Quantitative comparison of processes of oil- and water-based mud-filtrate invasion and corresponding effects on borehole resistivity measurements

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ABSTRACT

Some laboratory and qualitative studies have documented the influence of water-based mud (WBM) filtrate invasion on borehole resistivity measurements. Negligible work, however, has been devoted to studying the effects of oil-based mud (OBM) filtrate invasion on well logs and the corresponding impact on the estimation of petrophysical properties. We quantitatively compare the effects of WBM- and OBM-filtrate invasion on borehole resistivity measurements. We simulate the process of mud-filtrate invasion into a porous and permeable rock formation assuming 1D radial distributions of fluid saturation and fluid properties while other petrophysical properties remain constant. To simulate the process of mud-filtrate invasion, we calculate a time-dependent flow rate of OBM-filtrate invasion by adapting the available formulation of the physics of WBM-filtrate invasion. This approach includes the dynamically coupled effects of mud-cake growth and multiphase filtrate invasion. Simulations are performed with a commercial adaptive-implicit compositional formulation that enables the quantification of effects caused by additional components of mud-filtrate and native fluids. The formation under analysis is 100% water saturated and is invaded with a single-component OBM. Subsequently, we perform simulations of WBM filtrate invading the same formation assuming that it is hydrocarbon bearing, and compare the results to those obtained in the presence of OBM. At the end of this process, we invoke Archie’s equation to calculate the radial distribution of electrical resistivity from the simulated radial distributions of water saturation and salt concentration and compare the effects of invasion on borehole resistivity measurements acquired in the presence of OBM and WBM. Simulations confirm that the flow rate of OBM-filtrate invasion remains controlled by the initial mud-cake permeability and formation petrophysical properties, specifically capillary pressure and relative permeability. Moreover, WBM causes radial lengths of invasion 15%–40% larger than those associated with OBM as observed on the radial distributions of electrical resistivity. It is found also that, in general, flow rates of WBM-filtrate invasion are higher than those of OBM-filtrate invasion caused by viscosity contrasts between OBM filtrate and native fluids, which slow down the process of invasion. Such a conclusion is validated by the marginal variability of array-induction resistivity measurements observed in simulations of OBM invasion compared with those of WBM invasion.

INTRODUCTION

During the process of drilling wells for hydrocarbon exploration and production, drilling fluids sustain a pressure higher than that of formation fluids (pore pressure), thereby resulting in radial pressure overbalance. In porous and permeable rocks, overbalance pressure is responsible for mud-filtrate invasion. Solids contained in the mud deposit on the borehole wall, thereby developing a mud cake, whereas mud filtrate radially displaces in-situ fluids (water or hydrocarbons) away from the borehole wall (Dewan and Chenevert, 1993, 2001). Mud and rock properties control the dynamic growth of mud cake and time evolution of the process of fluid displacement (Wu et al., 2004). The interplay between mud and formation properties occurs with oil-based mud (OBM) and water-based mud (WBM), whereby the spatial distributions of fluids within the invaded formation affect measurements acquired with well-logging instruments, particularly resistivity measurements (La Vigne et al., 1997; George et al., 2004; Wu et al., 2004).
Borehole induction resistivity measurements commonly are acquired in the presence of OBM and fresh WBM. In the case of OBM, invading mud filtrate is miscible with formation oil. Such a fluid miscibility condition results in changes of fluid density and viscosity, thereby altering the apparent oil-phase mobility in the near-wellbore region (Malik et al., 2008; Salazar et al., 2007). On the other hand, WBM-filtrate invasion results in salt mixing between mud filtrate and connate water, which often gives rise to a large resistivity contrast between the near-wellbore region and the undisturbed (virgin) zone (George et al., 2004). Therefore, it becomes imperative to model accurately the effect of OBM and WBM on the invasion process and, subsequently, on borehole resistivity measurements acquired some time after the onset of invasion.

Dewan and Chenevert (2001) performed laboratory experiments with WBM to study mud-cake buildup and mud-filtrate invasion. Wu et al. (2004, 2005) numerically implemented the same theory and performed sensitivity analyses of mud cake and rock properties to the time evolution of the flow rate of invasion and mud-cake buildup. Warner and Rathmell (1997) performed laboratory experiments on rock-core samples invaded with OBM filtrate to determine the mechanisms controlling the rate of filtration. They concluded that mud cake controls the pressure gradient between the mud and rock sample, thereby controlling the radial length of invasion.

The objective of this paper is to compare quantitatively the processes of OBM- and WBM-filtrate invasion. For that purpose, and without loss of generality, we focus attention exclusively on the case of radial variations of fluid saturation and fluid properties while keeping porosity, permeability, relative permeability, and capillary pressure constant. Even though this approach does not account for the effect of gravity and permeability anisotropy, it provides adequate reference to compare the effects of two types of muds on the time evolution of the process of invasion.

We implement a 1D simulation method similar to that advanced by Wu et al. (2005) into a commercial multiphase and multicomponent reservoir simulator. This simulation method calculates the time evolution of the flow rate of mud-filtrate invasion and mud-cake thickness for OBM- and WBM-filtrate invasion, and accounts for the effects of capillary pressure and relative permeability. The simulation of OBM-filtrate invasion also considers the mixing between invading and native oil when the latter is present (Malik et al., 2008), or else assumes immiscible fluid displacement in the presence of a water-bearing rock formation.

Initially, OBM is regarded as a single-component hydrocarbon and, subsequently, as an emulsion formed by oil, surfactants, and water (Bourgoyne Jr. et al., 1986) invading an oil-bearing sand. Alteration of wettability caused by the presence of surfactants in the OBM (Van et al., 1988; Yan and Sharma, 1989) is simulated by changing capillary pressure and relative permeability in response to changes of component concentration (Carlson, 2003). On the other hand, the immiscible fluid-displacement process associated with WBM filtrate invading hydrocarbon-bearing sands includes the effect of salt mixing between filtrate and connate water.

For the purpose of illustration, the study focuses on a synthetic clastic formation invaded with either OBM or WBM filtrate for a range of assumed petrophysical and fluid properties. Comparative sensitivity analyses to mud and rock petrophysical properties for OBM and WBM invasion allow us to quantify the influence of parameters affecting the time-dependent flow rate of invasion. We calculate radial distributions of water saturation assuming transient, constant, or step flow rates to account for the effect of spurt loss at the onset of invasion. Finally, we simulate array-induction resistivity measurements from the radial distributions of resistivity in the presence of OBM- and WBM-filtrate invasion at different conditions of native fluid saturation.

The objective is to assess the influence of fluid and petrophysical properties on array-induction resistivity measurements for the two types of invasion processes. For the case of OBM filtrate invading the formation, we postulate examples of 100% water-saturated, partially oil-saturated, and partially gas-saturated conditions. In the case of WBM filtrate invading the formation, we consider partially oil-saturated and partially gas-saturated conditions. Simulations of borehole induction resistivity measurements reveal different variability of apparent-resistivity curves with multiple radial lengths of investigation for each case. These analyses shed light into the petrophysical and fluid properties affecting the radial distribution of fluids in the near-wellbore region and provide a general quantitative framework for interpretation of borehole electrical resistivity measurements in terms of fluid and petrophysical properties.

**NUMERICAL SIMULATION OF THE PROCESS OF MUD-FILTRATE INVASION**

We approach the simulation of the process of mud-filtrate invasion by assuming cylindrical coordinates with the origin at the center of a vertical borehole. Simulations enforce boundary source conditions on the wellbore with constant pressure to calculate the flow rate at each time step while simulating the process of invasion. Remaining vertical limits of the formation consist of impermeable zones with no-flow boundary conditions. The geometry used to simulate all the cases of study consists of a finite-difference grid with 51 nodes logarithmically spaced in the radial direction. Simulations assume a radially symmetrical (1D) system in a homogeneous isotropic formation with negligible gravity effects. We assume an arbitrary reservoir thickness equal to one meter. Moreover, we assume a wellbore with radius equal to 0.1454 m and a reservoir with external radius equal to 610 m. Figure 1 shows the 1D cylindrical grid used for the simulations considered in this paper.

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**Figure 1.** Finite-difference radial grid used in the numerical simulations. The shaded region in the borehole wall corresponds to mud cake formed during the process of mud-filtrate invasion. Variable N designates the number of gridblocks, \( P_m \), \( P_h \), and \( P_w \) are mud, water, and pressure across the mud cake, respectively; \( r_h \) and \( r_m \) are wellbore and mud-cake radius, respectively; and \( h_m \) is mud-cake thickness.
Simulations are performed with commercial adaptive-implicit software, known as STARS\(^3\) (CMG, 2006). Because STARS is a three-phase multicomponent fluid-flow simulator, the same software allows us to simulate invasion with OBM and WBM filtrate into either water- or hydrocarbon-bearing rock formations. We assume that WBM filtrate and connate water constitute the aqueous phase and that they might include salt components. On the other hand, the OBM filtrate is considered as a component of the oleic phase.

For invasion in the base case, OBM consists of a single oil pseudocomponent with hydrocarbons in the range of C\(_{14}\) to C\(_{18}\) lumped into a single component, namely, MC\(_{16}\). To study the effect of surfactants in the mud, OBM is described further as an emulsion of oil, surfactant, and water forming an oil phase. We assume a generic surfactant composed of a chain of 18 carbon atoms (La Scala et al., 2004; Lide, 2007), henceforth referred to as EMUL (Salazar et al., 2007). Native oil is formed by hydrocarbons with pseudocomponents in the range of C\(_{7}\) through C\(_{18}\).

Initially, we assume a single-phase native hydrocarbon lumped into a single component, namely, C\(_{7}\)-C\(_{18}\). For completeness, we also consider the case of an additional lighter hydrocarbon lumped into the pseudocomponent C\(_{7}\)-C\(_{18}\) and a gas-bearing formation with the single component CH\(_{4}\) invaded with WBM filtrate. Table 1 summarizes the critical properties of the surfactant and hydrocarbon pseudocomponents assumed in the simulations of mud-filtrate invasion considered in this paper.

### CALCULATION OF THE FLOW RATE OF INVASION

In the process of OBM-filtrate invasion, the magnitude and time evolution of the flow rate of invasion (\(q_{mf}\)) remains uncertain. Static filtration governs the initial mud-cake growth, whereas the role of dynamic filtration\(^4\) is to limit the growth itself (Chin, 1995; Dewan and Chenevert, 2001). In this paper, we assume that mud-cake reaches a limiting thickness and that mud-cake growth is dynamically coupled to the invasion model so that the growth rate is linked to the fluid invasion front that continuously penetrates the rock formation displacing in-situ fluids. The formulation for WBM-filtrate invasion assumes buildup of mud cake on the borehole wall. For the specific case of OBM-filtrate invasion, however, we note that Warner and Rathmell (1997) concluded that the filter cake “should be mostly a cake formed within the pore space of the rock.” In our work, and for the sake of consistency with the formulation of WBM-filtrate invasion, OBM-filtrate invasion also is assumed to build mud cake on the borehole wall.

The time evolution of the thickness of mud cake (\(h_{mc}\)) and flow rate of invasion depends on mud properties and formation properties, such as permeability (\(k\)), porosity (\(\phi\)), water saturation (\(S_w\)), capillary pressure (\(P_c\)), and relative permeability (\(k_r\)). Capillary pressure is a function of water saturation and is given by

\[
P_c(S_w) = P_{mw} - P_w,
\]

where \(P\) is pressure and the subscripts \(w\) and \(mw\) refer to wetting (water) and nonwetting (oil or gas) fluid phases, respectively. Dewan and Chenevert (2001) performed laboratory experiments of WBM filtrate invasion on rock samples and used Darcy’s equation to describe the flow rate of invasion, namely,

\[
q_{mf}(t) = \frac{k_{mc}(t)A}{\mu_{mf}} \left[ \frac{P_m - P_{mf}(t)}{h_{mc}(t)} \right],
\]

where \(t\) is time of invasion, \(k_{mc}\) is permeability of mud cake, \(A\) is the core’s cross-sectional area, \(\mu_{mf}\) is viscosity of filtrate, \(h_{mc}\) is thickness of mud cake, \(P_m\) is mud pressure, and \(P_w\) is pressure across the mud cake.

Permeability (\(k_{mc}\)) and porosity (\(\phi_{mc}\)) of mud cake are given by (Dewan and Chenevert, 1993)

\[
k_{mc}(t) = \frac{k_{mc0}}{P_{mc}^\nu(t)},
\]

and

\[
\phi_{mc}(t) = \frac{\phi_{mc0}}{P_{mc}^\delta(t)},
\]

respectively, where \(k_{mc0}\) is reference mud-cake permeability defined at 6.9 kPa differential pressure, \(\nu\) is a “compressibility” exponent in the range of 0.4–0.9, \(\phi_{mc0}\) is reference mud-cake porosity, and \(\delta\) is a multiplier in the range of 0.1–0.2 that captures the difference in power-law behaviors of porosity and permeability of compressed mud cake. We calculate the flow rate of invasion assuming the radial 1D geometry described in Figure 1.

The flow rate of mud-filtrate invasion is calculated using the method advanced by Wu et al. (2005) that enforces mass balance for the total flow rate through the entire 1D model. Therefore, the inlet filtrate flow rate is dynamically coupled to fluid flow in the invaded formation by the expression (Wu et al., 2005)

\[
q_{mf}(t) = \frac{2\pi h[r_{w}(t) - r_{mc}(t)]}{N}
\]

\[
= \sum_{i=2}^{N-1} \frac{k_{mc}(t)}{\mu_{mf}} \frac{P_{mf}(t) - P_{mf,i}(t)}{\ln(r_{i+1}) - \ln(r_i)} + \frac{k_{mc}(t)}{\mu_{mf}} \ln \left( \frac{r_{w}}{r_{mc}} \right) - \frac{k_{mc}(t)}{\mu_{mf}} \ln \left( \frac{r_{w}}{r_{mc}} \right)
\]

\[
= \frac{2\pi h[r_{w}(t) - r_{mc}(t)]}{N}
\]

\[
= \sum_{i=2}^{N-1} \frac{k_{mc}(t)}{\mu_{mf}} \frac{P_{mf}(t) - P_{mf,i}(t)}{\ln(r_{i+1}) - \ln(r_i)} + \frac{k_{mc}(t)}{\mu_{mf}} \ln \left( \frac{r_{w}}{r_{mc}} \right)
\]

Table 1. Summary of assumed pressure-volume-temperature (PVT) properties for in-situ hydrocarbon components and mud filtrate.

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical temperature</td>
<td>°C</td>
</tr>
<tr>
<td>Critical pressure</td>
<td>kPa</td>
</tr>
<tr>
<td>Mass density</td>
<td>kg/m(^3)</td>
</tr>
<tr>
<td>Molar weight</td>
<td>gmole</td>
</tr>
<tr>
<td>Viscosity</td>
<td>mPa.s</td>
</tr>
</tbody>
</table>

\(^3\)Mark of Computer Modeling Group.

\(^4\)During static filtration, the slurry being filtered remains static and mud cake thickens as filtration progresses. In dynamic filtration, the slurry is circulated over the mud cake, which is eroded and deposited at the same time.
where \( h \) is formation thickness; \( r \) is radial distance away from the wellbore center; \( N \) is the number of radial gridblocks; \( k \) and \( \mu \) are the permeability and viscosity of the fluid phases, respectively; \( r_w \) is wellbore radius; and \( r_{mc} \) is mud-cake radius.

If we assume that solid particles in the mud do not enter the formation, the time evolution of mud-cake radius is given by (Chin, 1995; Wu et al., 2005)

$$\frac{dr_{mc}}{dt} = \frac{f_s}{(1 - f_s)[1 - \phi_{mc}(t)]} \frac{q_{mf}(t)}{2\pi h \cdot r_{mc}(t)}$$ \hspace{1cm} (6)

where \( f_s \) is mud solid fraction. The maximum flow rate of invasion occurs at \( t = 0 \), where \( h_{mc} = 0 \) (\( h_{mc} = r_w - r_{mc} \)) and hence \( r_{mc} = r_w \). As mud cake builds up, \( h_{mc} \) monotonically increases and the flow rate decreases until reaching steady-state behavior once the mud cake has been formed completely. Equation 6 can be integrated numerically for each time step. However, for constant mud-cake properties, equation 6 can be solved analytically and \( r_{mc} \) becomes a function of \( \sqrt{t} \) (Wu et al., 2005).

The algorithm described above was implemented with an interface between MATLAB\textsuperscript{5} and STARS. Simulations begin by assuming constant pressure exerted by the mud at the borehole wall (the mud being in overbalance with the formation pore pressure). From STARS we obtain values of relative permeability, capillary pressure, and oil pressure as a function of water saturation at each gridblock and for each time step. In MATLAB\textsuperscript{5}, equation 1 is used to calculate water pressure, and equations 3–6 are solved at each time step to obtain flow rate of invasion and mud-cake thickness. We compared flow rates for WBM calculated with this method to those obtained with the University of Texas Texas Evaluation Tool box (UT-FET) (Ramírez et al., 2006). Results indicated a good agreement between the two calculations for times of invasion longer than one hour. At early times, the agreement varies between 80% and 90%, thereby confirming the reliability of the method.

**CASES OF STUDY**

The base case of study consists of a radially homogeneous synthetic clastic rock with porosity equal to 25%, permeability equal to 30 md, and irreducible water saturation equal to 8%. Connote water is assumed to exhibit salt concentration ([NaCl]) equal to 160,000 ppm, and the formation is assumed shale free, which allows the use of Archie’s (1942) equation to calculate resistivity from water saturation. We assess the flow rate of oil-based and water-based mud-filtrate invasion on the base case and later perform perturbations on base-case petrophysical properties to study their effect on the calculated flow rate of invasion. Accordingly, the base case of study is pursued as follows:

- Base-case OBM, wherein the rock is assumed 100% water saturated with null irreducible hydrocarbon saturation.
- Base-case WBM, with initial water saturation \( (S_{iw}) \) of the rock equal to 30% and residual oil saturation equal to 10%.

The objective of the above assumptions is to compare the time evolution of flow rate of oil invading movable water with that of water invading movable oil assuming immiscible fluid-flow displacement. Simulations of mud-filtrate invasion begin by assuming typical properties of WBM (Dewan and Chenevert, 2001; Wu et al., 2005) and OBM with null or low water saturation (Warner Jr. and Rathmell, 1997). For all the cases of study, the overbalance pressure is fixed at 2413 kPa. Moreover, we assume that: (1) OBM consists of the lumped pseudocomponent MC\textsubscript{16}, (2) the WBM is fresh with salinity equal to 3000 ppm, and (3) native oil consists of the lumped pseudocomponent C\textsubscript{7}C\textsubscript{18}. Table 2 summarizes the mud-cake and additional formation properties assumed in the simulation of mud-filtrate invasion.

### Table 2. Summary of mud-cake, fluid, and formation properties assumed in the simulations of the process of mud-filtrate invasion.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mud-cake reference permeability</td>
<td>md</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(( \mu )m\textsuperscript{2})</td>
<td>(2.96E–5)</td>
</tr>
<tr>
<td>Mud-cake maximum thickness</td>
<td>m</td>
<td>1.02E–2</td>
</tr>
<tr>
<td>Mud-cake reference porosity</td>
<td>fraction</td>
<td>0.25</td>
</tr>
<tr>
<td>Mud solid fraction</td>
<td>fraction</td>
<td>0.06</td>
</tr>
<tr>
<td>Mud-cake compressibility exponent</td>
<td>fraction</td>
<td>0.40</td>
</tr>
<tr>
<td>Mud-cake exponent multiplier</td>
<td>fraction</td>
<td>0.10</td>
</tr>
<tr>
<td>Filtrate salt concentration (WBM)</td>
<td>ppm</td>
<td>3000</td>
</tr>
<tr>
<td>Formation salt concentration</td>
<td>ppm</td>
<td>160,000</td>
</tr>
<tr>
<td>Mud hydrostatic pressure</td>
<td>kPa</td>
<td>27580</td>
</tr>
<tr>
<td>Initial formation pressure</td>
<td>kPa</td>
<td>25166</td>
</tr>
<tr>
<td>Wellbore radius</td>
<td>m</td>
<td>0.1454</td>
</tr>
<tr>
<td>Formation outer boundary</td>
<td>m</td>
<td>610</td>
</tr>
<tr>
<td>Maximum invasion time</td>
<td>days</td>
<td>3.00</td>
</tr>
<tr>
<td>Formation and fluid temperature</td>
<td>°C</td>
<td>100.6</td>
</tr>
<tr>
<td>Residual hydrocarbon saturation (oil or gas)</td>
<td>fraction</td>
<td>0.10</td>
</tr>
</tbody>
</table>

\textsuperscript{5}Mark of The MathWorks.

**Sensitivity analysis**

We apply the method described above to calculate the flow rate of invasion with OBM and WBM for the rocks under analysis. Subsequently, we perform perturbations to the petrophysical properties of base-case rock and mud cake to assess their effect on the flow rate of invasion. The first step of the analysis consists of performing perturbations to formation properties \( (P_r, k, \phi, \text{ and } S_{iw}) \), whereas the last two steps of the analysis consider perturbations to mud-cake petrophysical properties \( (k_{mc} \text{ and } \phi_{mc}) \).

**Effect of capillary pressure and relative permeability**

We assume a sandstone lithology represented by three rock types with different petrophysical properties as described in Table 3. Rock no. 1 represents a good-quality rock with large pore throat sizes, whereas rock no. 3 corresponds to poor-quality tight rock. Rock no. 2 is a medium-quality rock and describes the base case, which serves as reference for the sensitivity analysis. Capillary pressure curves are calculated using the Brooks-Corey (1966) model wherein we perform simultaneous changes of porosity-permeability (Leverett, 1941), irreducible water saturation, and values of exponents and coefficients involved in the equation.
Moreover, we use the Brooks-Corey equations in their parametric form to calculate saturation-dependent relative permeability (Delshad and Pope, 1989). Relative permeability curves are modified by altering the values of residual fluid-saturation (wetting and nonwetting phases), end points, and exponents involved in the equations of these curves. Appendix A describes the Brooks-Corey equations used to calculate relative permeability and capillary pressure curves. Table 3 summarizes the specific parameters used in the Brooks-Corey saturation-dependent equations for each rock type. Figure 2 describes the water-oil capillary pressure and relative permeability curves associated with the three rock types assumed in the analysis.

Figure 3 shows the calculated flow rate of invasion and mud-cake thickness for the rocks under consideration. The shapes of the simulated curves are very similar for OBM and WBM. Rock no. 1 (best quality) entails a very short spurt-loss\(^6\) duration (<1 s) compared with the poorest-quality rock with a spurt-loss duration shorter than 50 seconds. Moreover, mud cake thickens faster for the cases of rocks no. 1 and no. 2 than for rock no. 3. Finally, we observe that WBM entails higher flow rates of invasion and shorter mud-cake buildup times than OBM.

**Effect of absolute permeability and porosity**

We performed simulations for three values of absolute permeability, namely, 10 md, 30 md (base case), and 100 md. Capillary pressure and remaining petrophysical properties were kept the same as those of the base case. Figure 4 shows the effect of permeability on flow rate of invasion and mud-cake thickness. As in the previous analysis, large permeability is an indication of a good-quality rock, and hence the results of this case are similar to those obtained in the previous case. In general, the flow rate of invasion for WBM filtrate is larger than that of OBM, whereas the time of mud-cake buildup is shorter for WBM than for OBM. In addition, we observe that the higher the permeability, the shorter the spurt-loss time for OBM and WBM.

Individual perturbations of interconnected porosity (20%, 25%, and 30%) do not cause appreciable changes in the shape and magnitude of the flow rate of invasion. However, porosity does affect the radial length of invasion. The area under these curves represents the total volume of mud filtrate lost into the formation, which is not significantly different for the three modeled permeabilities. This implies that one cannot judge the absolute permeability of the formation based on the radial length of invasion by itself at late times (three days, in this case). The radial length of invasion will be, however, a function of porosity at late times. Conversely, the volume and subsequent length of invasion at early times will be a function of the absolute permeability of the formation as evidenced by the dramatically different areas under each curve at early times.

**Effect of initial water saturation**

The importance of this analysis lies in the fact that initial water saturation (\(S_{\text{init}}\)) affects the values of capillary pressure and relative permeability at early times in the simulation. Because the OBM case is 100% water saturated, and the initial water saturation for the WBM case is equal to 30%, the analysis is slightly different in the two cases. Figure 5 displays the results from this sensitivity analysis. Variations of \(S_{\text{init}}\) entail changes only on the maximum flow rate and time of spurt loss (\(t_{\text{spurt}}\)).

From the above simulations, we remark that the maximum flow rate of invasion is intrinsically dependent on phase relative permeability (in fact, it is a function of phase mobility). For this particular case, mud filtrate, native oil, and formation water have similar viscosities; hence the higher the magnitude of \(k_{\text{nw}}\) or \(k_{\text{ro}}\), the higher the maximum flow rate of invasion. When the rock is partially oil saturated of capillary pressure and relative permeability at early times in the simulation. Because the OBM case is 100% water saturated, and the initial water saturation for the WBM case is equal to 30%, the analysis is slightly different in the two cases. Figure 5 displays the results from this sensitivity analysis. Variations of \(S_{\text{init}}\) entail changes only on the maximum flow rate and time of spurt loss (\(t_{\text{spurt}}\)).

From the above simulations, we remark that the maximum flow rate of invasion is intrinsically dependent on phase relative permeability (in fact, it is a function of phase mobility). For this particular case, mud filtrate, native oil, and formation water have similar viscosities; hence the higher the magnitude of \(k_{\text{nw}}\) or \(k_{\text{ro}}\), the higher the maximum flow rate of invasion. When the rock is partially oil saturated.

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\(^6\)The instantaneous constant flow rate immediately after the onset of invasion.

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**Table 3. Summary of petrophysical properties and empirical parameters of the rock types considered in the sensitivity analysis of capillary pressure and relative permeability.**

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Rock no. 1</th>
<th>Rock no. 2</th>
<th>Rock no. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irreducible water saturation</td>
<td>fraction</td>
<td>0.04</td>
<td>0.08</td>
<td>0.15</td>
</tr>
<tr>
<td>Effective porosity</td>
<td>fraction</td>
<td>0.30</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>Absolute permeability</td>
<td>md ((\mu m^2))</td>
<td>300 ((0.296))</td>
<td>30 ((2.96E-2))</td>
<td>3 ((2.96E-3))</td>
</tr>
<tr>
<td>Empirical exponent for wetting phase</td>
<td>—</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Empirical exponent for nonwetting phase</td>
<td>—</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>End point for wetting phase</td>
<td>—</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>End point for nonwetting phase</td>
<td>—</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Empirical exponent for pore-size distribution</td>
<td>—</td>
<td>20</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Capillary pressure coefficient</td>
<td>Pa.cm</td>
<td>6.85</td>
<td>2.74</td>
<td>1.87</td>
</tr>
</tbody>
</table>

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Figure 2. Water-oil capillary pressure (a) and relative permeability (b) curves assumed for the three rock types in simulations of mud filtrate invasion. Rock no. 2 corresponds to the base case. Different entry pressures are assumed for each rock, whereas residual nonwetting-phase saturation is assumed equal to zero when the formation is 100% water saturated. Variables \(k_{\text{nw}}\) and \(k_{\text{ro}}\) designate the relative permeabilities of oil and water, respectively.
rated, relative permeability increases at low values of $S_{wor}$, and hence one observes a larger maximum flow rate and shorter time of spurt loss compared with values of relative permeability where $S_{wor} = S_{crw}$ (which is approximately the minimum total relative permeability). Such behavior is identical for OBM and WBM, and it is observed in Figure 5a when oil invades a 100% water-saturated rock (base-case OBM), and the flow rate is similar to the case in which the same formation exhibits $S_{wor} = 34\%$. This is explained by the fact that $S_w = 100\%$, $k_{rw} = 0.3$ and $k_{rwo} = 0$, whereas at $S_w = 34\%$, $k_{rw} = 0.01$ and $k_{rwo} = 0.28$, both having similar relative permeabilities.

**Effect of mud-cake permeability and porosity**

Variations of mud-cake properties entail amplitude and time variations of the flow rate of invasion and mud cake. We made independent perturbations of mud-cake permeability ($k_{mc}$) and porosity ($\phi_{mc}$) while keeping the maximum mud-cake thickness constant. Figure 6 shows the results of this sensitivity analysis for three orders of magnitude of $k_{mc}$. The maximum flow rate remains constant regardless of the values of mud-cake permeability. However, we observe that for large values of $k_{mc}$, mud cake forms relatively fast, and

---

7Critical water saturation, where $k_{rw} = k_{rwo}$.
the flow rate of invasion reaches steady-state conditions faster than for low values of $k_{mc}$. Even though mud cake forms rapidly, the flow rate of invasion is larger and time of spurt loss longer than for the case of low mud-cake permeability. Therefore, the total volume ($V_t$) of invading fluid, $V_t = \int q(t)\,dt$, increases with increasing mud-cake permeability.

The effect of mud-cake porosity on the flow rate of invasion and mud-cake thickness is less obvious than that caused by mud-cake permeability. Figure 7 shows the time evolution of the flow rate of invasion and mud-cake thickness for three values of mud-cake porosity. Large values of $\phi_{mc}$ entail faster mud-cake buildup and slightly lower values of $V_t$ than for low values of mud-cake porosity. Additional mud-cake properties affecting the flow rate of invasion are mud solid fraction, maximum mud-cake thickness, and mud-cake compressibility exponent. For muds containing a large solid fraction, mud cake builds up more rapidly because of the increasing speed of solid deposition (Wu et al., 2005), whereupon the total volume of invading fluid decreases. All the observations made for this analysis are equally valid for OBM and WBM.

Tables 4 and 5 summarize the sensitivity analyses performed in this section for OBM and WBM, respectively. Such tables compare the duration of spurt loss, maximum rate of mud-filtrate invasion, and total volume of invading mud filtrate after three days of invasion. In general, the total volume of invading mud filtrate is larger for WBM than for OBM. The latter behavior implies that the process of OBM invasion into a rock formation subject to immiscible flow ($S_{sw} = 1$) is slower than for the case of WBM. In the following section, we perform a more in-depth analysis of this observation.

**EFFECT OF THE FLOW RATE OF INVASION ON THE RADIAL DISTRIBUTIONS OF WATER SATURATION**

Knowledge of the flow rate of invasion is necessary to assess the radial distribution of water saturation as a function of time. Simulations of mud-filtrate invasion can be performed using a time-variable flow rate $[q_{mf}(t)]$ as shown in Figures 3–7, with an average equivalent constant flow rate ($q_{mfave}$) (Wu et al., 2004), or with an average equivalent step rate ($q_{mfstep}$) to account for the effect of instantaneous fluid loss at the onset of invasion (spurt loss). Such equivalent rates can be calculated with the equations

![Figure 5.](image)

**Figure 5.** Sensitivity of the processes of OBM- and WBM-filtrate invasion to initial water saturation ($S_{sw}$). Black solid curves identify the base cases ($S_{sw} = 1.0$ for OBM and $S_{sw} = 0.3$ for WBM). Parts (a) and (b) describe the time evolution of the flow rate of invasion; (c) and (d) describe the mud-cake thickness as a function of time of invasion. The maximum flow rate of invasion is a function of phase mobility, and $S_{sw}$ affects capillary pressure and relative permeability during spurt loss.

![Figure 6.](image)

**Figure 6.** Sensitivity of the processes of OBM- and WBM-filtrate invasion to mud-cake permeability ($k_{mc}$). Black solid curves identify the base cases ($k_{mc} = 0.03$ md). Parts (a) and (b) describe the time evolution of the flow rate of invasion; (c) and (d) describe the mud-cake thickness as a function of time of invasion. Flow rate of invasion and mud-cake buildup are affected by $k_{mc}$ after approximately 0.01 hours of invasion.
Sensitivity to Value

Figure 7. Sensitivity of the processes of OBM- and WBM-filtrate invasion to mud-cake permeability ($\phi_{mc}$). Black solid curves identify the base cases ($\phi_{mc} = 0.25$): (a) and (b) describe the time evolution of the flow rate of invasion; (c) and (d) describe the mud-cake thickness as a function of time of invasion. The effect of $\phi_{mc}$ on flow rate and mud-cake buildup is minimal compared with that of $k_{mc}$.

Table 4. Results of the sensitivity analysis of the flow rate of invasion of OBM filtrate invading a 1-m-thick clastic formation. Time of spurt loss ($t_{SL}$), maximum flow rate of invasion ($q_{mf}^{\text{max}}$), and the total volume of mud filtrate ($V_t$) injected after 3 days of invasion for all cases are compared to the base case (BC).

<table>
<thead>
<tr>
<th>Sensitivity to</th>
<th>Value</th>
<th>$t_{SL}$ [s]</th>
<th>$q_{mf}^{\text{max}}$ [m$^3$/d]</th>
<th>$V_t$ [m$^3$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock type</td>
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<td>1/4</td>
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<tr>
<td></td>
<td>2 (BC)</td>
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<td>0.163</td>
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<tr>
<td></td>
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<tr>
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<tr>
<td></td>
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</tr>
<tr>
<td>$S_{mc}$ [fraction]</td>
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<td>0.61</td>
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</tr>
<tr>
<td></td>
<td>0.30</td>
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<td></td>
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<tr>
<td>$\phi_{mc}$ [fraction]</td>
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<td>0.148</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>27</td>
<td>1.46</td>
<td>0.169</td>
</tr>
</tbody>
</table>

Table 5. Results of the sensitivity analysis of the flow rate of WBM filtrate invading a 1-m-thick clastic formation. Time of spurt loss ($t_{SL}$), maximum flow rate of invasion ($q_{mf}^{\text{max}}$), and total volume of mud filtrate ($V_t$) injected after 3 days of invasion for all cases are compared to the base case (BC).

<table>
<thead>
<tr>
<th>Sensitivity to</th>
<th>Value</th>
<th>$t_{SL}$ [s]</th>
<th>$q_{mf}^{\text{max}}$ [m$^3$/d]</th>
<th>$V_t$ [m$^3$]</th>
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<td>$k$ [md]</td>
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<tr>
<td>$\phi_{mc}$ [fraction]</td>
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<td>14</td>
<td>1.93</td>
<td>0.199</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>22</td>
<td>1.93</td>
<td>0.220</td>
</tr>
</tbody>
</table>
equal to 0.1, 0.4, and 0.5 for water, EMUL, and MC16, respectively. Surfactants contained in the OBM reduce the interfacial tension between oil and water (Skalli et al., 2006) and hence make the rock preferentially oil wet (Van et al., 1988; Yan and Sharma 1989).

As shown in Figure 9, such effects are accounted for by decreasing the capillary pressure (Salazar et al., 2007) and by simultaneously changing the value of critical water saturation and end points on the relative permeability curves (Carlson, 2003). We observe that the presence of surfactants in the OBM causes lower values of qinv,max and steeper slopes in the time variations (compared to the cases of WBM or OBM with no surfactants) after the instantaneous rate has taken place. Table 1 describes the properties for each fluid component assumed in the simulation.

Effect of the flow rate of invasion on water saturation

We simulated the processes of OBM- and WBM-filtrate invasion assuming the different modalities of flow rate described above, namely, constant, step, and time variable. Figure 10 shows radial distributions of water saturation for the base-case OBM ($S_{\text{inv}} = 100\%$), whereas Figures 11 and 12 show radial distributions of water saturation and salt concentration for the base-case WBM ($S_{\text{inv}} = 30\%$), respectively. Such radial distributions were calculated with the previously defined flow rates after 0.5 hours, 2.35 hours, 1 day, and 3 days of mud-filtrate invasion, in that order.

We observe that invasion with a constant or step rate yields the same radial distributions ($S_{\text{w}}$ and [NaCl]) because the duration of the instantaneous rate is too short to account for a large volume of invading fluid. The instantaneous fluid loss lasts 27 and 22 seconds for the OBM and WBM cases, respectively. When invading with a time-variable flow rate, we do observe a difference of 15% for the case of OBM (more oil) and 8% for the case of WBM (more water) in the shape of the radial distributions at late times (after one day of invasion). However, the total volume of fluids invading the formation is the same for constant, step, and variable rates, which causes the maximum radial length of invasion to be approximately the same for all cases. Based on this behavior, in the remaining examples of mud-filtrate invasion described below we assume a time-variable flow rate of invasion.

Effect of the flow rate of invasion on the radial length of invasion

Comparison of Figures 10 and 11 indicates that WBM filtrate entails radially deeper invasion than OBM filtrate. After three days of invasion, OBM filtrate penetrates 0.81 m radially into the formation, whereas WBM filtrate penetrates 1.12 m, which is 39% deeper than for the case of OBM. At early times in the invasion process (<12 hours), WBM filtrate penetrates 15% to 20% radially deeper than in the case of OBM filtrate.

Figure 8. Calculated flow rate of invasion as a function of time ($q_{\text{inv}}(t)$), shown in black triangles) for the various cases of OBM- and WBM-filtrate invasion. Constant ($q_{\text{inv}}$) shown in red and step ($q_{\text{step}}$ shown in blue) rates were calculated as average values over three days of mud-filtrate invasion. Part (a) describes the base case OBM (100% water saturated) invaded with single-component OBM (MC16); (b) describes the same case with 30% water and 70% oil; (c) describes the base case with initial water saturation ($S_{\text{inv}}$) equal to 30% with invading fluid formed by water and surfactant emulsified within MC16; finally, (d) describes the base case WBM with $S_{\text{inv}} = 30\%$ (WBM-filtrate invading a partially oil-saturated rock). In each part, $t_{\text{SL}}$ designates the duration of the spurt loss.

Figure 9. Water-oil capillary pressure (a) and relative permeability (b) curves assumed in the simulations of mud-filtrate invasion with water and surfactants emulsified in the OBM (water, EMUL, and MC16). Notice that capillary pressure decreases, the end points of relative permeability change, and critical water saturation is less than 50% compared with the base case shown in Figure 2.
We emphasize that in performing this comparison, we regarded both cases as subject to immiscible fluid-flow displacement. As shown in the next section, the same conclusion holds when considering mixing between OBM filtrate and native oil.

**EFFECT OF THE FLOW RATE OF INVASION ON THE RADIAL DISTRIBUTIONS OF ELECTRICAL RESISTIVITY**

We calculate radial distributions of electrical resistivity from the radial distributions of water saturation obtained from the numerical simulation of mud-filtrate invasion. Because we assume shale-free sandstone with high salt concentration in the connate water (160,000 ppm), Archie’s (1942) equation is a good approximation to calculate resistivity, namely,

\[
\frac{1}{R_t(r)} = S_w(r)^n \cdot \frac{d_m}{\alpha \cdot R_w(r)},
\]

(9)

where \( R_t \) is true formation resistivity; \( R_w \) is connate water resistivity; \( m = 2 \) and \( n = 2 \) are Archie’s cementation and saturation exponents, respectively; and \( \alpha = 1 \) is the tortuosity factor. For the case of OBM-filtrate invasion, \( R_w \) is assumed radially constant in the formation.

On the other hand, when WBM filtrate invades the rock, it mixes with formation water, whereby salt concentration fluctuates radially between that of mud filtrate (3000 ppm) and that of connate water. Moreover, \( R_w \) can be calculated as a function of temperature \( T \) and salt concentration using the equation advanced by Bigelow (1992) and Hallenburg (1998):

\[
R_w(r) = 0.0123 + \frac{3647.5}{([\text{NaCl}]^{0.955})^{0.364}} \cdot \frac{45.4}{T + 21.5},
\]

(10)

where [NaCl] is salt concentration in parts per million (ppm), and \( T \) is formation temperature in °C.
Modeling of borehole resistivity measurements

Calculated radial distributions of electrical resistivity are used to simulate array-induction resistivity measurements (AIT\(^8\)). The objective is to quantify the relative variability of apparent-resistivity measurements that exhibit multiple radial lengths of investigation (R10, R20, R30, R60, and R90). We emphasize that although the invasion modeling entails only radial variations of electrical resistivity, the simulation of borehole resistivity measurements does include the presence of the borehole. We assume a vertically homogeneous formation to avoid shoulder-bed effects, and focus our attention on variations of apparent resistivity for the various radial lengths of investigation. The simulation of AIT is performed with the numerical mode-matching method (NMM) (Chew et al., 1984; Zhang et al., 1999).

Resistivity modeling in the presence of OBM: Water and oil

Four cases of OBM-filtrate invading the base case were modeled by assuming immiscible, binary, and multicomponent fluid flow:

Case A: Single-component (MC\(_{16}\)) OBM invading a 100% water-saturated rock formation (immiscible fluid displacement). The native fluid is only water.

Case B: Single-component (MC\(_{16}\)) OBM invading a 70% partially oil-saturated rock formation (binary miscible fluid flow). Native fluids are water (irreducible and movable) and single-component oil (C\(_7\)C\(_{18}\)).

Case C: Multicomponent OBM (MC\(_{16}\), EMUL, and water) invading a 70% partially oil-saturated rock formation. Native fluids are water (irreducible and movable) and single-component oil (C\(_7\)C\(_{18}\)).

Case D: Multicomponent OBM (MC\(_{16}\), EMUL, and water) invading a 70% partially oil-saturated rock formation. Native fluids are water (irreducible and movable), and multicomponent oil (C\(_7\)C\(_6\) and C\(_7\)C\(_{18}\)) constituting a very light fluid.

We used the time-dependent flow rates described in Figure 8 to simulate the radial distribution of water saturation shown in Figure 13 after three days of OBM-filtrate invasion for the cases of study described above. Figure 14 shows the corresponding radial distributions of electrical resistivity for each case. We observe that case A exhibits the largest difference between flushed- and virgin-zone resistivities caused by the large amount of movable water displaced by the OBM filtrate, thereby entailing a radial length of invasion equal to 0.81 m. By contrast, the radial length of invasion is approximately equal to 0.61 m for cases B and C, and 0.69 m for case D. However, the low amount of movable water along with the mixing between OBM filtrate and native oil is responsible for the relatively small difference between flushed- and virgin-zone resistivities for the partially oil-saturated cases.

Moreover, the presence of surfactants causes a large viscosity contrast between filtrate and native oil, which results in a water bank near the wellbore. Such a water bank is responsible for the presence of a low-resistivity annulus in the invaded zone. The annulus is remarkable only for case D, because that case involves the largest viscosity contrast between native oil and OBM. In addition, the large viscosity contrast causes the invading surfactant to displace preferentially the connate water ahead of it, creating a bank of water manifested as an annulus, in which water saturation is greater than S\(_{min}\).

Figure 15 shows the corresponding simulated AIT measurements for case A, whereas Figure 16 compares the simulated AIT resistivity for case D with single-component OBM filtrate invading a partially oil-saturated rock formation with single-component (case C) and multicomponent (case D) native oil. Radial distributions were simulated with a time-dependent flow rate for each case. Figure 10 shows the results of single-component OBM filtrate invading a 100% water-saturated rock formation (case A). The cases were designed for the modeling of borehole resistivity measurements in the presence of OBM. The presence of surfactants (case D) causes a water bank in the invaded zone.

\(^8\)Mark of Schlumberger.
measurements for cases B, C, and D. In similarity with corresponding radial distributions of electrical resistivity, case A exhibits the largest variability of apparent resistivity for all the cases, in which the relative difference between R10 and R90 is approximately one order of magnitude. The presence of surfactants in the OBM, cases C and D, causes abrupt displacement of the connate water by filtrate, which is caused by a formation with Swi greater than Swir and hence entails higher resistivity values in the flushed zone than for the case of single-component OBM, as indicated by the shallowest-sensing apparent-resistivity curves. Even though initial water saturation is constant in cases B, C, and D, the presence of different components in OBM (surfactants) and formation oil (light hydrocarbons) causes different resistivity readings in the invaded zone, as indicated by the shallow- and intermediate-sensing apparent-resistivity curves (R20, R30, and R60). Moreover, the reading of R60 below R90 in case D is caused by the presence of the annulus observed on the radial distribution of electrical resistivity.

Resistivity modeling in the presence of OBM: Water and gas

For completeness, we modeled two additional cases of OBM filtrate invading the base case partially gas saturated and assuming a binary miscible fluid flow:

Case E: Single-component (MC16) OBM invading a 70% partially gas-saturated rock formation. Native fluids are water (irreducible and movable) and single-component gas (CH4).

Case F: Single-component (MC16) OBM invading a 70% partially gas-saturated rock at irreducible water-saturation conditions. Native fluids are irreducible water and single-component gas (CH4).

Accordingly, radial distributions of water saturation were calculated using time-dependent flow rates of OBM-filtrate invasion. Figure 17 shows these distributions after three days of OBM-filtrate invasion for cases E and F compared with the partially oil-saturated case B. Water saturation in the movable water case shows variability to as high as 0.7 m in the formation, which is similar to the radial length of invasion observed in case B. The bottom parts of Figure 17 show the simulated radial distributions of gas and oil saturation, in which we observe that the gas is fully flushed in the movable water case E and partially removed in the irreducible water case F. When the rock is at irreducible water-saturation conditions, the effect of capillary forces is much higher than when the same formation is within a transition saturation zone, hence preventing the native hydrocarbon to be fully flushed by the invading fluid. Between the radial distances of 0.23 m and 0.70 m, the three phases (water, oil, and gas) coexist.

Figure 18 shows the corresponding radial distributions of electrical resistivity for each case. We observe that the electrical resistivity for case E is approximately equal to 4.5 Ωm near the borehole wall and that it slowly decreases to 3.4 Ωm at about 0.23 m, exhibiting another reduction below 3.4 Ωm between the radial distances of 0.49 m and 0.70 m. Irreducible water saturation is altered only near the borehole wall because of the effect of overbalance pressure, and it remains constant away from the borehole. We observe that case B exhibits the largest difference between flushed- and virgin-zone resistivities monotonically decreasing from 6.2 Ωm on the borehole wall to 3.4 Ωm in the virgin zone with a radial length of invasion shorter than 0.61 m.

Figure 19 compares the simulated array-induction resistivity curves for cases E and F. The case with movable water entails radial variability of apparent-resistivity curves with a difference of only 0.3 Ωm between shallowest- and deepest-sensing curves with marginal variability between the intermediate- and deepest-sensing curves. Even though water saturation for the case of irreducible water is practically constant in the radial direction, the reduction of water saturation near the wellbore causes variations of the apparent-resistivity curves in the order of 0.15 Ωm between shallowest- and deepest-sensing curves. The latter situation is consistent with previously published field cases that report R10 reading higher than R90 in zones remaining at irreducible water saturation (La Vigne et al., 1997). Reduction of water saturation at irreducible water-saturation conditions is caused by reduction of capillary-bound water by the overbalance pressure, which is greater than the formation’s capillary pressure. As expected, Figure 19 also indicates a complete overlap between the shallowest- and deepest-sensing curves for case F.
Resistivity modeling in the presence of WBM

We modeled two cases of WBM filtrate invading the base case and assuming immiscible fluid-flow displacement:

- WBM filtrate invading a 70% partially gas-saturated rock formation. Native fluids are water (irreducible and movable) and single-component gas (CH₄).
- WBM filtrate invading a 70% partially oil-saturated rock formation. Native fluids are water (irreducible and movable) and single-component oil (C₇-C₁₈).

Time-dependent WBM-filtrate flow rates of invasion for each case are used to simulate invasion with fresh WBM filtrate into the base case during three days. Figure 20 shows that, at early times, the flow rate for the gas case is two orders of magnitude larger than that associated with the oil case. Furthermore, the relatively high mobility of formation fluid causes the spurt-loss time to become negligible for the case of native gas. As indicated in Table 2, salt concentration of mud filtrate is equal to 3000 ppm, whereas salt concentration of connate water is approximately 160,000 ppm. The latter situation is responsible for a large salinity contrast in the radial direction, which has a strong effect on the radial distribution of electrical resistivity.

Figures 21 and 22 show the radial distributions of water saturation and salt concentration simulated for each case after three days of WBM-filtrate invasion. Radial distributions of water saturation indicate that the movable gas is removed completely by mud filtrate in the near-wellbore region. By contrast, movable oil is removed par-
tially by mud filtrate, and the radial distributions are smoother than those simulated for the gas case. This behavior indicates that oil moves slower than gas during the process of invasion, and the effects of capillary pressure in the water-oil system are larger than those in the water-gas system.

We use radial distributions of water saturation and salt concentration to calculate the radial distributions of electrical resistivity after three days of WBM-filtrate invasion. Figure 23 compares radial distributions of electrical resistivity calculated for the gas- and oil-bearing formations. These radial distributions indicate that the invasion front penetrates radially deeper into the formation than in the case of OBM filtrate, even though the petrophysical properties are identical. The large contrast of salt concentration causes a large difference between flushed- and virgin-zone resistivities, whereupon a low-resistivity annulus ensues in the invaded zone. Moreover, we note that the annulus associated with the gas case exhibits a lower resistivity than for the case of native oil. Radial distributions of electrical resistivity calculated for the gas case show constant resistivity in the flushed zone resulting from full displacement of free gas by mud filtrate, which is not observed in the oil case.

Finally, we simulated the corresponding array-induction resistivity measurements described in Figure 24. Simulated apparent resistivities exhibit a large separation between the deepest- and shallow-
that miscibility between OBM filtrate and native oil slows down the process of invasion. The variability of fluid saturation in the invaded zone significantly affected the radial distribution of electrical resistivity.

Array-induction resistivity measurements simulated from the radial distributions of electrical resistivity are affected largely by invading fluids near the wellbore. In the case of OBM filtrate invading a wet zone, we observed a large separation between apparent-resistivity curves, similar to that observed for the case of WBM filtrate invading a movable hydrocarbon-saturated zone. Alterations of wettability resulting from the presence of surfactants in the mud increased the contrast between flushed- and virgin-zone resistivities for the cases of OBM filtrate invading a partially oil-saturated rock formation. The presence of surfactants in the OBM altered the shape of the flow rate of invasion as well as radial distributions of electrical resistivity, thereby affecting the response of array-induction resistivity measurements.

The large viscosity contrast between native oil and emulsified OBM resulted in a water bank, which formed a low-resistivity annulus in the invaded zone. Such an annulus arose when native oil consisted of very light liquid hydrocarbon components ($\mu_\text{c}<0.25$ mPa.s). Capillary pressure and relative permeability governed the shape and contrast of the radial distribution of electrical resistivity in the invaded zone. Simulations of OBM- and WBM-filtrate invasion into gas-bearing formations indicated that the effect of capillary pressure on the radial distributions of electrical resistivity was more remarkable in oil-bearing than in gas-bearing formations.

Sensitivity analyses to mud and rock properties described in this paper shed insight into the character of fluid distributions near the wellbore resulting from OBM- and WBM-filtrate invasion. Simulation of the process of invasion for three rock types confirmed the reliability of this simulation method to assess the effect of invasion on borehole resistivity measurements. These simulations can be used to improve the petrophysical interpretation of other logging measurements, including those acquired with nuclear and sonic tools.

**ACKNOWLEDGMENTS**

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**APPENDIX A**

**BROOKS-COREY’S CAPILLARY PRESSURE AND RELATIVE PERMEABILITY EQUATIONS**

We assume the model proposed by Brooks and Corey (1966) to calculate the saturation-dependent relative permeability and capillary pressure curves. In this model, wetting-phase ($k_{\text{w}}$) and nonwetting phase ($k_{\text{nw}}$) parametric relative permeability curves (Delshad and Pope, 1989) are respectively given by
\[ k_{rw} = k_{rw}^0 S_n^\varepsilon_w, \]  
\[ k_{rmw} = k_{rmw}^0 (1 - S_n)^\varepsilon_{rmw}, \]

where \( k_{rw}^0 \) and \( k_{rmw}^0 \) are wetting- and nonwetting-phase relative permeability end points, respectively, and \( \varepsilon_w \) and \( \varepsilon_{rmw} \) are empirical exponents for wetting and nonwetting phase, in that order. In equations A-1 and A-2, \( S_n \) is normalized wetting-phase saturation, given by

\[ S_n = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{nw}}, \]

where \( S_n \) is saturation and \( S_{wr} \) is irreducible saturation for the wetting phase, and \( S_{nw} \) is irreducible saturation for the nonwetting phase. Drainage capillary pressure is expressed as a function of porosity (\( \phi \)), permeability (\( k \)) (Leverett, 1941), and wetting-phase water saturation, namely,

\[ P_c = P_c^0 \sqrt[3]{\frac{\phi}{k}} (1 - S_w)^\varepsilon_w, \]

where \( P_c \) is capillary pressure between wetting and nonwetting phases, \( P_c^0 \) is the coefficient for capillary pressure, and \( \varepsilon_w \) is the pore-size distribution empirical exponent.

**ACRONYMS AND NOMENCLATURE**

- **WBM**: Water-based mud
- **STARS**: Steam, thermal, and advanced processes reservoir simulator
- **PVT**: Pressure-volume-temperature
- **CMG**: Computer Modeling Group
- **WOB**: Weight on bit
- **Rch**: Radial (or) central length
- **R60**: 60-inch radial length of investigation apparent resistivity, [Ωm]
- **R90**: 90-inch radial length of investigation apparent resistivity, [Ωm]
- **t**: Time of invasion, [hours], [days]
- **tSL**: Time of spurt loss, [s]
- **V\text{inj}**: Total volume of fluid injected, [m³]
- **δ**: Mud-cake exponent multiplier, []
- **σ**: Formation effective porosity, [fraction]
- **Φ**: Mud-cake porosity, [fraction]
- **Φ\text{max}**: Reference mud-cake porosity, [fraction]
- **μ**: Oil viscosity, [Pa.s]
- **μ\text{off}**: Mud-filtrate viscosity, [Pa.s]
ACRONYMS AND NOMENCLATURE

$\mu_{nw}$: Nonwetting phase viscosity, [Pa.s]

$\mu_w$: Wetting phase viscosity, [Pa.s]

$\nu$: Mud-cake compressibility exponent, [ ]

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