ (c.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.60 SiO$_2$, 0.40 H$_2$O</td>
<td>40</td>
<td>1.99</td>
<td>0.40</td>
<td>13.21</td>
<td>11.61</td>
</tr>
<tr>
<td>2</td>
<td>0.95 SiO$_2$, 0.05 H$_2$O</td>
<td>5</td>
<td>2.57</td>
<td>0.05</td>
<td>20.36</td>
<td>5.43</td>
</tr>
<tr>
<td>3</td>
<td>0.60 SiO$_2$, 0.40 CH$_4$</td>
<td>40</td>
<td>1.60</td>
<td>0.01</td>
<td>37.86</td>
<td>3.07</td>
</tr>
</tbody>
</table>
higher-$\Sigma$ materials. Indeed, we observe that the flux undergoes a contraction in the high-$\Sigma$ shale and a relaxation in the lower-$\Sigma$ sand layer. Although one can anticipate such changes on qualitative grounds, quantifying them is not trivial. Even though the gas-bearing sand layer has a lower $\Sigma$ than the oil-bearing layer, the dynamic variations are greater for the latter owing to the impact of the tool position with respect to the layer boundary.

Radial length of investigation

The above-defined FSFs describe the importance of particle distribution in the radial direction, which is essential to estimate the radial depth of investigation of PNC measurements. Figure 6 shows the radial J-factor (Sherman and Locke, 1975) as a function of the radial distance from the borehole wall for the six different cases reported in Table 5. For 90% of the response, the J-factor exhibits variations of 3.8 cm between low- and high-$\Sigma$ formations. It is found that the size and

Table 4. Summary of the assumed petrophysical properties for the eight cases whose FSFs are reported in Figure 4. The formations consist of a homogeneous sandstone layer, saturated either with fresh water (formation A) or with 200-kppm salt water (formation B). Slow-down length $L_s$, diffusion coefficient $D$, and intrinsic capture cross section $\Sigma_{\text{int}}$ were calculated with SNUPAR.

<table>
<thead>
<tr>
<th>$\phi$ (%)</th>
<th>Formation</th>
<th>$L_s$ (cm)</th>
<th>$D$ (cm)</th>
<th>$\Sigma_{\text{int}}$ (c.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A</td>
<td>20.4</td>
<td>1.02</td>
<td>5.43</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>20.7</td>
<td>1.03</td>
<td>9.32</td>
</tr>
<tr>
<td>15</td>
<td>A</td>
<td>16.0</td>
<td>0.63</td>
<td>7.20</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>16.3</td>
<td>0.65</td>
<td>18.86</td>
</tr>
<tr>
<td>25</td>
<td>A</td>
<td>14.4</td>
<td>0.46</td>
<td>8.97</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>14.6</td>
<td>0.47</td>
<td>28.4</td>
</tr>
<tr>
<td>35</td>
<td>A</td>
<td>13.5</td>
<td>0.36</td>
<td>10.73</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>13.7</td>
<td>0.37</td>
<td>37.93</td>
</tr>
</tbody>
</table>

Figure 4. Impact of $\Sigma$ on the spatial sensitivity of PNC measurements acquired with the Longhorn tool in homogeneous formations. Blue and red lines represent the thermal-neutron FSFs for a homogeneous sand layer, saturated either with fresh water or with 250-kppm salt water, respectively. Porosity varies from 5% (top left panel) to 35% (bottom right panel). Orange lines at 0 and 35 cm indicate the positions of source and detector, respectively. Simulations assume a common borehole configuration for all three cases: 25.4-cm-diameter borehole, 17.8-cm-diameter casing, and 250-kppm borehole fluid. Normalization was implemented according to equation 6.

Figure 5. Comparison of MCNP-generated thermal-neutron FSFs for the layered formation (solid red line) against the FSF for a homogeneous formation (dashed black line), for cases 1 (left panel) and 2 (right panel). Orange lines at 0 and 35 cm indicate the locations of source and detector, respectively. Normalization was implemented in agreement with equation 6.
properties of the borehole do not cause significant impact on the J-factor. The total length of investigation equals 26 cm, which is similar to the result of 26.7 cm reported by Hopkinson et al. (1974). Careful consideration of radial length of investigation is recommended to prevent misleading interpretations of PNC logs in invaded formations (Allen et al., 1965), although cased-hole cases are less likely to be influenced by invasion (Clavier et al., 1971a).

**METHOD**

Equation 7 is the basis for simulating $\Sigma$ logs. Finding a solution of this equation based on the linear, first-order Born approximation, as already successfully done for neutron and density measurements by Mendoza et al. (2010), is not applicable to our study. Because FSFs are highly dependent on the neutron absorption properties of layered media, the recursive solution of equation 7 is not efficient with the FSF for a homogenous formation (linear iterative refinement of a first-order Born approximation). Instead, we develop a solution based on the distorted Born approximation wherein each measurement point is associated with a FSF specific to the local multilayer model in the vicinity of the measurement point, here termed $\text{FSF}_{LT}(z_0, \Sigma(c_0))$. Generating these local FSFs via Monte Carlo simulations requires time-consuming computations, hence the need for a faster method. Likewise, generating these FSFs in advance and constructing a comprehensive library of FSFs in heterogeneous formations is not practical either: one would need to compute as many FSFs as there are heterogeneous cases ($\Sigma$, bed thickness, bulk density, and tool position with respect to the layered formation are all variable parameters). The resulting method would be cumbersome and difficult to implement.

The procedure adopted in this paper for the calculation of local FSFs in conjunction with equation 7 is based on an initial FSF derived for a homogenous (base-case) formation (which also includes borehole, casing, and cement environmental effects) via Monte Carlo-simulation methods. The initial FSF remains constant for all measurement points as long as the borehole environmental conditions are uniform. Subsequently, the latter FSF is modified to account for local layer diffusion effects using an analytical correction. Assuming that diffusion theory is valid within the local domain of application simplifies the Boltzmann transport equation, thereby allowing us to account for formation heterogeneities analytically. Relevant details about this correction procedure, the underlying assumptions, as well as comparisons between Monte Carlo-computed and diffusion-corrected FSFs, are included in Appendix A. Upon correcting the FSFs for local, multilayer diffusion effects, we use equation 7 to determine $\Sigma$ at each tool position, from which we numerically simulate the PNC log. The analytical nature of the correction of multilayer diffusion effects enables its implementation within negligible CPU times on a standard computer.

**SYNTHETIC CASES**

The two synthetic examples described below are intended as benchmarking exercises to appraise the reliability and accuracy of the procedure described above to rapidly simulate PNC measurements.

**Synthetic case 1: Turbidite formation**

We first evaluate the performance of the simulation method for the case of a turbidite sequence that alternates oil-saturated sand beds and shale layers. Sand layers include bed

![Figure 6. Radial J-factors of the FSFs for six different cases. Red and blue lines indicate the geometric factors for oil-bearing sand and shale formations, respectively. Solid lines identify the case of a 25.4-cm-diameter openhole. Dotted lines show the case of a 25.4-cm-diameter borehole and 17.8-cm-diameter casing configuration. Dashed lines assume a 30.5-cm-diameter borehole and 24.4-cm-diameter casing. The green line identifies the radial length of investigation in the sense of Sherman and Locke (1975), i.e., the 90% value of the radial J-factor.](image)

**Table 5. Summary of the assumed petrophysical and borehole geometry properties for the six cases whose radial J-factors are reported in Figure 6. Borehole salinity of 250 kppm is common to all six cases. The intrinsic capture cross section $\Sigma_{int}$ was calculated with SNUPAR.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Lithology</th>
<th>$\Sigma_{int}$ (c.u.)</th>
<th>Borehole diameter (cm)</th>
<th>Casing diameter (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20% porosity oil-bearing clean sandstone</td>
<td>9.4</td>
<td>25.4</td>
<td>Openhole</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>25.4</td>
<td>17.8</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>30.5</td>
<td>24.4</td>
</tr>
<tr>
<td>4</td>
<td>10% porosity shale saturated with 200-kppm salt water</td>
<td>49.1</td>
<td>25.4</td>
<td>Openhole</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>25.4</td>
<td>17.8</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>30.5</td>
<td>24.4</td>
</tr>
</tbody>
</table>
thickness values ranging from 24.4 to 48.8 cm, whereas shale bed thickness is constant and equal to 24.4 cm. Both sides of the multilayer model include end members of the sedimentary sequence as thick beds. Tables 6 and 7 summarize the lithology, fluid, and geometrical properties assumed for this case of study.

Longhorn PNC-tool measurements are simulated along the cased and cemented borehole every 7 cm using MCNP. For depth-matching purposes, we use the geometric center of the FSFs to define the location of the measurement point, thereby generating field-like PNC measurements that serve as reference. These Monte Carlo simulations required more than 180 hours of CPU time (for 47 depth sample points, assuming 4 hours per run) on a Linux server with Intel Itanium 2 1.4-GHz microprocessors.

We use equation 7 together with the FSF-correction procedure (reported in Appendix A) to simulate numerically the PNC log for this case. Figure 7 compares the corresponding results against those obtained with MCNP. Less than 1 s of CPU time was necessary to obtain PNC measurements in excellent agreement with their Monte Carlo-derived counterparts. The simulation method reliably reproduces the effects of thin beds and the ensuing rapid variations of the $R$ log; shoulder-bed effects are reproduced with a difference of less than 2.5 c.u.

**Table 6.** Summary of the assumed values of total porosity $\phi$, saturation $S$, formation water salt concentration $C_{w,f}$, bulk density $\rho_b$, intrinsic capture cross section $\Sigma_{int}$ (from SNUPAR) and expected measured capture cross section $\Sigma_{dif}$ (from MCNP simulations of the pure, infinite-extent formation in the presence of the borehole) for the sand and shale components of synthetic case 1.

<table>
<thead>
<tr>
<th>Lithology</th>
<th>$\phi$ (%)</th>
<th>$S$</th>
<th>$C_{w,f}$ (kppm)</th>
<th>$\rho_b$ (g/cm$^3$)</th>
<th>$\Sigma_{int}$ (c.u.)</th>
<th>$\Sigma_{dif}$ (c.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shale</td>
<td>10</td>
<td>$S_w = 1$</td>
<td>100</td>
<td>2.99</td>
<td>39.52</td>
<td>40.84</td>
</tr>
<tr>
<td>Sand</td>
<td>20</td>
<td>$S_w = 0.2$</td>
<td>100</td>
<td>2.29</td>
<td>9.38</td>
<td>13.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$S_o = 0.8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 7.** Summary of the assumed hydrocarbon densities, borehole fluid properties, well geometry and sampling rate for synthetic cases 1 and 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation oil: carbon weight fraction</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>Formation oil: hydrogen weight fraction</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>Formation oil density</td>
<td>0.82</td>
<td>g/cm$^3$</td>
</tr>
<tr>
<td>Formation gas density</td>
<td>0.017</td>
<td>g/cm$^3$</td>
</tr>
<tr>
<td>Borehole fluid salt concentration</td>
<td>225</td>
<td>kppm</td>
</tr>
<tr>
<td>Borehole fluid capture cross section</td>
<td>111.2</td>
<td>c.u.</td>
</tr>
<tr>
<td>Borehole diameter</td>
<td>25.4</td>
<td>cm</td>
</tr>
<tr>
<td>Casing diameter</td>
<td>17.8</td>
<td>cm</td>
</tr>
<tr>
<td>Sampling rate</td>
<td>7</td>
<td>cm</td>
</tr>
</tbody>
</table>

**Synthetic case 2: Complex multilayer formation**

We now examine the capabilities of the method with a 5.2-m synthetic formation consisting of successive sandstone, shale, shaly sand, and carbonate beds, saturated with gas, oil, fresh water, or saline water. In this example, $\Sigma$ spans the range from 12.41 to 45.55 c.u., with bed thicknesses between 24 and 76 cm. Variations of $\Sigma$ from one bed to another are from 6.63 (between beds 1 and 2) to 32.65 c.u. (between beds 4 and 5). Table 8 summarizes the petrophysical information about this case, whereas Table 7 reports the borehole fluid properties, well geometry information, and hydrocarbon densities involved.

Figure 8 compares the numerically simulated log against the Monte Carlo reference log that was obtained in a fashion similar to that reported for synthetic case 1, for a total of 300 hours of CPU time (for 74 depth sample points). Our simulation method required less than 1 second of CPU time to reliably reproduce the variations of the measured $\Sigma$ log, yielding a maximum difference of 2 c.u. with respect to the Monte Carlo-simulated measurements for the transitions between similar materials. By contrast, the transition between the low-$\Sigma$ bed 4 and the high-$\Sigma$ bed 5 exhibits an error of up to 5 c.u.
It is important to emphasize that the simulations obtained with Monte Carlo methods as described above yield count rates that were subsequently processed to calculate the corresponding PNC logs (as described in a previous section of this paper). By contrast, our simulation method directly yields $R$ values.

**CONCLUSIONS**

We have developed a new numerical method to simulate PNC logs in seconds of CPU time that successfully substitutes time-prohibitive Monte Carlo methods. The agreement obtained against MCNP simulations indicates that the new simulation method properly captures the physics of PNC logging and diffusion effects that take place between dissimilar beds. Benchmarking examples included the cases of a thinly bedded, turbiditic formation, as well as a complex multilayer formation, which encompassed most fluids and lithologies encountered in practice, with the presence of borehole, casing, and cement.

The simulation algorithm initially solves the coupled neutron-and-photon transport equation with Monte Carlo methods to determine the base-case, late-time thermal-neutron flux sensitivity function included in our formulation of $\Sigma$. We invoked the distorted Born approximation to evaluate the ensuing expression for $\Sigma$ and validated the use of 1D neutron diffusion theory to approximate the spatial variations of the late-time, thermal-neutron FSFs in vertically heterogeneous formations. The correction procedure applied to measurements acquired across large contrasts of porosity and saturating fluids (i.e., with variable slowing-down lengths) resulted in maximum differences of 3 c.u. in the subsequent estimation of $R$. When applied to two benchmarking examples, the agreement between numerically simulated logs and measurements was typically within 2 c.u. of error, on average.

Although neutron measurements could be affected by substantial borehole environmental effects, our procedure does not explicitly account for them and does not require their calculation. Instead, we focus our efforts to the late-time formation signal, thereby implicitly assuming a high-$\Sigma$ borehole fluid configuration, which is typical of PNC logging.

Table 8. Summary for synthetic case 2 of the assumed lithology, bed thickness $2H$, nonshale porosity $\phi_s$, shale porosity $\phi_{sh}$, volumetric concentration of shale $C_{sh}$, water saturation $S_w$, formation water salt concentration $C_{wF}$, nature of the hydrocarbons $HC$, bulk density $\rho_b$, slowing-down length $L_s$ (from SNUPAR), intrinsic capture cross section $\Sigma_{int}$ (from SNUPAR), and expected measured capture cross section $\Sigma_{diff}$ (from MCNP), from top to bottom of the formation.

<table>
<thead>
<tr>
<th>#</th>
<th>Lithology</th>
<th>$2H$ (cm)</th>
<th>$\phi_s$ (%)</th>
<th>$\phi_{sh}$ (%)</th>
<th>$C_{sh}$</th>
<th>$S_w$</th>
<th>$C_{wF}$ (kppm)</th>
<th>$HC$</th>
<th>$\rho_b$ (g/cm$^3$)</th>
<th>$L_s$ (cm)</th>
<th>$\Sigma_{int}$ (c.u.)</th>
<th>$\Sigma_{diff}$ (c.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Limestone</td>
<td>70.1</td>
<td>5</td>
<td>—</td>
<td>0.4</td>
<td>150</td>
<td>Gas</td>
<td>2.60</td>
<td>22.1</td>
<td>8.32</td>
<td>12.41</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Dolomite</td>
<td>67.1</td>
<td>12</td>
<td>—</td>
<td>0.65</td>
<td>250</td>
<td>Oil</td>
<td>2.68</td>
<td>14.8</td>
<td>14.60</td>
<td>19.04</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Shale</td>
<td>39.6</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>200</td>
<td>—</td>
<td>2.96</td>
<td>10.5</td>
<td>41.86</td>
<td>45.55</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Sandstone</td>
<td>24.4</td>
<td>20</td>
<td>—</td>
<td>0.4</td>
<td>50</td>
<td>Oil</td>
<td>2.30</td>
<td>15.1</td>
<td>9.38</td>
<td>12.90</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Shale</td>
<td>76.2</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>200</td>
<td>—</td>
<td>2.96</td>
<td>10.5</td>
<td>41.86</td>
<td>45.55</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Shaly sand</td>
<td>61</td>
<td>10</td>
<td>10</td>
<td>0.6</td>
<td>0.5</td>
<td>100</td>
<td>2.36</td>
<td>15.0</td>
<td>11.99</td>
<td>15.21</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Shaly sand</td>
<td>45.7</td>
<td>15</td>
<td>10</td>
<td>0.3</td>
<td>0.7</td>
<td>150</td>
<td>2.40</td>
<td>14.8</td>
<td>17.75</td>
<td>20.95</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Sandstone</td>
<td>30.5</td>
<td>20</td>
<td>—</td>
<td>0.4</td>
<td>50</td>
<td>Oil</td>
<td>2.30</td>
<td>15.1</td>
<td>9.38</td>
<td>12.90</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Sandstone</td>
<td>45.7</td>
<td>25</td>
<td>—</td>
<td>1</td>
<td>220</td>
<td>—</td>
<td>2.28</td>
<td>14.6</td>
<td>30.63</td>
<td>31.69</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Sandstone</td>
<td>57.9</td>
<td>25</td>
<td>—</td>
<td>1</td>
<td>0</td>
<td>—</td>
<td>2.24</td>
<td>14.4</td>
<td>8.96</td>
<td>11.65</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8. Comparison of numerically simulated (solid red line) and measured $\Sigma$ logs (solid green line, calculated with MCNP) for synthetic case 2. Actual layer-by-layer values are shown by the dashed blue line. Petrophysical information about beds 1–10 is reported in Table 8.
Even though the results presented in this paper were determined for a specific PNC-tool configuration, the base-case FSF can be readily adapted to any commercial data-acquisition configuration (source-detector spacing, tool diameter, etc.). Also, the simulation method could be extended to 2D and 3D geometries to account for deviated and horizontal wells, dipping beds, and invasion effects on measurements.

Our fast numerical method may be implemented in conjunction with inversion techniques to develop a quantitative petrophysical interpretation of Σ logs. Such a procedure could improve the interpretation of thinly bedded formations and complex lithologies, to calculate layer-by-layer values of Σ. Finally, the method is fast enough to enable the quantitative integration of PNC logs with other open- and cased-hole logs.

ACKNOWLEDGMENTS

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APPENDIX A

FSF-CORRECTION PROCEDURE BASED ON 1D DIFFUSION THEORY

Layered formations introduce model heterogeneities that cause perturbations on the FSFs. We calculate such perturbations under a set of assumptions. First, common subsurface materials exhibit a sufficiently low capture cross section compared with their total cross section; therefore, it is valid to consider that scattering is more likely to take place than absorption. Second, angular variations of the neutron distribution are linear and we deal exclusively with isotropic scattering. Because the physics of PNC measurements involves mostly thermal neutrons, the diffusion equation (Duderstadt and Hamilton, 1976) may accurately describe the conservation of such monoenergetic neutrons under the conditions mentioned above, given by

\[- \frac{\partial^2 \phi(r)}{\partial z^2} + \frac{1}{L_d^2} \phi(r) = \frac{s}{D}, \quad (A-1)\]

where \( \phi(r) \) is thermal-neutron flux at position \( r \), \( L_d \) is diffusion length, \( D \) is diffusion coefficient, and \( s \) is neutron source strength.

Also, we consider the PNC measurements to be acquired sufficiently late in time so that the contributions from the highly absorbing borehole and casing become negligible. Table A-1 describes the thermal decay times for typical formations, which are larger than those associated with both borehole fluid and casing, even for the case of high-Sigma shale lithology. Assuming that diffusion takes place solely in the vertical direction with no radial effect due to the borehole (we consider neither dipping beds nor mud-filtrate invasion in this study), the neutron conservation equation simplifies to the 1D diffusion equation, given by

\[- \frac{\partial^2 \phi(z)}{\partial z^2} + \frac{1}{L_d^2} \phi(z) = \frac{s}{D}, \quad (A-2)\]

where \( \phi(z) \) is thermal-neutron flux at depth \( z \).

We subsequently implement the principle of superposition for the linear differential equation \( A-2 \), which enables us to decompose a complex multilayer problem into elementary single-layer subproblems. Such a procedure is equivalent to successively considering each layer as a source of thermal neutrons that diffuse into the surrounding layers. The initial distribution is given by the Monte Carlo-derived, late-time thermal-neutron FSF \( \phi_{\text{homog}} \); this function describes the flux of thermal neutrons in a homogeneous formation at late times, and already accounts for (a) the transport of high-energy neutrons and (b) all residual borehole and radial effects.

Accordingly, for each subproblem \( k \), the diffusion-corrected FSF is calculated from

\[
\begin{align*}
\left( \text{FSF}_{\text{corrected}} \right)_k &= \left( \phi_{\text{homog}} \right)_k \exp \left( \frac{- T_{\text{start}}}{t_k} \right) \\
&\times \left\{ \sinh \left( \frac{h_k}{L_d} \right) \exp \left( - \frac{z_k}{L_d} \right), \quad z_k \in [H_k; \infty[ \right\} \\
&\times \left\{ 1 - \exp \left( - \frac{h_k}{L_d} \right) \cosh \left( \frac{z_k}{L_d} \right), \quad z_k \in [-H_k; H_k] \right\} \\
&\times \left\{ \sinh \left( \frac{h_k}{L_d} \right) \exp \left( \frac{z_k}{L_d} \right), \quad z_k \in [-\infty; -H_k] \right\},
\end{align*}
\]

where \( \phi_{\text{homog}} \) is Monte Carlo-computed, late-time thermal-neutron FSF for a homogeneous formation similar to layer \( k \); \( T_{\text{start}} \) is the delay necessary for borehole-effect cancellation; \( z_k \) is relative depth for subproblem \( k \) (the reference origin is at the center of layer \( k \)); and \( H_k, L_d \), and \( t_k \) are half-thickness, diffusion length, and thermal decay time, respectively, for layer \( k \). We sum the individual \( \left( \text{FSF}_{\text{corrected}} \right)_k \) for all the subproblems \( k \) to obtain the FSF associated with the multilayer problem and normalize it to honor equation 6.

Figures A-1 and A-2 show the thermal-neutron FSF calculated with this fast, approximate diffusion correction procedure (assuming knowledge of all formation properties), compared against the FSF computed directly from MCNP for a variety of layered formations whose petrophysical properties are reported in Table A-2. Values of \( \Sigma \) span the range from 10.6 to 49.6 c.u., with beds as thin as 24 cm. The correction procedure successfully reproduces the effects on the FSFs of multilayer formations with similar slowing-down lengths \( L_s \) (Figure A-1), despite the multiple transitions across distinct materials. This results in a difference of less than 2 c.u. on the estimated \( \Sigma \). The presence of the gas-bearing formations (represented in yellow) requires a specific treatment due to the lower hydrogen index, hence the larger value of \( L_s \) compared with oil- and water-saturated formations. To that end, we apply the previously described corrections to another FSF \( \phi_{\text{homog}} \) previously calculated for the case of a homogeneous gas-bearing formation with similar value of \( L_s \). Upon correction for presence of gas, the calculation procedure leads to approximate FSFs (Figure A-2), whose subsequent use gives rise to a difference of up to 3 c.u. on the estimated \( \Sigma \).
Table A-1. Thermal decay time $\tau$ for various materials (calculated with SNUPAR).

<table>
<thead>
<tr>
<th>Material</th>
<th>$\tau$ (\us)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation 1</td>
<td>690</td>
<td>Clean 15% porosity sandstone, saturated with 100-kppm salt water ($S_w = 0.3$) and methane ($S_g = 0.7$)</td>
</tr>
<tr>
<td>Formation 2</td>
<td>291</td>
<td>Clean 20% porosity limestone, saturated with 150-kppm salt water ($S_w = 0.5$) and 0.82-g/cm$^3$ oil ($S_o = 0.5$)</td>
</tr>
<tr>
<td>Formation 3</td>
<td>93.6</td>
<td>30% porosity illite saturated with 250-kppm salt water</td>
</tr>
<tr>
<td>Borehole fluid</td>
<td>37.0</td>
<td>250-kppm salt water</td>
</tr>
<tr>
<td>Casing</td>
<td>21.0</td>
<td>Steel</td>
</tr>
</tbody>
</table>

Figure A-1. Comparison of late-time, thermal-neutron FSFs simulated with the diffusion-approximation correction (dashed red line) and MCNP (solid blue line), for formations 1, 2, and 3 (shale, water-bearing sand, and oil-bearing sand, respectively), which exhibit similar slowing-down lengths. We initialized the correction procedure using the base-case FSF, represented by the dashed black line. Orange lines at 0 and 35 cm indicate the positions of source and detector, respectively.

Figure A-2. Comparison of late-time, thermal-neutron FSFs simulated with the diffusion-approximation correction (dashed red line) and MCNP (solid blue line), for formations with different slowing-down lengths. We initialized the correction procedure using the regular base-case FSF (dashed black line) that is tailored to low-$L_s$ formations (materials 1–3). For the gas-bearing formation (material 4), we made use of a specific gas-formation FSF. Orange lines at 0 and 35 cm indicate the positions of source and detector, respectively.
Table A-2. Summary of the assumed lithology, total porosity $\phi$, saturation $S$, formation water salt concentration $C_{w}$, bulk density $\rho_{b}$, slowing-down length $L_{s}$ (from SNUPAR), intrinsic capture cross section $\Sigma_{intrinsic}$ (from the SNUPAR), and expected measured capture cross section $\Sigma_{exp}$ (from the MCNP), for the eight cases whose FSFs are reported in Figures A-1 and A-2.

<table>
<thead>
<tr>
<th>#</th>
<th>Lithology</th>
<th>$\phi$ (%)</th>
<th>$S$</th>
<th>$C_{w}$ (kppm)</th>
<th>$\rho_{b}$ (g/cm$^{3}$)</th>
<th>$L_{s}$ (cm)</th>
<th>$\Sigma_{intrinsic}$ (c.u.)</th>
<th>$\Sigma_{exp}$ (c.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Shale</td>
<td>35</td>
<td>$S_w = 1$</td>
<td>215</td>
<td>2.21</td>
<td>12.4</td>
<td>48.2</td>
<td>49.6</td>
</tr>
<tr>
<td>2</td>
<td>Sand</td>
<td>35</td>
<td>$S_w = 1$</td>
<td>200</td>
<td>2.27</td>
<td>14.6</td>
<td>28.4</td>
<td>32.9</td>
</tr>
<tr>
<td>3</td>
<td>Sand</td>
<td>30</td>
<td>$S_w = 0.2$</td>
<td>200</td>
<td>2.12</td>
<td>13.9</td>
<td>14.3</td>
<td>18.2</td>
</tr>
<tr>
<td>4</td>
<td>Sand</td>
<td>25</td>
<td>$S_w = 0.1$</td>
<td>200</td>
<td>1.80</td>
<td>30.0</td>
<td>6.47</td>
<td>10.6</td>
</tr>
</tbody>
</table>

**NOMENCLATURE**

$A$ = Amplitude of PNC signal, [cps/cps]  
$C$ = Volumetric concentration, [ ]  
$C_{w}$ = Salt concentration (NaCl equivalent) of connate water, [kppm]  
$D$ = Diffusion coefficient, [cm$^{-2}$s$^{-1}$]  
$H$ = Half bed thickness, [cm]  
$HI$ = Hydrogen index, [ ]  
$L_{d}$ = Diffusion length, [cm]  
$L_{s}$ = Slowing-down length, [cm]  
$M(t)$ = Relative photon count, [cps/cps]  
$r$ = Position vector  
$s$ = Neutron source strength, [particles/cm$^{3}$]  
$S$ = Saturation, [ ]  
$t$ = Time, [$\mu$s]  
$z$ = Vertical direction, [cm]  
$\rho_{b}$ = Bulk density, [g/cm$^{3}$]  
$\Sigma$ = Macroscopic capture cross section, [c.u.]  
$\tau$ = Thermal decay time, [$\mu$s]  
$\phi$ = Porosity, [%]  
$\varphi$ = Thermal-neutron flux, [particles/cm$^{2}$s$^{-1}$]  
$BH$ = Borehole  
$\text{dif} = \text{Expected measurement for the pure, infinite-extend formation in presence of borehole}$  
$\text{exp} = \text{Experimental}$  
$F = \text{Formation}$  
$h = \text{Hydrocarbons}$  
$\text{homog} = \text{Homogeneous}$  
$\text{int} = \text{Intrinsic}$  
$k = \text{Layer } k$  
$LT = \text{Late time}$  
$ma = \text{Matrix}$  
$s = \text{Non-Shale}$  
$sh = \text{Shale}$  
$\text{sim} = \text{Simulated}$  
$w = \text{Water}$  
$0 = \text{Arbitrary}$  
$\text{cps} = \text{Count Per Second}$  
$c.u. = \text{Capture Unit (1 c.u. = } 10^{-3} \text{ l/cm)}$  

**REFERENCES**


Fast simulation of PNC borehole measurements


