INVERSION OF SECTOR-BASED LWD DENSITY MEASUREMENTS ACQUIRED IN LAMINATED SEQUENCES PENETRATED BY HIGH-ANGLE AND HORIZONTAL WELLS

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ABSTRACT

We show that inversion processing improves the petrophysical interpretation of logging-while-drilling (LWD) density measurements acquired in high-angle and horizontal (HA/HZ) wells. Our interpretation method consists of first detecting bed boundaries from short-spacing (SS) detector and bottom-quadrant compensated densities by calculating their variance within a sliding window. Subsequently, a correlation algorithm calculates dip and azimuth from the density image. Depth shifts that vary azimuthally and depend on relative dip angle, together with the effective penetration length (EPL) of each sensor, refine previously selected bed boundaries. Subsequently, inversion combines all the sector-based density measurements acquired at all measurement points along the well trajectory to estimate layer densities. We implement the inversion with a recently developed linear approximation that accurately and efficiently simulates borehole density measurements.

To verify the reliability and applicability of the inversion method, we first use forward simulations to generate synthetic density images from a model constructed from field data. Results indicate that inversion improves the interpretation of azimuthal density data as it consistently reduces shoulder-bed effects. We appraise inversion results obtained from field measurements by quantifying the corresponding integrated porosity-feet yielded by inversion methods in comparison to standard techniques that use simple cutoffs on field-processed compensated density. Integrated porosity-feet of inverted synthetic density measurements increase by 4.4% with respect to non-inverted field measurements. By comparison, integrated porosity-feet from inversion results that include only bottom sectors improve by 27.8% with respect to that calculated with field-compensated, bottom-quadrant density measurements. Experience shows that an additional benefit of inversion methods is their ability to detect and quantify the inaccuracies attributed to increasing tool standoff in the upper sectors of the measurement.

INTRODUCTION

Conventional processing of LWD density measurements in HA/HZ wells may not yield results with sufficient spatial resolution to estimate actual layer density. This situation commonly arises when using standard compensation (spine-and-rib method) of single-detector density measurements acquired in thin laminations. Unlike in vertical wells, it has been shown that enhanced-resolution processing does not improve the resolution of compensated density in HA/HZ wells (Radtke et al., 2006; Mendoza et al., 2006). Accurate estimation of true stratigraphic thickness (TST) and density-derived porosity is essential for reliable calculations of net pay. Because existing standard and enhanced-resolution compensation methods were designed for vertical wells, other authors have expressed a need for compensation techniques suitable for HA/HZ wells. Recent publications on Monte-Carlo simulation of azimuthal density measurements propose alternative post-processing techniques of raw density images acquired across thin laminations in HA/HZ wells (Uzoh et al., 2009; Yin et al, 2008).

Our approach is to utilize inversion methods to calculate more accurate layer properties for subsequent use in petrophysical interpretations. The proposed inversion technique eliminates shoulder-bed effects due to the high apparent dip observed in HA/HZ wells and improves the estimation of bed petrophysical properties, specifically porosity and fluid saturation, based on simulation of nuclear measurements.

Historically, the lack of fast and reliable numerical simulation methods constrained the applicability of inversion methods for petrophysical interpretation of nuclear measurements. Patchett and Wiley (1994) used an iterative inverse modeling procedure to determine porosity, water saturation, and lithology with a forward modeling algorithm that simulated nuclear logs based on elemental composition of the rocks and fluids.

The objective of this paper is to implement linear inversion methods to improve the estimation of layer densities and TST based on LWD azimuthal density measurements acquired in HA/HZ wells. To that end, we make use of fast numerical simulation procedures that approximate the response of nuclear borehole measurements using the concept of Monte Carlo-derived flux sensitivity functions (FSFs). It has been shown that this rapid simulation method accurately approximates density measurements for arbitrary rock-fluid mixtures in the presence of shoulder beds and dipping layers (Mendoza et al., 2009a and b). We implement two-dimensional (2D) inversion to simultaneously account for azimuthal and axial variations of formation density detected by measurements acquired in HA/HZ wells. Because LWD density instruments acquire continuous azimuthal measurements around the perimeter of the borehole, we include multiple azimuthal sectors in the inversion. This approach provides redundancy of measurements, thereby enabling more reliable estimations of layer density and thickness than from quadrant measurements.

As a starting point, we select bed-boundaries from sector-based compensated and SS (Short Spacing)-detector density measurements. We use an algorithm that detects inflection points of density measurements as the location of bed boundaries. Subsequently, we use a correlation algorithm to estimate dip and azimuth from density azimuthal sector measurements. With the estimated angles, we refine the location of the selected boundaries for each azimuthal sector. Next, we perform multi-layer linear inversion of density based on three independent inversion procedures. The first two procedures assume as input data the unfiltered or raw and then filtered SS sector-based measurements combined with the LS (Long Spacing)-detector density measurements. The third procedure uses only fully-compensated azimuthal density as input. Inversions performed with synthetic density images appraise the accuracy and reliability of the inversion methods. Furthermore, using field data we evaluate the practical implementation of inversion on LWD density measurements. We show that inversion consistently reduces shoulder-bed effects and that it enables the detection of inaccurate densities due to excessive tool standoff in the upper sectors. Comparison of integrated porosity-feet yielded by inversion methods quantifies their relative worth with respect to conventional compensation techniques.

**INVERSION METHOD**

Numerical simulation of borehole density measurements invokes the concept of Monte Carlo-derived flux sensitivity functions and fast numerical approximations described by Mendoza et al. (2009a). Inversion methods introduced in this paper focus on measurements acquired with a commercial LWD density tool. Accordingly, we consider an azimuthal binning scheme divided into 16 sectors, each subtending an angle of 22.5 degrees from the center of the tool (Evans et al., 1995, Holenka et al., 1995). Furthermore, we assume that a measurement acquired with a given sector represents the average angular LWD density measurement within that sector (Uzoh et al., 2007).

Inversion procedures consider four principal stages of analysis: (1) selection of layer boundaries from sector-based compensated or single SS-detector density measurements, (2) estimation of dip and azimuth, (3) linear inversion of layer densities, and (4) forward simulation of density measurements based on a model constructed from inverted densities and bed boundaries. Differences between simulated and measured densities quantify the reliability and consistency of inversion results. The procedure can be repeated with a different selection of bed boundaries to further appraise the reliability and non-uniqueness of the results.

**Dip-Angle Estimation.** In analogy to dipmeter processing used with micro-resistivity measurements, we implement a fixed-interval correlation technique to estimate dip from density images (Vincent et al., 1979). This method consists of cross-multiplication between pairs of curves at all possible positions within a specified window to ascertain maximum coherence. Maximum coherence follows from the minimization of residuals between pairs of density measurement sectors, i.e., \( \sum_i |d_i - \bar{d}_i|^2 \), where \( \bar{d}_i \) is the vector of density measurements and the Roman subscripts designate sector number. The number of shift positions multiplied by the depth sampling rate of the measurements gives the corresponding measured-depth offset between different sectors. In the first step, the top azimuthal sector is depth shifted against all other azimuthal density measurements to calculate the coherence per depth shift. At zero azimuth, the top azimuthal sector correlates with the bottom sector at maximum depth
offset. Calculated depth offsets along with their calculated coherence between pairs of azimuthal sectors are stored as indexed entries of a matrix. The second step consists of searching for depth shifts that exhibit the maximum coherence. Maximum coherence can occur between different pairs of azimuthal sectors depending on the direction of drilling relative to formation dip and azimuth. These depth shifts form a sinusoid whose amplitude \( A \) is used to estimate relative dip angle, \( \theta \), with the equation (Plumb and Luthi, 1989)

\[
\theta = \tan^{-1} \left( \frac{A}{D + 2\text{EPL}} \right),
\]

where \( D \) is borehole diameter and \( \text{EPL} \) is referred to as the effective penetration length. The correlation procedure requires specification of several input parameters including correlation depth interval, step distance, and search angle (Figure 1). Correlation depth interval is the depth window used in the comparison of logs for correlation. Step distance is the depth shift implemented for multiple correlation intervals. Search angle controls the length of the maximum displacement between pairs of azimuthal measurements. In the presence of highly-laminated sequences, the above processing parameters must include depth intervals that encompass several bed boundaries and a search angle that allows sufficient vertical displacement between sectors. Testing with synthetic measurements yielded maximum differences of 0.5 degrees in estimated dip angles with respect to actual values.

**Detection of Bed Boundaries.** Bed boundaries are determined from density measurements by calculating the variance of the log within a sliding window, and by placing a bed boundary wherever the variance increases above a pre-specified threshold (Uzoh et al., 2007). Because of its higher vertical resolution, we use SS-detector density measurements for bed boundary detection. Subsequently, depth shifts that vary azimuthally and depend on relative dip angle, together with the EPL of each sensor, refine previously selected bed boundaries (Uzoh et al., 2009). The FSFs assume the source location as the measurement point whereas the inversion technique is performed in TVD space; therefore, individual-sector field measurements acquired with the LS and SS sensors require a depth shift. These depth-shifts have a measurement point, \( \text{EPL} \) and an azimuthal component, with the general equation given by

\[
\Delta \text{TVD} = \text{Source to Sensor}_{mp} + \left[ \text{EPL}_{\text{sensor}} \frac{\cos \beta}{\tan \theta} \right] \cos \alpha, \tag{2}
\]

where \( \text{Source to Sensor}_{mp} \) is the distance between the source and the sensor measurement point, \( \beta \) is borehole inclination angle, and \( \theta \) and \( \alpha \) are formation relative dip and azimuth, respectively. Figure 2 shows the geometrical conventions assumed for the modeling of log-based boundary TVD shifts and the total TVD shift applied to the layer boundary estimated with SS-detector density to correlate with that of the LS-detector density, where \( \Delta \text{TVD}_{i} = (\text{LS}_{mp} - \text{SS}_{mp}) \cos \beta \). In the case of null dip (layering parallel to the borehole axis), the total TVD shift is equal to the corresponding EPL. Values of EPL for SS and LS sensors are equal to 1.2 and 1.8 inches, respectively (Uzoh et al., 2009).

**Inversion.** The objective of inversion is to estimate layer-by-layer densities of the model previously constructed with bed-boundary location and dip angle estimation. Sector-based inversion of single-detector density measurements is posed as the minimization of a quadratic cost function, given by

\[
C(\mathbf{x}) = \frac{1}{2} \left\| \mathbf{f}(\mathbf{x}) \right\|^2 + \lambda^2 \left\| \mathbf{x} - x_0 \right\|^2, \tag{3}
\]

where \( \mathbf{x} \) is a vector containing the unknown layer densities, and \( \lambda \) is a stabilization parameter that controls the weight given to the residual (data misfit) norm relative to a prescribed reference model, \( x_0 \). The value of \( \lambda \) increases for decreasing values of well inclination angle across thinly-laminated sequences where non-uniqueness in the solution of \( \mathbf{x} \) increases. We choose \( x_0 \) with constant entries equal to the value of measured density averaged over the depth interval considered for inversion. The term \( \left\| \mathbf{x} - x_0 \right\|^2 \) incorporates a-priori assumptions about the expected solution with the
objective of reducing non-uniqueness in the estimation of \( \bar{x} \) (Hansen, P. C., 1998). Entries of the unknown vector of layer densities,

\[
\begin{bmatrix}
x_1 \\
\vdots \\
x_i \\
\vdots \\
x_T 
\end{bmatrix},
\]

are bounded as \( 0 \leq x_i \leq \varepsilon \), where \( x_i \) is the \( m \)-th density layer, \( \varepsilon \) is the maximum value of density allowed in the solution, and the subscript \( T \) designates the total number of layers. The vector of data residuals \( \overline{e}(\bar{x}) \) is written as

\[
\bar{e}(\bar{x}) =
\begin{bmatrix}
e_1(\bar{x}) \\
\vdots \\
e_m(\bar{x}) \\
\vdots \\
e_M(\bar{x})
\end{bmatrix}
= 
\begin{bmatrix}
d_1(\bar{x}) - d_1^0 \\
\vdots \\
d_m(\bar{x}) - d_m^0 \\
\vdots \\
d_M(\bar{x}) - d_M^0
\end{bmatrix} = \overline{d}(\bar{x}) - \overline{d}^0,
\]

where the entry \( d_m(\bar{x}) \) is the \( m \)-th value of density simulated from a model \( \bar{x} \) constructed from inverted layer densities, and \( d_m^0 \) is the \( m \)-th density measurement. The term \( \overline{d}(\bar{x}) \) is the vector of numerically simulated sector-based density measurements. Accordingly, the vector of measurements \( \overline{d}^0 \) contains blocks of indexed density measurements acquired with SS and LS detectors and includes all azimuthal sectors for all measurement points along the well trajectory. For each azimuthal sector, blocks containing indexed LS-density measurements follow similar blocks of SS-density values i.e.,

\[
\overline{d}(\bar{x}) =
\begin{bmatrix}
\rho_{SS,j,M} \\
\vdots \\
\rho_{SS,j,m} \\
\rho_{SS,j,m} \\
\vdots \\
\rho_{SS,j,M} \\
\rho_{SS,j,M} \\
\vdots \\
\rho_{LS,j,M} \\
\vdots \\
\rho_{LS,j,M} \\
\rho_{LS,j,M}
\end{bmatrix}
\equiv \overline{K} \cdot \bar{x},
\]

where the subscripts \( j \) and \( m \) designate the \( j \)-th azimuthal sector and the \( m \)-th measured density value, respectively. The subscript \( J \) designates the total number of azimuthal sectors and is equal to 16. Similarly, \( M \) denotes the total number of data points (the number of measurement points along the well trajectory). Because of the assumption of linearity between measurements and layer densities, it follows that \( \overline{d}(\bar{x}) = \overline{K} \cdot \bar{x} \), where the matrix \( \overline{K} \) is constructed with flux sensitivity functions (FSFs). Blocks containing integrated 2D FSFs for SS- and LS-detectors oriented in the direction of each azimuthal sector are included as rows in \( \overline{K} \) (details about the entries of \( \overline{K} \) are included in the Appendix) that weight the vector \( \bar{x} \) to reproduce indexed density measurements included in vector \( \overline{d}^0 \). The solution of Equation (3) is given by

\[
\overline{K} \cdot \overline{K} + \lambda I \overline{K} \cdot \bar{x} = \overline{K} \cdot \overline{d}^0 + \lambda \overline{x},
\]

where the superscript \( t \) indicates transposition.

\[ \text{Figure 2: Geometrical conventions for the detection of bed boundaries based on dip angle, effective penetration length (EPL), and source-detector spacing. In the figure, S is source location, whereas LS and SS are the locations of measurement points for long- and short-spaced detectors, respectively. The subscript mp designates measurement points, } \beta \text{ is borehole inclination angle and } \theta \text{ is formation relative dip angle. } \Delta TVD_{1,SS} \text{ and } \Delta TVD_{1,LS} \text{ are true vertical depth-shifts from source to SS- and LS-detector measurement points, respectively. } \Delta TVD_{2,SS} \text{ and } \Delta TVD_{2,LS} \text{ are true vertical depth-shifts from SS- and LS-detector measurement points, respectively to the intersection between a boundary and the borehole. } X_{SS} \text{ and } X_{LS} \text{ are distances between the apparent intersection of a bed boundary and the actual boundary intersection with the borehole. The figure assumes measurements acquired with a bottom azimuthal sector.} \]

**Forward Modeling.** We simulate azimuthal density measurements from the inverted layer densities with the fast linear FSF approximation introduced by Mendoza et al. (2009a). To that end, we use Monte Carlo-derived SS- and LS-detector FSFs for a commercial LWD density tool. Simulations use 2D FSFs (radial and vertical) integrated azimuthally to weight the angular average density of every sector at each fixed depth-
point. We assume that the tool is pressed against the wall of an 8.5-inch borehole for all tool locations around the perimeter of the wellbore (i.e., numerical simulations do not include tool standoff). Subsequently, we use commercial post-processing of single-detector densities to calculate compensated and enhanced (alpha-processed) densities. With simulated azimuthal density measurements we construct images for SS- and LS-detector measurements as well as for compensated densities. Comparison of images constructed with simulations to those constructed with field measurements quantifies the ability of inversion to reproduce density values for each azimuthal sector along a given interval of measured depth.

**Figure 3** is a flow chart of the linear inversion procedure implemented with simulated SS and LS-detector density measurements. For field density measurements where the SS detector densities are filtered, we implement an inversion procedure that incorporates density averaging over a measured depth interval that is proportional to the size of commercial filters. For inversion of fully-compensated density measurements, an iterative procedure of forward simulations yields densities of layers previously determined from the location of bed boundaries. Inversion of single-detector densities uses SS-detector density measurements for boundary location. Similarly, inversion of compensated density uses compensated density to determine the location of bed boundaries.

**INVERSION RESULTS**

**Benchmark Examples.** We consider a multi-layer formation model consisting of alternating 2.0 and 2.6 g/cm³ density layers of varying thickness. Simulations of density measurements with the FSF approximation across the multi-layer formation comprise cases of wells of 60, 70, and -80 degrees of inclination with respect to the vertical. Negative angles indicate simulations for up-dip direction of drilling and positive angles correspond to down-dip LWD simulations. We use the numerically simulated densities for each case as input data for inversion.

**Figure 4** shows inversion results obtained from density measurements simulated across a measured-depth section of the multi-layer formation model penetrated by a -80-degree deviated well. We also consider inversion results obtained across similar measured-depth sections in the same formation model penetrated by wells deviated 60 and 70 degrees. Common observations about the three inversion cases are that, in general, inversion improves the assessment of layer densities across thick layers and relatively high angles of well inclination. Because the number of measurements decreases across thin layers and for smaller well inclination angles, non-uniqueness increases in the estimation of layer densities. Therefore, inversion of density measurements across thin layers in low-angle wells requires larger values of the regularization parameter \( \lambda \) to control the smoothness of the solution. For a measured depth interval of 20 feet, inversion from a 60-degree well required a regularization parameter equal to 0.8 whereas inversion for 70 and -80-degree wells required values of 0.4 and 0.2, respectively.

To appraise the reliability of inversions, we calculate density errors between simulations and synthetic data averaged over a measured-depth section. **Figure 5** compares residuals between inversion-based simulations and input synthetic data as a function of
azimuthal sector. The azimuthal influence of the errors is due to the fact that at high apparent dip, all sectors are not simulating the same mix of low and high density layers over the same depth interval (Figure 4). The errors shown in Figure 5 represent the maximum discrepancy of the inversion in thinly-bedded formations. For both SS- and LS-detectors, simulations for the 80-degree up-dip well exhibit smaller differences compared to simulations for 60- and 70-degree wells. Average density errors for 60- and 70-degree well simulations vary across azimuthal sectors ranging from -0.006 g/cm$^3$ to 0.095 g/cm$^3$ and -0.005 g/cm$^3$ to 0.005 g/cm$^3$ for the SS- and LS-detectors, respectively. Density errors for simulations for the -80-degree well have maximum values of -0.005 g/cm$^3$ and -0.0015 g/cm$^3$ for the SS- and LS-detectors, respectively. In general, maximum differences for SS-detector, inversion-based simulations are under ±0.01 g/cm$^3$ and for the LS-detector; residuals are under ±0.005 g/cm$^3$.

Simulations confirm that compensated and alpha-processed (Flaum, et al., 1987) densities in HA/HZ wells are practically the same (Uzoh et al., 2007). Alpha processing imposes the vertical resolution of the short-spaced detector onto the compensated density over an interval of measured depth. Because the smaller radial geometric factor controls the effective vertical resolution in HA/HZ wells, differences of effective resolution of both SS-detector and compensated density are not as large in HA/HZ wells as they are in vertical wells. Similarly, filtering of SS- and LS-detector density averages the measurements over a relatively short measured-depth interval, whereby the influence of filtering is not significant for inversion. In the following sections we focus our attention to field measurements processed with standard commercial compensation techniques.

**Field Case of Study.** To verify the reliability and applicability of inversion methods described in previous sections, we first perform inversions of density measurements numerically simulated from a synthetic model constructed from field data. Subsequently, we perform the inversion on actual LWD field data images. The case under analysis corresponds to a hydrocarbon field located in West Africa. For testing, we select a depth interval located below the free oil-water contact. The formation consists of alternating 1 to 2-foot TST laminations of siltstones, argillaceous siltstones, and calcite-cemented siltstones.

![Figure 4: Example of inversion of synthetic measurements for the case of an 80-degree deviated well: Measured densities were numerically simulated across a multi-layer model consisting of alternations of thin layers with densities equal to 2.0 and 2.6 g/cm$^3$. The assumed borehole diameter is 8.5 inches and the direction of drilling is down-dip. Estimated well inclination angle was -79.47 degrees. Curves describe: azimuthal sector density yielded by inversion (piecewise-constant black line), simulations from inversion results (dashed red curves), and input measurements (continuous blue curve). Top panels show results from SS-detector inversion and simulations; Bottom panels show results from LS-detector inversion and simulations. Blue numbers describe azimuthal location around the perimeter of the borehole of each sector. Inversion was performed with a regularization parameter, $\lambda$, equal to 0.2 and with a constant reference model, $\bar{\rho}$, equal to 2.3 g/cm$^3$. Refer to equations (3) through (5) for details about the inversion method.](image)
Figure 5: Comparison of sector-based inversion results across azimuthal sectors for cases of synthetic density measurements acquired in wells of 60, 70, and -80 degrees of inclination. Colored curves show density differences between azimuthal density simulations from inversion and density azimuthal measurements averaged across the same depth interval. Refer to Figure 4 for details about the assumed multi-layer model and inversion parameters.

Figure 6 displays LWD measurements acquired in a highly-deviated well across the depth section under analysis in true vertical depth (TVD). Measurements shown in Figure 6 include gamma-ray (GR), resistivity, bottom-sector photoelectric effect (PEF), bottom-sector compensated density, well inclination, and compensated density image. Density measurements were acquired with the same commercial LWD density tool assumed in both inversions and simulations. For this interval, well inclination fluctuates between 78 and 82 degrees from the vertical, the direction of drilling is down-dip, and bit size is equal to 8.5 inches.

Inversion of Field-Based Synthetic Density. At the outset, we construct a model from bottom-sector compensated density measurements. Due to gravity, in HA/HZ wells the tool makes better contact with the wellbore at the bottom-sector location. Consequently, bottom-sector compensated density measurements are the least affected by borehole environmental effects and are closer to true bed densities. Track 4 of Figure 6 describes measured compensated density (blue curve) and the constructed model (piecewise-constant red line). We use a procedure called zonation to calculate layer densities. Firstly, this procedure detects bed-boundary locations in the measured density whenever the variance, calculated within a sliding window, increases above a threshold value. Secondly, densities for each layer included in the model are calculated based on the selected bed-boundary location. These layer densities are either maximum, minimum, or average density across beds, depending on the variability of density measurements within each bed.

We use the multi-layer model constructed from compensated bottom quadrant density measurements to simulate LWD SS- and LS-detector density images. In so doing, we use linear approximation procedures for fast simulation of density measurements (Mendoza et al., 2009a). Simulations use FSFs specifically calculated for the commercial LWD density tool that acquired the field measurements. Subsequently, we calculate compensated density, enhanced-resolution density, and density correction ($\Delta \rho$), using commercial post-processing techniques of single-detector densities.

Numerical simulations obtained for the field-based model are used as input measurements for inversion. Accordingly, inversion procedures described above assume synthetic density data as the vector of measurements, $\bar{d}$, which includes all azimuthal sectors and SS- and LS-density values. Minimization of the quadratic cost function defined by equation (3) yields azimuthal layer densities, $\bar{x}$, which are subsequently used as input for fast forward simulation of density images.

Density images are constructed from the combination of sector density measurements around the perimeter of the wellbore. Figures 7 and 8 compare density-derived images displayed in measured depth, for SS-, and LS-detectors, respectively. Figures display images in the following order from left to right: field density measurements, synthetic density, inversion results, simulated density, density difference, and percent difference between simulated and synthetic density images. Colors describe density values corresponding to simulations and measurements. Sectors located in the upper sides of the borehole are displayed in the left and right sides of the images, whereas bottom sectors are displayed in the center of the images.
exhibits continuous boundaries and density values for each bed across azimuthal sectors. In addition, inversion shows higher density contrasts between layers of varying densities and thicknesses. Compensated SS- and LS-detector density images show sinusoids of larger amplitude than inversion. This effect is due to (1) the inverted image is represented as being at the borehole wall interface, and (2) field measurements have an apparent EPL for SS, LS, and compensated measurements (Uzoh et al, 2007).

Because inversion exhibits continuous bed boundaries (bed thickness) around azimuthal sectors, the calculated value of porosity-feet is constant for intervals of measured depth. By contrast, azimuthal variations of bed-boundary locations detected with compensated density yield azimuthally variable porosity-feet. Using a cutoff of 2.2 g/cm³, values of integrated porosity-feet across the selected measured-depth interval, averaged over bottom quadrant azimuthal sectors, are equal to 25.08 and 26.19 feet for compensated density and for inversion, respectively.

Percent errors between simulations and field-based synthetic data quantify the ability of inversion to reproduce input measurements. Figure 9 shows that maximum differences between simulated and synthetic SS-detector density images averaged over the measured-depth interval under analysis are 0.02 g/cm³. Similarly, for LS-detector and compensated density images, maximum average differences are 0.007 g/cm³ and 0.003 g/cm³, respectively. Comparison of simulated SS- and LS-detector density images to original synthetic images indicates better results in upper sectors than in side and bottom sectors. Errors in upper sectors (sectors 1 and 16) are equal to 0.43 g/cm² and 0.15 g/cm², for SS and LS detectors, respectively. Figure 10 shows that inversion improves porosity-feet estimations compared to compensated bottom-quadrant density.

**Figure 6:** Logging while drilling (LWD) measurements acquired in a highly deviated well displayed in true vertical depth (TVD). Measurements were acquired with the same commercial tool assumed in the inversion and simulation examples studied in this paper. Starting from the left, panel 4 shows compensated bottom sector density measurements (blue curve) used for the construction of a field-based model (piece-wise constant red line). Well inclination in panel 5 fluctuates between 78 and 82 degrees and the direction of drilling is up-dip. Panel 6 shows a density image constructed with compensated density measurements.

**Inversion of Field Density Measurements.** Using the same inversion procedures described above for synthetic density data, we perform inversion on field measurements. We consider the same depth interval described in Figure 6. In contrast to synthetic density measurements numerically simulated from a field-based model, field measurements are less accurate in the upper sectors in the presence of large tool standoffs. The LWD density tool is preferentially eccentered toward the bottom side of the borehole due to gravity. Hence, single-detector density measurements yield density values that vary azimuthally depending on tool standoff. Because bottom-sector density measurements are the least affected by tool standoff, inversion should preferentially weight density measurements acquired with these sectors.
Figure 7: Density images obtained from azimuthal measurements of short-spacing (SS) detector density. From left to right, panels describe: field-measurement image, synthetic density image (simulated from a model constructed with field measurements), density inversion image (constructed from sector-based layer densities inverted from the synthetic density image), density image simulated from inverted layer densities, and density and percent difference between input and simulated (simulated from inverted layer densities) density images.

Figure 8: Density images obtained from azimuthal measurements of long-spacing (LS) detector density. From left to right, panels describe: field-measurement image, synthetic density image (simulated from a model constructed with field measurements), density inversion image (constructed from sector-based layer densities inverted from the synthetic density image), density image simulated from inverted layer densities, and density and percent difference between input and simulated (simulated from inverted layer densities) density images.
Given that our simulations do not include tool standoff, we perform several inversions winnowing out sectors that are the most affected by tool standoff. The first case includes only density measurements acquired with bottom sectors (sectors 8 and 9). For comparison, additional cases include sectors 6 through 11 and all azimuthal sectors. To that end, we construct the vector of measurements $\mathbf{d}$ in Equation (6) with SS- and LS-densities from the selected azimuthal sectors. In all cases, we assume a constant well inclination angle of 77.28 degrees calculated from field measurements using the correlation technique described earlier. This well inclination and the assumption of horizontal layering are equivalent to an apparent dip relative to the borehole axis ($\theta$ in Equation 2) of 12 to 13 degrees.

Figures 11 through 13 show inversion results for the case that includes only bottom sectors (sectors 8 and 9). The density image derived from inversion exhibits azimuthally continuous layer thicknesses and densities. Simulated SS- and LS-detector density images neglect borehole environmental effects. Consequently, single-detector and compensated density images simulated from inversion results show more azimuthally continuous layers than field measurements. This effect causes the upper simulated sector densities to not adequately reproduce the measured density images. Figures 11 and 12 show that simulations performed from inversion results of field density measurements acquired with bottom sectors (sectors 8 and 9) yield minimum depth-averaged errors of 3% and 1.5% for SS- and LS-detector density images, respectively. These minimum percent errors are associated with those sectors included in the inversion. Density simulations across sectors where field measurements are the most affected by tool standoff (upper sectors) yield maximum errors. These errors are as large as 25% and 9% for SS- and LS-detectors, respectively. Large errors reflected on the SS-detector correspond to measurements of low density affected by tool standoff. By contrast, compensated density images simulated from inversion results yield maximum average percent errors of 2.5% with respect to field measurements.

Figure 9: Comparison of sector-based inversion results across azimuthal sectors. The top panel shows the case of synthetic density measurements simulated with a model constructed from field measurements. The bottom panel shows the case of inversion from field measurements. Colored curves show average density differences between azimuthal densities simulated from inversion and azimuthal density measurements. Averages were calculated across the same depth interval. Refer to Figures 7, and 8, and 11 through 13 for details about the corresponding density images.

Figure 10: Comparison of integrated values of porosity-feet calculated from inversion and from compensated density for the case of synthetic azimuthal density simulated with a model constructed from field data. The blue line describes integrated porosity-feet calculated from inversion, whereas the straight red line describes the bottom-quadrant integrated porosity-feet calculated from synthetic compensated density. Letters along the vertical axis designate upper sectors (U), right sectors (R), bottom sectors (B), and left sectors (L). Refer to Figures 7 and 8 for details about inversions and simulations.
Figure 11: Density images obtained from azimuthal measurements of short-spacing (SS) detector density. From left to right, panels describe: field-measurement image, density-inversion image (constructed from sector-based layer densities inverted from the field-density image), density image simulated from inverted layer densities, and density and percent difference between field and simulated (simulated from inverted layer densities) density images. Inversion was performed with bottom sectors only (sectors 8 and 9).

Figure 12: Density images obtained from azimuthal measurements of long-spacing (LS) detector density. From left to right, panels describe: field-measurement image, density-inversion image (constructed from sector-based layer densities inverted from the field-density image), density image simulated from inverted layer densities, and density and percent difference between field and simulated (simulated from inverted layer densities) density images. Inversion was performed with bottom sectors only (sectors 8 and 9).
Figure 13: Density images obtained from azimuthal field measurements of compensated density. From left to right, panels describe: field-measurement image, density-inversion image (constructed from sector-based layer densities inverted from the field-density image), density image simulated from inverted layer densities, and density and percent difference between field and simulated (simulated from inverted layer densities) density images. Inversion was performed with bottom sectors only (sectors 8 and 9).

Figure 14: Density images simulated from inverted layer densities of azimuthal field-compensated density measurements. Left-hand panels show the density image simulated from inverted layer densities using only bottom sectors (sectors 8 through 9), and the corresponding percent difference between field and simulated (simulated from inverted layer densities) density images. Center panels show the density image simulated from inverted layer densities using bottom sectors (sectors 6 through 11), and the corresponding percent difference between field and simulated (simulated from inverted layer densities) density image. Right-hand panels show the density image simulated from inverted layer densities using all sectors (sectors 1 through 16), and the corresponding percent difference between field and simulated (simulated from inverted layer densities) density images.
Figure 14 compares compensated density images simulated from inversion results for the case that includes only bottom sectors (sectors 7 and 8), bottom, right, and left sectors (sectors 6 through 11), against results for the case that includes all azimuthal sectors. Salient observations from the above inversion exercise are that including density measurements adversely affected by environmental effects yields inaccurate estimations of layer densities and thicknesses. Inclusion of upper-sector density measurements that are affected by tool standoff into the inversion increases percent errors in the simulated compensated density images.

Figures 15 shows that the integrated value of porosity-feet from inversion that includes only bottom sectors exhibits relative improvement with respect to that calculated from bottom-sector compensated density measurements. A false apparent increase of porosity-feet occurs for the case when inversion is performed with all sectors. This latter false increase of porosity-feet is due to erroneous low values of density resulting from tool standoff.

In all the examined cases, regardless of input measurements, the inversion-derived integrated porosity-feet was consistently larger than the integrated porosity-feet calculated from field measurements. This result is primarily due to the implicit correction of shoulder-bed effects by the inversion process and illustrates the superiority of inversion for petrophysical and hydrocarbon reserves calculations. In addition, a cutoff is normally implemented on field measurements that further reduces the corresponding integrated porosity-feet.

CONCLUSIONS

Because inversions performed on numerically simulated measurements assumed the same tool configuration and 2D geometrical characteristics of the model as did the simulations, the accuracy of results depended solely on a-priori estimated parameters (i.e., boundary location and dip angle) and non-uniqueness of unknown layer densities. Simulations of synthetic density measurements did not incorporate borehole environmental effects. Estimated dip angles for synthetic measurements yielded maximum differences of 0.5 degrees with respect to actual values. Simulations based on inversion results in wells deviated 60, 70 and 80 degrees reproduced SS-detector azimuthal density with less than 0.01 g/cm\(^3\) difference with respect to input measurements. Simulations for LS-detector and compensated density reproduced all azimuthal sector densities with less than 0.005 g/cm\(^3\) and 0.0025 g/cm\(^3\) error, respectively. Because of increased measurements per layer in HA/HZ wells, redundancy of data decreased non-uniqueness for inversion thereby improving the estimation of layer densities. In all the example cases, inversion consistently reduced shoulder-bed effects. In addition, inversion enabled the quantitative assessment of the reliability and internal consistency of field measurements in the presence of large tool standoffs in the upper sectors, shown as differences between simulations and measurements. Inversion of synthetic data contaminated with random Gaussian noise yielded results comparable to those of the synthetic case described above. We found that the main source of inversion biases was the selection of bed-boundary locations.

Inversion of synthetic density images, numerically simulated from a model constructed from field measurements, yielded accurate simulations of density images. The field-based model was constructed from bottom-sector compensated measurements, which were close to values of formation density, and did not include borehole environmental effects. Comparison of inversion-based, density-image simulations to input density measurements yielded percent errors of less than 0.02 g/cm\(^3\) for SS-, LS-detectors, and less than 0.003 g/cm\(^3\) for compensated density. Additionally, integrated porosity-feet calculated with field-based inversion results increased by 4.4% with respect to that calculated with synthetic compensated density.
For the case of inversions performed on field measurements, we considered field-delivered sector-based densities acquired with the same LWD tool used in our simulations. Because density images constructed from single-detector field measurements were influenced by borehole environmental effects, and simulations did not include tool standoff, we preferentially weighted azimuthal sectors that were the least affected by tool standoff in the inversion. Density images simulated from inversion results that included only bottom sectors reproduced these sectors with less than 3% error with respect to field measurements. Simulations of single-detector density across the upper sectors yielded large values of data misfit because tool standoff was not included in the inversion. However, compensated density images simulated from inversion results yielded average errors smaller than 0.02 g/cm³ across all azimuthal sectors. Incorporating additional sectors of field measurements that exhibited non-zero tool standoff effects did not, in general, improve inversion results. Future work will include the effect of tool standoff in both simulation and inversion of density measurements.

Calculations of integrated porosity-feet over the analyzed measured depth interval quantified the relative worth of inversion compared to standard compensated post-processing of single detector density images. In all cases, inversion yielded a constant value of porosity-feet across azimuthal sectors while compensated density yielded an azimuthally variable value. Integrated porosity-feet values averaged over bottom-quadrant sectors from compensated density images were approximately 6.5 feet or 27.8% less than values calculated from inversion results that included only bottom sectors across the same measured depth interval. By comparison, integrated porosity-feet calculated from inversion results that included additional azimuthal sectors increased by 27% and 64% with respect to that calculated from compensated density measurements. Including density measurements with large tool standoff effects in the inversion may result in biased porosity-feet estimations. For best practices, we recommend that those sectors not be included in the inversion.

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REFERENCES


Mendoza, A., Torres-Verdin, C., and Preeg, W. E., 2009b, Linear iterative refinement method for the rapid


**SYMBOLS AND NOMENCLATURE**

- $\bar{o}$: Null off-diagonal blocks in the matrix of coefficients
- $\alpha$: Azimuth angle, deg.
- $A$: Amplitude of borehole image sinusoid, ft
- $\beta$: Borehole inclination angle, deg.
- $C$: Quadratic cost function, g/cm$^3$
- $D$: Borehole diameter, in
- $\bar{d}$: Vector of density measurements, g/cm$^3$
- $\Delta\rho$: Density correction, g/cm$^3$
- $\bar{e}$: Vector of data residuals, g/cm$^3$
- $EPL$: Effective penetration length, in
- $FSF$: Flux sensitivity function
- $GR$: Gamma-ray, GAPI
- $HA/HZ$: High-Angle/Horizontal
- $\bar{K}$: Coefficient matrix constructed with flux sensitivity functions
- $\lambda$: Stabilization (regularization) parameter
- $LS$: Long-spaced
- $LWD$: Logging while drilling
- $PEF$: Photoelectric factor, b/e
- $\theta$: Formation dip angle, deg.
- $SS$: Short-spaced
- $TST$: True stratigraphic thickness, ft
- $TVD$: True vertical depth, ft
- $TVT$: True vertical thickness, ft
- $\bar{x}$: Vector of the unknown layer densities, g/cm$^3$

**Superscripts**

- $0$: Prescribed reference model
- $t$: Transposition
- $mp$: measurement point

**Subscripts**

- $0$: Indexed density measurement
- $\varepsilon$: Maximum value of density allowed in the solution, g/cm$^3$
- $I, II$: Azimuthal sector number
- $j$: Index for sectors
- $J$: Total number of sectors
- $LS$: Long-spaced detector
- $m$: Index for density measurement points
- $M$: Total number of density measurement points
- $SS$: Short-spaced detector
- $t$: Index for layer number
- $T$: Total number of layers
APPENDIX: STRUCTURE OF THE COEFFICIENT MATRIX

We constructed the matrix of coefficients included in equations (6) and (7) with integrated 2D flux sensitivity functions (FSFs) specifically calculated for commercial LWD density measurements. Entries of the coefficient matrix $\mathbf{K}$ exhibit the following structure:

$$
\mathbf{K} = \begin{bmatrix}
FSF_{SS}^{1,1,1} & \ldots & FSF_{SS}^{1,1,j} & \ldots & FSF_{SS}^{1,1,T} \\
\vdots & \ddots & \vdots & & \vdots \\
FSF_{SS}^{j,1,1} & \ldots & FSF_{SS}^{j,1,j} & \ldots & FSF_{SS}^{j,1,T} \\
\vdots & \ddots & \vdots & & \vdots \\
FSF_{SS}^{J,1,1} & \ldots & FSF_{SS}^{J,1,j} & \ldots & FSF_{SS}^{J,1,T} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
FSF_{LS}^{1,1,1} & \ldots & FSF_{LS}^{1,1,j} & \ldots & FSF_{LS}^{1,1,T} \\
\vdots & \ddots & \vdots & & \vdots \\
FSF_{LS}^{j,1,1} & \ldots & FSF_{LS}^{j,1,j} & \ldots & FSF_{LS}^{j,1,T} \\
\vdots & \ddots & \vdots & & \vdots \\
FSF_{LS}^{J,1,1} & \ldots & FSF_{LS}^{J,1,j} & \ldots & FSF_{LS}^{J,1,T} \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}
\end{bmatrix}
= 0
$$

In the above expression, superscripts $SS$ and $LS$ designate short- and long-spaced detectors, respectively. Likewise, subscripts $j$, $m$, and $t$ designate $j$-th sector number, $m$-th measurement point (along the well trajectory), and $t$-th layer, respectively. Additionally, Subscripts $J$, $M$, and $T$ designate total number of sectors, total number of measurement points, and total number of layers, respectively. The $m$-th pair of $SS$ and $LS$ FSF blocks included as rows in the matrix describes coefficients used for the solution of vector $\mathbf{K}$ by minimizing least-squares residuals for the $m$-th measurement i.e., $d_n(x) - d_n^*$. Each entry $(j,m,t)$ in the matrix $\mathbf{K}$ corresponds to the $SS$- or $LS$-FSF integrated over the measured depth across the $t$-th layer, azimuthally oriented to the $j$-th sector, and located at the depth of the $m$-th measurement point. Because the depth location of the measurement point of the FSF progressively shifts with increasing $m$ or row index, $\mathbf{K}$ includes null off-diagonal blocks, identified with the symbol $0$.