Enhanced dispersion analysis of borehole array sonic measurements with amplitude and phase estimation method

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Summary

We introduce a new non-parametric matched-filterbank spectral estimator, referred to as Amplitude and Phase Estimation (APES), to perform dispersion analysis of borehole array sonic measurements. This method extracts the dispersion characteristics of all wave modes by applying an APES filter to array sonic spectral data and converting the estimated wavenumber to slowness. The implemented adaptive filter in APES ensures that the output signal be sufficiently close to a sinusoid with a designated wavenumber in space domain, which constrains the interference from other wavenumber components and suppresses the noise gain. Consequently, the resolution and signal-noise-ratio of dispersion analysis is significantly enhanced. Dispersion fitness functions processed with APES indicate clearer and narrower ridges with minimum presence of alias. At each frequency, dispersions of all modes can be identified without knowledge \textit{a priori} of the exact number of modes. More importantly, the new method is not computationally intensive compared to existing dispersion analysis methods. Processing examples with synthetic and field data are presented and compared with the weighted spectral semblance (WSS) method to demonstrate the applicability and advantages of this method.

Introduction

Dispersion analysis has become increasingly important in modern array sonic data processing. In practice, most wave modes excited in borehole environments are dispersive. An incomplete list includes: (1) pseudo-Rayleigh and Stoneley modes from a monopole source; (2) leaky P and flexural modes from a dipole source; (3) screw and other dispersive waves from multi-pole sources (quadrupole or hexpole). Results from dispersion analysis have found many applications in geophysical reservoir characterizations such as shear slowness and borehole-fluid slowness inversion (Yang et al., 2011) and radial profiling of formation shear velocities (Tang and Patterson, 2010).

Numerous parametric and non-parametric algorithms have been proposed for dispersion analysis and have achieved some limited success. For parametric algorithms, the predictive processing method in frequency domain (Lang et al., 1987; Tang, 1997; Ma et al., 2009) based on Prony’s parameter estimation model requires the number of modes as an input and the Maximum-Likelihood-based methods (Hsu and Baggeroer, 1986) are computationally intensive. On the other hand of non-parametric algorithms, phase minimization or coherency maximization techniques (Nolte and Huang, 1997) and WSS method (Tang and Cheng, 2004) can only identify one mode at a given frequency; non-dispersive processing methods for filtered narrow frequency bands (Rao and Toksoz, 2005) yield a 3D data volume consuming large amounts of memory and computational time. Challenges still remain in developing a new method that preserves processing accuracy with lower computational cost without knowing the number of modes \textit{a priori}. In addition, there is still room for improvement in the resolution and signal-noise-ratio (SNR) of dispersion analysis from all the above-mentioned methods.

In this paper, we introduce a non-parametric spectral estimation method, Amplitude and Phase Estimation method (APES) (Li and Stoica, 1996), which uses a data-dependent (or adaptive) filter (Stoica et al., 1999). This method does not require the number of modes as an input in order to process array sonic data in the f-x domain, and generates a dispersion fitness function diagram from which a set of dispersion curves of different modes can be extracted at each single frequency. We will subsequently process both the synthetic data and field data with this method to demonstrate its applicability and advantages.

APES Theory and Method

Assume that the array sonic spectral data consists of a number of propagating wave modes with noises, as given by

\[ X_\alpha(\omega) = \sum_{p=1}^{P} a_p e^{i\omega k_p d} + v_n, \]  

(1)

where \( P \) is the number of modes, \( k_p \) is the \( p \)th mode’s linear wavenumber in \( x \) direction, \( a_p \) and \( S_p \) are amplitude and slowness of the \( p \)th mode, \( d \) is distance between two adjacent receivers, and \( v_n \) is noise.

An M-tap FIR filter can pass a signal with designated wavenumber component (\( k_i \)) without distortion and filter other wavenumber components and noises. Let

\[ X_\mathbf{w}(\omega) = \left[X_\alpha, X_{\alpha}, ..., X_{\alpha} \right]^T \]  

(2)

stand for the signal vector and

\[ \mathbf{w} = \left[w_1, w_2, ..., w_M \right]^T \]  

(3)
Dispersion analysis with APES method

Indicate the coefficients of the filter, where \([\bullet]^T\) denotes transpose operation. The output of the filter is  
\[ y_k = w^T X_n = X_n^H w, \]  
where \([\bullet]^{-1}\) for conjugate transpose and \([\bullet]^*\) for conjugate operation.

Define  
\[ a(k) \triangleq a_w(k) = \begin{bmatrix} e^{j\omega k_1} \\ \vdots \\ e^{j\omega k_P} \end{bmatrix}, \quad S_w = \begin{bmatrix} \alpha_{1,1} e^{j\omega k_1} \\ \vdots \\ \alpha_{P,1} e^{j\omega k_P} \end{bmatrix}, \quad v_n = \begin{bmatrix} v_n \\ v_{n+1} \\ \vdots \\ v_{n+M-1} \end{bmatrix} \]
and \(A = [a(k_1), a(k_2), \ldots, a(k_P)]\). Then the matrix form of equation (5) is written as  
\[ X_n = AS_w + v_n \in \mathbb{C}^{M \times 1}. \]  
The requirements of our filter output are to be close to a sinusoid with wavenumber \(k_i\), to restrain the interference from other wavenumbers and to reduce the noise gain in the data. So this problem is a constraint optimization problem with objective function and constraint as following,  
\[
\min_{w,a} \left\{ J(w,a) \right\} \triangleq E \left\{ w^H X_n - e^{j\omega k} \right\} \quad \text{s.t.} \quad w^H a(k) = 1. \tag{6}
\]
For real sonic data with \(N\) continuous observations, we regroup them as \(N - M + 1\) series \(\{X_{n-1}, X_n, \ldots, X_{n+M-1}\}\). Then the objective function in (6) can be expressed as an average of these series. Using \(k\) instead of \(k_i\), we get  
\[
\min_{w,a} \left\{ J(w,a) \right\} \triangleq \frac{1}{N-M+1} \sum_{n=M}^{N} \left\{ w^H X_n - e^{j\omega k} \right\} \quad \text{s.t.} \quad w^H a(k) = 1, \tag{7}
\]
where \(k\) is an arbitrary wavenumber and \(a\) is the complex amplitude, correspondingly. Define  
\[ g(k) = \frac{1}{N-M+1} \sum_{n=M}^{N} X_n e^{-j\omega k}, \tag{8} \]
\[ \tilde{R} = \frac{1}{N-M+1} \sum_{n=M}^{N} X_n^H X_n, \tag{9} \]
then,  
\[ J(w,a) = |\alpha - w^H g(k)|^2 + w^H \tilde{R} w - w^H g(k) g^H(k) w. \tag{10} \]

Now the solution to the minimization of equation (10) is  
\[ \hat{a}(k) = w^H g(k) \]  
and the new optimization problem is  
\[
\min_{w} w^H \hat{R} w, \quad \text{s.t.} \quad w^H a(k) = 1, \tag{12}
\]
where  
\[ \hat{R} = \tilde{R} - g(k) g^H(k). \]  
Therefore, the optimum weight vector and amplitude can be obtained as  
\[
\hat{w}_{\text{APES}} = \frac{\hat{a}^{-1}(k) a(k)}{a^H(k) \hat{a}^{-1}(k) a(k)} \tag{14}
\]
\[
\hat{a}(k) = \frac{a^H(k) \hat{a}^{-1}(k) g(k)}{a^H(k) \hat{a}^{-1}(k) a(k)} \quad k \in \left[ -\frac{1}{d}, \frac{1}{d} \right]. \tag{15}
\]

For equation (15), when \(k = k_p, (p = 1, 2, \ldots, P)\), \(\hat{a}(k)\) will go to a local maximum, and at other wavenumbers where \(k \neq k_p\), \(\hat{a}(k)\) will be close to zero. Therefore, using \(\hat{a}(k)\) as a fitness function of dispersion analysis, it will be of much higher resolution and the picked slowness will be more accurate.

The standard workflow of applying the APES method to perform dispersion analysis is described as follows:

- Choose the filter length \(M\).
- Calculate the self-correlation matrix \(\hat{R}\) of the array sonic spectral data \(X\), using equation (9).
- Construct \(g(k)\) and \(\hat{a}(k)\) using equations (8) and (14) for a given wavenumber \(k\).
- Calculate the dispersion fitness function based on equation (15) and pick up the peaks of this function to get the wave modes’ wavenumbers \(k_p\).
- Convert wavenumber \(k_p\) into slowness using \(k_p = \omega S_p\) and obtain the dispersion curves for different modes.

The filter length \(M\) affects the resolution of the dispersion analysis results. Generally, a larger \(M\) entails higher resolution and vice versa. The ridges and peaks obtained from the above procedures usually contain some periodic alias which could be eliminated by post-processing. However, in this work we only present raw processing results from the APES dispersion analysis algorithm and WSS method without any post-processing.

**Synthetic Examples**

Two noise-free synthetic examples are created to illustrate the application of the APES method in array sonic data processing. In both examples, we use dipole measurements operating at a center frequency of 12 kHz. The first example is simulated in a fast formation and second in a slow formation. The formation and borehole fluid properties are listed in Table 1.

<table>
<thead>
<tr>
<th>Formation</th>
<th>(v_s) (m/s)</th>
<th>(v_p) (m/s)</th>
<th>(\rho) (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borehole Fluid</td>
<td>1500</td>
<td>--</td>
<td>1.00</td>
</tr>
<tr>
<td>Fast Formation</td>
<td>4500</td>
<td>2650</td>
<td>2.4</td>
</tr>
<tr>
<td>Slow Formation</td>
<td>2500</td>
<td>1300</td>
<td>1.75</td>
</tr>
</tbody>
</table>

![Table 1: Parameters of borehole fluid and formations](image)
Dispersion analysis with APES method

Assume the source and receivers are ideally aligned in the center of the borehole (diameter = 10 inch). The source is located at the bottom of the tool and the receiver array is placed 9.5 ft above the source with receivers equally spaced at 0.5 ft intervals. The simulated waveforms and the processed frequency spectra (Fig. 1) indicate that both fast and slow formations exhibit multiple modes. In fast formations, several flexural modes of different orders are identified. In slow formations, leaky P mode is also present which is attributed to the high center frequency of dipole source (Fig. 1c and 1d).

Processing results from both methods indicate there are three flexural modes (1st, 2nd, and 3rd order) identified from fast formation measurements and two different modes (1st order flexural mode and leaky P modes) from slow formation measurements. The dispersion fitness function for the 1st order flexural mode processed from the APES method (see Fig. 2a) still has good continuity after 10 kHz although with weak energy while the WSS generates discontinuous fitness function after 10 kHz (see Fig. 2c). Generally, with APES results, the ridge of the fitness function is narrower and more continuous, especially during the overlapped frequency band of two different modes. Moreover, the presence of alias peaks is significantly suppressed in APES method.

Field Example No.1: Soft Formation

The array sonic data in the first field example is acquired in unconsolidated siliciclastic reservoir (soft formation) with a Schlumberger DSI tool which uses 8 receivers equally spaced at 0.5 ft intervals. We process waveform data acquired by P&S monopole source and upper dipole source. The distance from source to the first receiver is 9 ft in monopole case and 11 ft for the dipole case.

Figure 3 compares the dispersion analysis results by APES and WSS methods for the array sonic data acquired from P&S Monopole measurements. Both methods indicate the presence of P and ST modes while there is no shear mode in the waveform data because the formation is soft. Figure 4 compares the dispersion analysis results of array sonic data acquired with dipole measurements by APES and WSS methods. In both monopole and dipole cases, the dispersion fitness function from APES method has higher resolution (narrower ridges) and SNR than that from WSS method.

We first apply Fast Fourier Transformation (FFT) on the two data sets, and then use the APES method to obtain dispersion fitness function as illustrated in Fig. 2a and 2b. For comparison, we process the same data sets with the WSS method and show the results in Fig. 2c and 2d.

Figure 1: Waveforms simulated with dipole measurements operating at a center frequency of 12 kHz excited in (a) fast and (b) slow formations and processed spectra for (c) fast and (d) slow formations.

Figure 2: Comparison of dispersion fitness functions from APES (a and b) and WSS (c and d) methods.

Figure 3: (a) Waveforms, (b) frequency spectra, (c) APES dispersion fitness function, and (d) WSS dispersion fitness function for the array sonic data acquired from P&S Monopole measurements in a soft formation.

Figure 4: Comparison of dispersion fitness functions from APES and WSS methods for array sonic data acquired from P&S Monopole measurements. Both methods indicate the presence of P and ST modes while there is no shear mode in the waveform data because the formation is soft.
Dispersion analysis with APES method

Figure 4: (a) Waveforms, (b) frequency spectra, (c) APES dispersion fitness function, and (d) WSS dispersion fitness function of array sonic data acquired with DSI sonic tool in upper dipole mode in a soft formation.

Field Example No.2: Carbonate Formation

The second field example considers dispersion analysis in a very hard carbonate formation. The waveforms are acquired by a Schlumberger Sonic Scanner tool which uses 13 receivers equally spaced at 0.5 ft intervals. The data sets for processing are acquired by P&S monopole measurements and one of the X-dipole measurements. The distance between source and the first receiver is 11 ft for the monopole case and 8.5 ft for the X-dipole case.

Figure 5 compares the dispersion analysis results by APES and WSS methods for the array sonic data acquired with P&S Monopole measurements. Both methods indicate the presence of P, S, and ST modes. Figure 6 compares the dispersion analysis results by APES and WSS methods for the array sonic data acquired from one of the X-dipole measurements. Similar to the previous analysis, the dispersion fitness functions from APES method are of higher resolution and SNR.

Conclusions

APES method significantly enhances the resolution and SNR of dispersion analysis of borehole array sonic measurements by implementing a data-dependent (adaptive) filter to restrain interferences from other wavenumber components and suppress the noise gain. This method yields dispersion fitness functions that are smoother and more continuous. In addition, APES method overcomes the major limitation of these parametric methods, which is that the number of modes has to be known a priori while outperforming most non-parametric methods in computational efficiency. Also, it can identify dispersion of all modes at each single frequency. Both synthetic and field examples confirm the advantages of APES method in obtaining high-quality dispersion analysis.

Acknowledgments

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