COMBINED INVERSION OF BOREHOLE RESISTIVITY AND SONIC MEASUREMENTS TO ESTIMATE WATER SATURATION, POROSITY AND DRY-ROCK ELASTIC MODULI IN THE PRESENCE OF INVASION

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ABSTRACT

Water saturation, porosity and dry-rock (skeleton) elastic moduli are often evaluated independently using measurements that obey different physical principles. Such an interpretation methodology does not take into account possible deterministic or statistical relationships between different physical measurements when probing the same rock formation. We describe a new inversion-based method that combines resistivity and sonic borehole measurements to estimate the four properties simultaneously. The objective of the combined inversion is to suppress ambiguity in the estimation of in-situ properties thereby improving the accuracy and reliability of the results over traditional methods.

INTRODUCTION

Water saturation, $S_w$, and porosity, $\phi$, play important roles in well-log interpretation and formation evaluation, while dry-rock bulk modulus, $K_d$, and dry-rock shear modulus, $\mu_d$, are indispensable parameters for fluid substitution in the study of seismic properties of rocks. Generally, $S_w$ is computed using Archie’s equation with prior knowledge of $\phi$. Porosity is commonly calculated from other measurements, such as neutron and/or density logs, and is often biased by environmental factors as well as by specific assumptions about matrix and fluid properties. This introduces additional biases in the estimation of water saturation. To reduce such an adverse effect, $\phi$ can be introduced as an additional unknown while evaluating water saturation. However, it is found that resistivity measurements are only sensitive to the product $\phi \cdot S_w$. In other words, resistivity measurements alone can not render separate estimates of $S_w$ and $\phi$. One has to include additional data in the estimation method to circumvent this lack of sensitivity.

For the case of elastic properties of rocks, the implementation of fluid substitution procedures requires specific knowledge about $K_d$ and $\mu_d$. Commonly, $K_d$ is derived from velocity measurements performed on controlled-humidity dried core samples. It can also be calculated using lithology-dependent empirical relationships based on well-log data. The first method is cost prohibitive as it often requires the testing of a significant number of core samples. Likewise, empirical relationships for $K_d$ are often established for specific rocks models within specific pressure regimes and hence cannot be generalized for different lithology transitions and/or depths of burial. One has to exercise caution when applying these relationships in the absence of core measurements.

Advances in numerical modeling techniques make it possible to accurately quantify environmental effects on sonic measurements. This provides the possibility of correcting velocity and density for these effects with an inversion-based method. When velocity and density are
properly estimated, one can use these estimates to calculate $K_d$ and $\mu_d$. Alternatively, one may resort to Gassmann’s equation to estimate $K_d$, which in turn requires knowledge of $S_w$ and $\phi$. Recalling that $\phi^n S_w^n$ can be determined from the inversion of borehole resistivity data, if we further assume that the fluid components are known, then $S_w$ and $\phi$ can be derived from the volume-average equation of density together with knowledge of the product $\phi^n S_w^n$.

Inversion of array-sonic measurements has proven challenging due to both the large data size involved and the inherent strong nonlinearity of the problem. Different inversion methods have been proposed to estimate rock elastic properties from sonic measurements. Travel-time tomography is based upon a high-frequency approximation and utilizes first arrivals; it has been well-accepted by formation-evaluation practitioners due to its efficiency and robustness (Hornby, 1993). Full-wave inverse scattering methods have also been proposed to use the information contained in sonic waveforms and has gained some attention over the past years (Tarantola, 1984, Mora, 1987, and Chi et al., 2004). Recently, Sinha et al. (2005) introduced linear inversion methods to estimate radial variations of shear slowness from flexural and Stoneley dispersions. Their work was based on the extraction of dispersion information together with a linear perturbation technique that assumes small variations of elastic properties from a depth-dependent background.

The inversion method developed in this paper falls into the category of full-wave nonlinear inverse scattering. In so doing, we introduce a modified preconditioned conjugate gradient method to minimize the quadratic cost function and to expedite the inversion. In addition, we use the average trace normalization method to reduce the dependence of waveforms on the amplitude spectrum of the sonic source.

In the following sections, we first introduce the conceptual basis of the proposed inversion method to combine borehole induction and sonic measurements for the in-situ estimation of elastic moduli of rocks. The method is based on the presence of radial zones of invaded and virgin fluid distributions. These distinct zones of fluid saturation provide the necessary measurement sensitivity (and hence degrees of freedom) to estimate $K_d$ and $\mu_d$ as well as porosity and fluid saturation. Subsequently, we appraise the accuracy and reliability of the inversion method on several synthetic examples that include slow and fast sandstone formations. The examples are constructed based on actual field measurements. Even though the combined inversion method is readily applicable to the interpretation of field measurements, in this paper we focus our attention exclusively to noisy synthetic measurements.

**METHOD**

We assume a radial one-dimensional (1D) model to develop the numerical simulation and inversion components of the study. As described in Figure 1, the radial profile of invasion includes a borehole and flushed and virgin zones.

To better quantify the invasion profile, we use array induction and sonic data as the input measurements to the inversion. The assumed induction and sonic tools are AIT-H* and Sonic Scanner*, respectively. Array induction measurements include 16 signals with different radial lengths of investigation. We use only 8 in-phase signals, which exhibit high radial resolution and long-enough radial length of investigation for general problems. Figure 2 shows the configuration of the assumed sonic tool. It provides a combination of monopole and dipole waveforms in the frequency range from 100 Hz to 10 kHz. In total, there are 5 transmitters and 13 receivers, thereby yielding a large data set to describe the mechanical properties of the surrounding formation in great detail. However, in this study we only make use of data acquired with the lower near monopole transmitter. The field excited by a monopole source attenuates slower than that of a dipole source, and hence is more amenable to inversion. In addition, this choice of source has higher radial resolution than using the far monopole source, thereby providing a data set more sensitive to the variation of components of pore-filling fluid included in the invasion profile.

![Figure 1](image-url)  
*Trademark of Schlumberger
We use the numerical-mode-matching method (Chew et al., 1984; Zhang et al., 1995; Zhang et al., 1999) to simulate induction measurements. This is a hybridization of the 1D finite element method in the radial direction with an analytic solution in the axial direction. Generally, this method is used to simulate induction measurements in the presence of axially-symmetric two-dimensional (2D) formations. For our problem, the implementation is relatively simple because there is no reflection and transmission effects are required in the formulation. For the simulation of the sonic tool we use the generalized reflection-transmission method (Chen et al., 1996; Chi and Torres-Verdín, 2004). This is a frequency-wavenumber domain method applicable to the simulation sonic waveforms acquired with a multi-pole source in cylindrically layered elastic media.

As emphasized earlier, in this paper we are concerned with the response of a monopole source. To model time-domain sonic data, the monopole source is driven with a Ricker wavelet excitation. The selected Ricker wavelet, shown in Figure 3, has a central frequency of 8 kHz.

In this study, there are two inverse problems to solve. One is the inversion of array induction data; the other is the inversion of sonic data. Each of the two inverse problems is formulated as the minimization of the quadratic cost function

\[
C(m) = \frac{1}{2} \left( \left\| d(m) - d^s \right\|^2 + \alpha \left\| m - m_g \right\|^2 \right),
\]

where \(d^s\) are the measurements, \(d(m)\) are the synthetic data, \(m\) designates the model parameters, \(m_g\) is the reference model, and \(\alpha\) is the regularization (stabilization) parameter. For the inversion of array induction data, we use a logarithm transformation to describe both data and unknown parameters instead of their original values. Numerical experiments show that this transformation helps to capitalize on the quasi-linearity of the induction problem. The distorted Born iterative method (DBIM) (Chew and Liu, 1994) is then used to solve the problem iteratively. For the inversion of sonic waveforms, we first normalize the data using the method of average trace normalization (Appendix A), by which we reduce the dependence of the data on the source spectrum. We then use a preconditioned conjugate gradient method (PCG) (Appendix B) to solve the inverse problem with the normalized data. It is noted, however, that there exist alternative methods described in the open technical literature which could also be used for sonic inversion (Tarantola, 1984; Hornby, 1993; Chi et al., 2004; Sinha, 2005).

In both DBIM and PCG, the derivatives of the measurements with respect to model parameters are required to provide a feasible search direction at each iteration. These derivatives are computed using the finite-difference approximation

\[
\frac{\partial d}{\partial m} \approx \frac{d(m + \Delta m) - d(m)}{\Delta m},
\]

where \(d\) and \(m\) designate a datum and a model parameter, respectively, \(\Delta m\) is the increment on \(m\), and is equal to 0.01\(m\). For the induction problem, \(d\) is apparent conductivity, and \(m\) is either \(\phi\), \(S_{w0}\), \(S_i\), or \(r_{so}\). Here, \(S_{w0}\) and \(S_i\) designate the water saturation of flushed and virgin zones, respectively, and \(r_{so}\) is the invasion radius. For the sonic inversion problem, \(d\) is pressure, and \(m\) designates either density, compressional velocity, or shear velocity of both flushed and virgin zones, i.e., \(\rho_{so}\), \(\rho_{s}\), \(V_{p,so}\), \(V_{p,s}\), \(V_{s,so}\) and \(V_{s,s}\).

For the inversion of induction measurements, the data set includes 8 apparent conductivity measurements. Model parameters are \(\phi\), \(S_{w0}\), \(S_i\) and \(r_{so}\). Values of \(\phi\), \(S_{w0}\) and \(S_i\) are related to \(\sigma_{so}\) and \(\sigma_i\) through Archie’s equation, namely,
\[
\sigma_{sw} = \frac{1}{a} \phi^n S_w^a \sigma_w, \quad (3a)
\]

and

\[
\sigma_i = \frac{1}{a} \phi^n S_i^a \sigma_w. \quad (3b)
\]

In the above expressions, \( \sigma_w, \sigma_{sw}, \) and \( \sigma_i \) are the electrical conductivities of formation water, flushed zone, and virgin zone, respectively. Moreover, \( a, m, \) and \( n \) are the cementation coefficient, cementation exponent, and saturation exponent, respectively. We assume that \( \sigma_w \) is known from independent measurements.

For the inversion of sonic measurements, input data are the micro seismogram in the time-domain (sonic waveforms). Sonic waveforms are first transformed to the frequency-domain. Then the components ranging from 1 kHz to 10 kHz with spacing of 0.5 kHz are selected as input measurements for the inversion. In total, there are 19 frequency components input to the inversion. The reason why we choose frequency samples this way is to use as much as possible information from the formation while keeping the computational overhead at an affordable level. Because each frequency component consists of both real and imaginary parts, the number of data finally doubles to 38. Model parameters involved are \( \rho_{sw}, \rho_f, V_{p,sw}, V_{p,f}, \)

\( V_{s,sw} \) and \( V_{s,f} \).

We use the multiplicative regularization technique introduced by Habashy and Abubakar (2004) to choose the regularization parameter \( \alpha \) included in the inversion of induction measurements. Moreover, we choose the model \( m \) obtained in the previous step as the reference model \( m_s \) for the current step. For the inversion of sonic measurements, we simply let \( \alpha = 0 \).

From Archie’s equations, we note that it is the product of \( S_w \) and \( \phi \) that is well resolved by the apparent conductivity \( \sigma_{sw} \), not \( S_w \) and \( \phi \) themselves. In fact, any combination of \( S_w \) and \( \phi \) satisfying the two equations

\[
\phi^n S_w^a = a \sigma_{sw} \sigma_w^{-1}, \quad (4a)
\]

and

\[
\left( \frac{S_w}{S_i} \right)^c = \frac{\sigma_{sw}}{\sigma_i}, \quad (4b)
\]

become a minimum of the cost function. These two equations describe a spatial curve in the three-dimensional (3D) space of the triplet \( (\phi, S_w, S_i) \). Therefore, the estimation of \( S_w \) and \( \phi \) is essentially ill-posed. However, the estimation of \( \phi^n S_w^a \) is generally well-posed considering that we have 8 apparent conductivity measurements with increasingly long radial lengths of investigation while the formation exhibits a step profile of invasion.

We first perform the inversion of array induction data, from which we obtain good estimates of \( \phi^n S_w^a \) and \( r_w \). We then fix \( r_w \) and perform the inversion of sonic data. The estimated density is used to calculate \( \phi \). We assume that the pore fluid consists of two phases, either water-oil, or water-gas. According to the volume average equation for density (Smith et al., 2003), one has

\[
\rho = \rho_g + \phi (\rho_{hc} - \rho_g) + \phi S_w (\rho_w - \rho_{hc}), \quad (5)
\]

where \( \rho_g, \rho_w \) are the density of mineral matrix and formation water, respectively, and \( \rho_{hc} \) is either \( \rho_{oil} \) or \( \rho_{gas} \). This equation, augmented with the knowledge of \( \phi^n S_w^a \), yields \( S_w \) and \( \phi \) with a non-linear equation solver. From the estimated value of \( \phi \), one can compute \( k_d \) via (Mavko et al., 2003)

\[
k_d = \frac{K_{sat} \left( \frac{\phi K_s}{K_{fl}} + 1 - \phi \right) - K_o}{\phi \frac{K_{sat} K_s}{K_{fl}} - 1 - \phi}, \quad (6)
\]

where \( K_{sat} \) is the bulk modulus of saturated rock, \( K_s \) is the bulk modulus of the mineral matrix, and \( K_{fl} \) is the bulk modulus of the pore fluid. In Eq. (6), \( K_{fl} \) is computed via Reuss’ average (Smith et al., 2003), namely,

\[
K_{fl}^{-1} = S_w K_w^{-1} + (1 - S_w) K_{hc}^{-1}, \quad (7)
\]

where \( K_w \) is the bulk modulus of the pore-filling water, and \( K_{hc} \) is the bulk modulus of the pore-filling hydrocarbon. The variable \( K_{hc} \) can be either \( K_{oil} \) or \( K_{gas} \). Moreover, \( K_{sat} \) is derived from the estimated
density and velocity by way of the expressions (Mavko et al., 2003)

\[ K = \rho V_p^2 - \frac{4}{3} \mu, \quad \text{(8a)} \]

and

\[ \mu = \rho V_s^2. \quad \text{(8b)} \]

We note that \( \rho, V_p \), and \( V_s \) can be either \( \rho_{\text{m}}, V_{p,\text{m}} \), and \( V_{s,\text{m}} \), or \( \rho, V_p, \) and \( V_s \).

NUMERICAL EXAMPLES

In this study, we discuss the application of the combined inversion method to synthetic data generated for clean sandstones. Table 1 describes the assumed values of density, bulk, and shear moduli of quartz, the matrix mineral of sandstone. Table 2 describes the assumed values of density and bulk modulus of water, oil and gas filling the pore space.

**Table 1. Assumed values of density, and bulk and shear moduli of quartz.**

<table>
<thead>
<tr>
<th>( \rho (\text{g/cm}^3) )</th>
<th>( K (\text{GPa}) )</th>
<th>( \mu (\text{GPa}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.65</td>
<td>37</td>
<td>44</td>
</tr>
</tbody>
</table>

**Table 2. Assumed values of density and bulk modulus of water, oil and gas.**

<table>
<thead>
<tr>
<th>( \rho (\text{g/cm}^3) )</th>
<th>( K (\text{GPa}) )</th>
<th>( \mu (\text{GPa}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.089</td>
<td>2.38</td>
<td>0.67</td>
</tr>
<tr>
<td>0.749</td>
<td>0.67</td>
<td>0.0208</td>
</tr>
<tr>
<td>0.103</td>
<td>0.0208</td>
<td></td>
</tr>
</tbody>
</table>

The formation water conductivity, \( \sigma_w \), and Archie’s constants \( a, m, \) and \( n \) are assumed known from independent measurements. We choose \( \sigma_w = 10 \, \text{S/m} \) and consider the case of clean sandstones with \( a=1, m=2 \) and \( n=2 \). The dry-rock elastic moduli \( K_d \) and \( \mu_d \) are calculated using the porosity relationships (Mavko et al., 2003)

\[ K_d = K_m \left( 1 - \frac{\phi}{\phi_c} \right), \quad \text{(9a)} \]

\[ \mu_d = \mu_m \left( 1 - \frac{\phi}{\phi_c} \right). \quad \text{(9b)} \]

Here, \( K_m \) and \( \mu_m \) are the bulk and shear moduli of the mineral matrix, and \( \phi_c \) is critical porosity. For sandstones, \( \phi_c = 40% \) (Mavko et al., 2003). To model a slow sandstone, we choose \( \phi = 37.5% \). Table 3 lists the resulting values of \( K_d \) and \( \mu_d \).

**Table 3. Assumed values of dry-rock elastic moduli for the fast and slow sandstone formations considered in this paper.**

<table>
<thead>
<tr>
<th>( \phi ) (%)</th>
<th>( K_d ) (GPa)</th>
<th>( \mu_d ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast Sandstone</td>
<td>25</td>
<td>13.875</td>
</tr>
<tr>
<td>Slow Sandstone</td>
<td>37.5</td>
<td>2.3125</td>
</tr>
</tbody>
</table>

Given \( \sigma_w \) and \( \phi, \sigma_m, \) and \( \sigma_l \) are computed using Archie’s equations. They are input to the forward solver together with the assumed values of \( S_{\text{m}}, S_l \) and \( r_{\text{m}} \) to generate a synthetic data set, \( \sigma_a \). With two values of porosity, we obtain two array-induction data sets, shown in Figure 4. In all example cases considered below, the values \( S_{\text{m}}, S_l \) and \( r_{\text{m}} \) are uniformly set to 0.8, 0.2, and 0.25 m, respectively. In addition, borehole radius and borehole conductivity are fixed at 0.1 m and 1 S/m, respectively.

**Figure 4. Synthetic apparent resistivities (reciprocal of apparent conductivity) simulated for two different cases of clean sandstone formations.** The upper and lower panels show apparent resistivities associated with fast and slow sandstones, respectively.

For the inversion of sonic waveforms, we consider four data sets depending on porosity and the type of hydrocarbon filling the pore space of the rock. We first compute \( \rho_{\text{m}}, \rho_l \) and \( \rho \), using Eq. (5), then compute \( K_{\text{sat},\text{m}}, K_{\text{sat},l} \) using Eq. (6), finally compute \( V_{p,\text{m}}, V_{p,l}, V_{s,\text{m}} \), and \( V_{s,l} \) using Eqs. (8a) and (8b). Table 5 describes the computed parameters. Subsequently, we perform numerical simulation to calculate four sets of sonic seismograms and their corresponding frequency spectra, shown in Figure 5.

**Figure 5.**

We first perform the inversion on the two resistivity induction data sets contaminated with different levels of noise.
Table 4. Summary of results obtained from the inversion of array-induction data.

<table>
<thead>
<tr>
<th></th>
<th>Noise-free</th>
<th>ε = 2%</th>
<th>ε = 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fast sandstone formation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>φ</td>
<td>0.293</td>
<td>0.289</td>
<td>0.277</td>
</tr>
<tr>
<td>S_w</td>
<td>0.684</td>
<td>0.661</td>
<td>0.653</td>
</tr>
<tr>
<td>S_r</td>
<td>0.171</td>
<td>0.175</td>
<td>0.186</td>
</tr>
<tr>
<td>φS_w</td>
<td>0.200</td>
<td>0.191</td>
<td>0.180</td>
</tr>
<tr>
<td>φS_r</td>
<td>0.050</td>
<td>0.051</td>
<td>0.051</td>
</tr>
<tr>
<td>r_i</td>
<td>0.250</td>
<td>0.262</td>
<td>0.279</td>
</tr>
<tr>
<td><strong>Slow sandstone formation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>φ</td>
<td>0.404</td>
<td>0.397</td>
<td>0.388</td>
</tr>
<tr>
<td>S_w</td>
<td>0.743</td>
<td>0.738</td>
<td>0.729</td>
</tr>
<tr>
<td>S_r</td>
<td>0.186</td>
<td>0.191</td>
<td>0.198</td>
</tr>
<tr>
<td>φS_w</td>
<td>0.300</td>
<td>0.293</td>
<td>0.283</td>
</tr>
<tr>
<td>φS_r</td>
<td>0.075</td>
<td>0.076</td>
<td>0.077</td>
</tr>
<tr>
<td>r_i</td>
<td>0.250</td>
<td>0.257</td>
<td>0.266</td>
</tr>
</tbody>
</table>

For all cases, we initialize the inversion with values of φ, S_w, and S_r equal to 0.45, 0.6, and 0.4, respectively. Both borehole radius and borehole conductivity are taken as known parameters and are not included in the inversion. When necessary, noise is added as follows:

\[ d \leftarrow d \left(1 + \beta \right), \]  \hspace{1cm} (10)

where \( d \) is any of the eight apparent conductivity measurements, and \( \beta \) is the noise level, for which we choose the two values of 2% and 5%.

Table 4 summarizes the corresponding inversion results. We note that the estimates of \( S_w \) and \( S_r \) are not good even in the presence of noise-free data. However, we find that the product of \( \phi \) and \( S_w \) or \( S_r \) is close to the true value. In fact, with different initial values of \( \phi \), we arrive at different values for \( S_w \) and \( S_r \), but \( \phi S_w \) and

![Figure 5. Synthentic sonic waveforms and their frequency spectra associated with four different formation models. (a) and (b) are waveforms for the fast oil- and gas-bearing sandstone, respectively. (c) and (d) are waveforms for the slow oil- and gas-bearing sandstone, respectively.](image)

![Figure 6. Space of plausible porosity-water saturation solutions for the case of a fast sandstone formation. All feasible solutions reside along the curve extending from (0.2, 1, 0.25) to (1, 0.2, 0.05) in the space of triplets \((\phi, S_w, \sigma)\).](image)
ϕS remain constant. In these cases, the product ϕS is easily obtained because of our previous assumption that that m = n = 2.

Figure 6 shows possible solution of ϕ, Sωx, and Sx to the inversion of noise-free data for the case of the fast sandstone formation. Any point along the 3D curve is a solution to the inversion. This behavior emphasizes our previous argument about the non-uniqueness of the determination of ϕ, Sωx, and Sx using induction measurements alone. The estimate of rωx is generally

![Graph](a)

![Graph](b)

![Graph](c)

**Figure 7.** Evolution of data misfit as a function of iteration for the case of array-induction data acquired in a fast sandstone formation. (a) shows the evolution of the RMS difference between simulated and measured array-induction data. (b) shows the evolution of the inverted parameters as a function of iteration. (c) shows the misfit between simulated and measured array-induction data at the end of the minimization.

![Graph](a)

![Graph](b)

![Graph](c)

**Figure 8.** Evolution of data misfit as a function of iteration for the case of array-induction data acquired in a slow sandstone formation. (a) shows the evolution of the RMS difference between simulated and measured array-induction data. (b) shows the evolution of the inverted parameters as a function of iteration. (c) shows the misfit between simulated and measured array-induction data at the end of the minimization.
good. In the presence of noise-free data, \( r_{wo} \) is close to the true values. Figures 7 and 8 describe the iteration history of the minimization procedure. We note that in all example cases no more than 15 iterations are needed to arrive at the results shown in Table 4.

We then input the values of \( r_{wo} \) obtained from the inversion of induction data to the inversion of sonic waveforms. For the two cases of fast formation, the values of \( r_{wo} \) are 0.25, 0.262, and 0.279, respectively, while for the two cases of slow formation, the values are 0.25, 0.257, and 0.266, respectively. Table 6 describes the values of \( \rho \), \( V_p \) and \( V_s \) used to initialize the inversion. We note that these values are not far from their respective true values. In practice, such a choice of initial values is feasible because fairly good estimates of \( V_p \) and \( V_s \) can be obtained via slowness-time coherence processing or dispersion analysis of sonic waveforms, while good estimates of \( \rho \) can be obtained from neutron and/or density logs. Actually, the initial values of \( \rho \), \( V_p \) and \( V_s \) obtained via these methods can be better than the ones used here, especially those included in the first row of the fast oil-bearing case. When necessary, different levels of noise are added to the spectrum of sonic data (\( p_{ijp}^{\text{true},\text{obs}} \)) as done for induction data before initiating the inversion procedure.

Table 7 lists the inverted values of \( \rho \), \( V_p \) and \( V_s \). Again, as in the inversion of induction data for \( \phi_{s,x} \) and \( r_{wo} \), these values converge to their respective true values while the level of noise added to the data becomes gradually small. Close inspection of the results indicates that, in general, the estimates of \( V_s \) are the best among all the inverted properties. The accuracy of the inverted values of \( \rho \) is lower than the accuracy of the inverted values of \( V_p \) and \( V_s \); the largest relative error of 6.7% is observed for \( \rho \) in the fast oil-bearing case when the noise level is 5%. Relative errors for other estimates are below 5% under the same conditions. Figures 9 through 12 shows details of the inversion procedure where we observe that the PCG approach works equally well for both fast and slow formation cases, thereby providing confidence in using the inversion results for the computation of porosity and dry-rock elastic moduli.

After completing the inversion of sonic waveforms, we turn our attention to Eqs. (5), (8a) and (8b) to calculate \( \phi \), \( K_{sat} \) and \( u_d \). Subsequently, we use \( \phi \), \( K_{sat} \) to calculate \( K_d \) via Eq. (6).

Figure 9. Evolution of data misfit as a function of iteration for the case of array-sonic data acquired in a fast sandstone formation. (a) shows the evolution of the RMS difference between simulated and measured array-sonic data. (b) shows the evolution of the inverted parameters as a function of iteration. (c) shows the misfit between simulated and measured array-sonic data at the end of the minimization.
Figure 10. Evolution of data misfit as a function of iteration for the case of array-sonic data acquired in a fast gas-bearing sandstone formation. (a) shows the evolution of the RMS difference between simulated and measured array-sonic data. (b) shows the evolution of the inverted parameters as a function of iteration. (c) shows the misfit between simulated and measured array-sonic data at the end of the minimization.

Figure 11. Evolution of data misfit as a function of iteration for the case of array-sonic data acquired in a slow oil-bearing sandstone formation. (a) shows the evolution of the RMS difference between simulated and measured array-sonic data. (b) shows the evolution of the inverted parameters as a function of iteration. (c) shows the misfit between simulated and measured array-sonic data at the end of the minimization.
We note that the inverted property values in the flushed and virgin zones can both be used to calculate \( \phi, \ K_d \) and \( u_d \), whereupon we have two values for each of them although they are both single-valued. As a final result, we can use either of them depending on their confidence levels or else use their average.

Table 8 summarizes the final inversion results. In general, the estimate of \( K_d \) is very good in all cases including noise-contaminated data sets. From Eq. (8b), we know that

\[
\frac{\Delta u_d}{u_d} = \frac{\Delta \rho}{\rho} + 2 \frac{\Delta V}{V_a}.
\]  

(11)

Close examination of the changes of \( \rho \) and \( V_a \) with noise level indicates that \( \rho \) and \( V_a \) vary in opposite directions in all cases. Therefore, errors in the two variables caused by the presence of noise tend to cancel each other according to the above equation. This behavior explains why the estimate of \( K_d \) is close to the true value even in the presence of very noise data sets.

Examination of the inverted values of \( \phi \) indicate that, on occasion, the estimates are insensitive to the level of noise, although sometimes they are also greatly affected by the presence of noise. Sensitivity analysis via Eq. (6) shows that

\[
\Delta \phi = \frac{1}{\rho_w - \rho_g} \Delta \rho \frac{\rho_w - \rho_{wc}}{\rho_{wc} - \rho_g} \Delta (\phi S_w).
\]  

(12)

Obviously, in oil-bearing formations, the contribution from the second term on the right-hand side of Eq. (12) is comparatively small, hence \( \Delta \phi \approx \Delta \rho \left( \frac{\rho_w - \rho_g}{\rho_{w0} - \rho_g} \right) \).

That is, the error in \( \phi \) is mainly determined by that of \( \rho \). In the three fast oil-bearing cases, we note that \( \rho_w \) is close to its true value. Accordingly, \( \phi_{w0} \) is very close to 0.25, the true value. The corresponding estimate of \( \phi_t \) is not good, whereas the error in the estimate of \( \phi_t \) is correspondingly large.

However, for cases of gas-bearing formations the contribution from the second term of Eq. (12) becomes significant and cannot be ignored. When the variations of \( \rho \) and \( \phi S_w \) with noise level are in line with each other, the corresponding errors tend to cancel each other (see the estimate of \( \phi_{w0} \) in the two gas-bearing cases). When the two variations are opposite to each other the error in \( \phi \) becomes large. Theoretically, this behavior should be observed in the estimate of \( \phi_t \) obtained for the two gas-bearing cases. However, from Table 4, we observe that the variation of \( \phi S_w \) is significantly small, hence its contribution is negligible, and the error in \( \phi_t \) is basically determined by that of \( \rho \).

Comparison of errors in the estimates of \( \phi_t \) and \( \rho_t \) obtained for the two gas-bearing cases shows that they do follow each other with a change of noise level.

The error in the estimate of \( S_w \) is closely related to that of \( \phi S_w \) and \( \phi \), namely,

\[
\frac{\Delta S_w}{S_w} = \frac{\Delta (\phi S_w)}{\phi S_w} - \frac{\Delta \phi}{\phi}.
\]  

(13)

The analysis of the variation of \( S_w \) is similar to that of \( \phi \). Accordingly, when changes of \( \phi S_w \) and \( \phi \) are in the same direction, the error in \( S_w \) becomes small; otherwise, the error in \( S_w \) is large. However, when \( \phi S_w \) is significantly small, the error is mainly controlled by that of \( \phi \). Comparison of the columns of \( \phi \) and \( S_w \) shows that the two properties exhibit exactly the opposite behavior, thereby proving our argument. For \( S_w \), given that the variation of \( \phi \) and \( S_w \) are opposite in this region, we observe that the estimate of \( S_w \) is generally better than that of \( S_t \).

We also note that the values of \( S_w \) and \( \phi \) obtained with the combined inversion approach are more accurate than those estimated with the induction measurements alone. This observation indicates that the inclusion of information from sonic inversion is critical to suppress ambiguity in the inversion of \( S_w \) and \( \phi \).

The sensitivity analysis of \( K_d \) is complex due to the nonlinear dependence of this parameter on \( \phi S_w \) and \( \phi \). In general, the estimate for the case of fast formations is better than that of slow formations, and it is better in gas-bearing cases than in oil-bearing cases.

**CONCLUSIONS**

We developed a new method for the simultaneous estimation of water saturation, porosity and dry-rock elastic moduli. The estimation effectively combines the information content available in both array-induction measurements and sonic waveforms acquired in the presence of a step profile of radial mud-filtrate.
invasion. Moreover, the estimation enforces a deterministic relationship between common formation properties included in Biot-Gassmann’s fluid-substitution equations and Archie’s saturation-resistivity equations.

Application of the combined inversion to noisy synthetic data sets confirms that the method provides reliable and accurate estimates of porosity, water saturation, and dry-rock elastic moduli for cases of both fast and slow formations which can be either oil- or gas-bearing. In general, the estimate of dry-rock shear modulus is accurate in all cases even in the presence of 5% measurement noise. Also, it was found that the estimate of dry-rock shear modulus in fast formations was generally better in slow formations than in fast formations. Estimates of water saturation and porosity exhibit a desirable behavior in the presence of noisy measurements manifested by a decrease of accuracy with an increase in the level of measurement noise.

The inclusion of a density estimate yielded by the inversion of sonic data provides an independent relationship which is necessary to reduce non-uniqueness in the determination of water saturation and porosity from induction data. On the other hand, induction data provide the sensitivity to water saturation and porosity necessary to obtain reliable estimates of dry-rock elastic moduli, which otherwise would be difficult to estimate given that sonic data are less sensitive than induction data to variations of porosity and fluid saturation.

In the future, we will investigate the applicability of the combined inversion method to cases of low-porosity sandstone formations as well as carbonate formations. For the cases of shaly-sand and shale-laminated formations, we anticipate the use of different deterministic relationships between porosity, saturation, and dry-rock elastic moduli that can account for the presence of shale. New developments will be reported in a forthcoming paper.

ACKNOWLEDGEMENTS

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REFERENCES

Chen, X., Y. Quan, and J. M. Harris, 1996, Seismogram synthesis for radially layered media using the

Figure 12. Evolution of data misfit as a function of iteration for the case of array-sonic data acquired in a slow gas-bearing sandstone formation. (a) shows the evolution of the RMS difference between simulated and measured array-sonic data. (b) shows the evolution of the inverted parameters as a function of iteration. (c) shows the misfit between simulated and measured array-sonic data at the end of the minimization.


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Carlos Torres-Verdin received a Ph.D. degree in Engineering Geoscience from the University of California, Berkeley, in 1991. During 1991–1997 he held the position of Research Scientist with Schlumberger-Doll Research. From 1997–1999, he was Reservoir Specialist and Technology Champion with YPF (Buenos Aires, Argentina). Since 1999, he has been with the Department of Petroleum and Geosystems Engineering of The University of Texas at Austin, where he currently holds the position of...
Associate Professor. He conducts research on borehole geophysics, well logging, formation evaluation, and integrated reservoir characterization. Torres-Verdín has served as Guest Editor for Radio Science, and is currently a member of the Editorial Board of the Journal of Electromagnetic Waves and Applications, and an associate editor for Petrophysics (SPWLA) and the SPE Journal. He is co-recipient of the 2003 and 2004 Best Paper Award by Petrophysics, and is recipient of SPWLA’s 2006 Distinguished Technical Achievement Award.

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**APPENDIX A: Average Trace Normalization Method**

The average trace normalization method (Zhou and Greenhalgh, 2003) is designed to reduce the dependence of the inversion on the source spectrum when the latter is uncertain. Let $N_\omega$ and $N_R$ denote the number of frequencies and the number of receiver stations, respectively. The original measurement data and synthetic data are denoted as $p_{ij,obs}^{\text{syn}}$ and $p_{ij}^{\text{syn}}$, respectively, where the index $i$ designates the specific frequency sample and ranges from 1 to $N_\omega$, and the index $j$ designates the receiver station and ranges from 1 to $N_R$. If the source spectrum is $S(\omega)$, one has

$$p_{ij}^{\text{syn}} = S(\omega_j)G(\omega_j, z_{r,j}) = S_i G_{ij},$$

where $G_{ij}$ is the response of a unit source at the $j$-th receiver station when the frequency is $\omega_j$. To eliminate the effect of the source spectrum on the data, the average trace normalization method redefines the input data as (Zhou and Greenhalgh, 2003)

$$d_{ij}^{\text{norm}} = \frac{1}{N_R} \sum_j h_{ij} G_{ij} = \frac{G_{ij}}{G_i}.$$

The same transformation is applied to the measurements, that is,

$$d_{ij,obs}^{\text{norm,obs}} = \frac{1}{N_R} \sum_j p_{ij,obs}^{\text{norm,obs}}.$$

Subsequently, the inversion is implemented on the transformed (normalized) data.

Derivatives of the normalized data with respect to model parameters are given by

$$\frac{\partial p_{ij}^{\text{norm}}}{\partial m_k} = \frac{1}{G_i} \left[ \frac{\partial G_{ij}}{\partial m_k} - \frac{G_{ij}}{G_i} \frac{1}{N_R} \sum_j \frac{\partial G_{ij}}{\partial m_k} \right].$$

Experience shows that the increase in the total computer cost when calculating the above derivatives (with respect to the calculation of derivatives from the original data) is minimal. Therefore, the transformation of data is not taxing on the computation of Fréchet derivatives required by the inversion.
APPENDIX B: Preconditioned Conjugate Gradient Method

The preconditioned conjugate gradient method used in the paper is adapted from the algorithm suggested by Mora (Mora, 1987). It is described as follows:

For $n = 1$ to $\infty$, $C_n = C(m_n)$, exit if converged,

\[ d_n = \nabla d(m_n), \quad \Delta d_n = d_n - d^*, \quad \Delta m_n = m_n - m_* , \]

\[ g_n = \nabla d^T (m_n) \Delta d_n + \lambda \Delta m_n , \]

\[ p_n = L_n^{-1} g_n , \quad L_n = \text{diag} \left\{ \frac{\partial d}{\partial m_j} \right\} , \]

\[ c_n = p_n + \beta_n c_{n-1} , \quad c_i = p_i , \]

\[ \beta_n = \max \left\{ \beta_R^n , 0 \right\} , \quad \beta_R^n = \frac{p_n^T ( g_n - g_{n-1} )}{p_{n-1}^T ( g_{n-1} )} , \]

\[ m_{n+1} = m_n - \lambda_n c_n , \]

\[ \lambda_n = \frac{c_n^T g_n}{c_n^T \nabla d^T (m_n) c_n + \alpha c_n^T c_n} , \]

\[ C_{n+1} = C(m_{n+1}) ; \]

if $C_{n+1} > C_n$, $c_n = p_n$, solve for $\lambda_n$ such that

\[ C(m_n - \lambda_n g_n) - C_n < -\gamma_2 p_n^T g_n . \]

Here, $L$ is a point-Jacobi preconditioner. The Polak-Ribiere method is used to choose the conjugate direction $c_n$. A conjugate direction is not necessarily a descent direction; therefore, when $C_{n+1} > C_n$, we restart the procedure (by letting $c_n = p_n$) to ensure the proper enforcement of Armijo’s rule (Kelly, 1999).

Table 5. Values of density, compressional velocity, and shear velocity assumed for the synthetic models considered in this paper.

<table>
<thead>
<tr>
<th>CASE</th>
<th>$\rho_o$ (g/cm$^3$)</th>
<th>$\rho_t$ (g/cm$^3$)</th>
<th>$V_{p,o}$ (m/s)</th>
<th>$V_{p,t}$ (m/s)</th>
<th>$V_{s,o}$ (m/s)</th>
<th>$V_{s,t}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast oil-bearing</td>
<td>2.243</td>
<td>2.192</td>
<td>4126.48</td>
<td>4112.04</td>
<td>2712.39</td>
<td>2743.76</td>
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<td>Fast gas-bearing</td>
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<td>2.063</td>
<td>4037.38</td>
<td>4172.91</td>
<td>2732.13</td>
<td>2828.39</td>
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<tr>
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<td>1.963</td>
<td>2152.80</td>
<td>1988.00</td>
<td>1161.30</td>
<td>1183.72</td>
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<td>1.991</td>
<td>1.769</td>
<td>1766.75</td>
<td>1847.88</td>
<td>1175.35</td>
<td>1246.88</td>
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</table>

Table 6. Values of density, compressional velocity, and shear velocity used to initialize the inversion examples considered in this paper.

<table>
<thead>
<tr>
<th>CASE</th>
<th>$\rho_o$ (g/cm$^3$)</th>
<th>$\rho_t$ (g/cm$^3$)</th>
<th>$V_{p,o}$ (m/s)</th>
<th>$V_{p,t}$ (m/s)</th>
<th>$V_{s,o}$ (m/s)</th>
<th>$V_{s,t}$ (m/s)</th>
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</thead>
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<td>Fast oil-bearing</td>
<td>2.3</td>
<td>2.3</td>
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<td>4500</td>
<td>3100</td>
<td>3100</td>
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<tr>
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<td>2.3</td>
<td>4200</td>
<td>4200</td>
<td>2900</td>
<td>2900</td>
</tr>
<tr>
<td>Slow oil-bearing</td>
<td>2.1</td>
<td>2.1</td>
<td>2200</td>
<td>2200</td>
<td>1200</td>
<td>1200</td>
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<tr>
<td>Slow gas-bearing</td>
<td>2.0</td>
<td>2.0</td>
<td>1900</td>
<td>1900</td>
<td>1300</td>
<td>1300</td>
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</table>
### Table 7. Estimated values of density, compressional velocity, and shear velocity for the inversion examples considered in this paper.

<table>
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<tr>
<th>CASE</th>
<th>Noise-free</th>
<th>$\varepsilon = 2%$</th>
<th>$\varepsilon = 5%$</th>
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<th>$\varepsilon = 2%$</th>
<th>$\varepsilon = 5%$</th>
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</thead>
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<tr>
<td>Fast oil-bearing</td>
<td>$\rho_{o0}$ (g/cm$^3$)</td>
<td>2.243</td>
<td>2.192</td>
<td>4126.44</td>
<td>4111.92</td>
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<td>$\rho_{t0}$ (g/cm$^3$)</td>
<td>2.232</td>
<td>2.127</td>
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<td>4162.83</td>
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<td>$V_{p,o}$ (m/s)</td>
<td>2.223</td>
<td>2.043</td>
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<td>2.063</td>
<td>4037.41</td>
<td>4173.03</td>
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<td>4151.76</td>
<td>2733.46</td>
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<td>$V_{s,t}$ (m/s)</td>
<td>2.196</td>
<td>1.974</td>
<td>4080.22</td>
<td>4148.03</td>
<td>2725.63</td>
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<td>Fast gas-bearing</td>
<td>$\rho_{o0}$ (g/cm$^3$)</td>
<td>2.039</td>
<td>1.963</td>
<td>2152.78</td>
<td>1988.00</td>
<td>1161.30</td>
</tr>
<tr>
<td></td>
<td>$\rho_{t0}$ (g/cm$^3$)</td>
<td>1.991</td>
<td>1.769</td>
<td>1766.76</td>
<td>1847.90</td>
<td>1175.34</td>
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<td>1.953</td>
<td>2189.40</td>
<td>2010.28</td>
<td>1160.79</td>
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<td>$V_{s,t}$ (m/s)</td>
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<td>Slow oil-bearing</td>
<td>$\rho_{o0}$ (g/cm$^3$)</td>
<td>13.874</td>
<td>13.878</td>
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<td>16.500</td>
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<td>$\rho_{t0}$ (g/cm$^3$)</td>
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<td>14.436</td>
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<td>$\phi_{o}$ (%)</td>
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<td>0.800</td>
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<tr>
<td></td>
<td>$\phi_{t}$ (%)</td>
<td>2.004</td>
<td>0.250</td>
<td>0.800</td>
<td>0.200</td>
<td>0.250</td>
</tr>
<tr>
<td>Slow gas-bearing</td>
<td>$\rho_{o0}$ (g/cm$^3$)</td>
<td>14.698</td>
<td>14.121</td>
<td>16.259</td>
<td>0.257</td>
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<td>$\rho_{t0}$ (g/cm$^3$)</td>
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<td>13.272</td>
<td>16.059</td>
<td>0.251</td>
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<td></td>
<td>$\phi_{o}$ (%)</td>
<td>2.004</td>
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<td>0.800</td>
<td>0.200</td>
<td>0.250</td>
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<tr>
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<td>$\phi_{t}$ (%)</td>
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<td>0.800</td>
<td>0.200</td>
<td>0.250</td>
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<tr>
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<td>$S_{o}$ (%)</td>
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<tr>
<td></td>
<td>$S_{t}$ (%)</td>
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<td>0.250</td>
<td>0.800</td>
<td>0.200</td>
<td>0.250</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\phi}$ (%)</td>
<td>2.004</td>
<td>0.250</td>
<td>0.800</td>
<td>0.200</td>
<td>0.250</td>
</tr>
<tr>
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<td>$K_{d,o}$ (GPa)</td>
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<td>2.312</td>
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<td>Slow gas-bearing</td>
<td>$\mu_{d,o}$ (GPa)</td>
<td>2.313</td>
<td>2.313</td>
<td>2.750</td>
<td>0.375</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td>$\mu_{d,t}$ (GPa)</td>
<td>3.919</td>
<td>2.479</td>
<td>2.652</td>
<td>0.409</td>
<td>0.364</td>
</tr>
</tbody>
</table>

### Table 8. Estimated values of water saturation, porosity, and dry-rock elastic moduli for the inversion examples considered in this paper.

<table>
<thead>
<tr>
<th>CASE</th>
<th>Noise-free</th>
<th>$\varepsilon = 2%$</th>
<th>$\varepsilon = 5%$</th>
<th>Noise-free</th>
<th>$\varepsilon = 2%$</th>
<th>$\varepsilon = 5%$</th>
</tr>
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<tr>
<td>Fast oil-bearing</td>
<td>$K_{d,o}$ (GPa)</td>
<td>13.873</td>
<td>16.500</td>
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<td>0.250</td>
<td>0.800</td>
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<td>$K_{d,t}$ (GPa)</td>
<td>14.082</td>
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<tr>
<td></td>
<td>$S_{t}$ (%)</td>
<td>2.004</td>
<td>0.250</td>
<td>0.800</td>
<td>0.200</td>
<td>0.250</td>
</tr>
<tr>
<td></td>
<td>$\tilde{\phi}$ (%)</td>
<td>2.004</td>
<td>0.250</td>
<td>0.800</td>
<td>0.200</td>
<td>0.250</td>
</tr>
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<td>$K_{d,o}$ (GPa)</td>
<td>2.735</td>
<td>2.746</td>
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<td>0.380</td>
<td>0.752</td>
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<td>2.199</td>
<td>2.739</td>
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<td>0.380</td>
<td>0.752</td>
</tr>
<tr>
<td>Slow gas-bearing</td>
<td>$\mu_{d,o}$ (GPa)</td>
<td>2.313</td>
<td>2.750</td>
<td>0.375</td>
<td>0.375</td>
<td>0.800</td>
</tr>
<tr>
<td></td>
<td>$\mu_{d,t}$ (GPa)</td>
<td>1.972</td>
<td>2.745</td>
<td>0.390</td>
<td>0.380</td>
<td>0.752</td>
</tr>
</tbody>
</table>