Field Examples of the Combined Petrophysical Inversion of Gamma-ray, Density, and Resistivity Logs acquired in Thinly-bedded Clastic Rock Formations

Jorge A. Sanchez-Ramirez, Carlos Torres-Verdín, David Wolf, Gong Li Wang, Alberto Mendoza, Zhipeng Liu, and Gabriela Schell

ABSTRACT
Limited vertical resolution of logging tools causes shoulder-bed effects on borehole measurements, therefore biases in the assessment of porosity and hydrocarbon saturation across thinly-bedded rock formations. Previously, a combined inversion procedure was developed for induction resistivity and density logs to improve the petrophysical assessment of multi-layer reservoirs. In this paper, we include the inversion of gamma-ray (GR) logs in the interpretation method and evaluate three field cases that comprise hydrocarbon-saturated Tertiary turbidite sequences. On average, inversion results yield 19% better agreement to core measurements and lead to 28% increase in hydrocarbon reserves when compared to standard well-log interpretation procedures. The most critical step for reliable and accurate inversion results is the detection/selection of bed boundaries. Inversion of field data also indicates that the minimum bed thickness resolvable with combined inversion is approximately 0.7ft (0.21m), and that inflection points of density logs are the best option for bed-boundary detection. Combined inversion also allows the detection of noisy, inconsistent, and inadequate measurements, including cases of abnormal measurement-correction biases otherwise difficult to diagnose on processed logs.

INTRODUCTION
Superposition of individual-bed responses near layer boundaries causes shoulder-bed effects on borehole measurements. The corresponding effect on petrophysical interpretation depends on both the physical property being measured and the difference between vertical resolution of the measurement and bed thickness. Deterministic inversion is commonly used to reduce shoulder-bed effects on borehole measurements. In so doing, bed boundaries are first detected and continuous logs are transformed into piecewise-constant vertical distributions of properties, i.e. continuous logs are transformed into blocky-log representations for subsequent petrophysical analysis.

Although the resolution of logging tools depends on the source-sensor arrangement, logging speed, sensor resolution, source strength, volume of investigation, borehole conditions, etc., there are typical values specified in the literature (Dewan, 1983; Passey et al., 2006). On the other hand, layers are generally designated as thick or thin depending on whether their thickness is above or below, respectively, of the vertical resolution of conventional logs. In this paper, we designate layers following Campbell’s (1967) thickness-based classification and the methods that are commonly used to interpret their log responses. Figure 1 shows the classification of layers that we have adopted in our study, as well as typical values of vertical resolution assumed for various logging tools.

Shoulder-bed effects in very thick layers are regarded negligible because in those cases the vertical resolution of the measurements is shorter than the thickness of the layers; i.e. well logs have the resolution necessary to resolve true layer petrophysical properties. Standard log analysis is deemed accurate to estimate hydrocarbon reserves.

The thickness of thick and medium beds is comparable to the vertical resolution of most conventional well logs, whereby logs can be adversely affected by shoulder beds to the point of rendering standard log analysis unreliable.
Deterministic inversion can be used to reduce shoulder-bed effects and improve the estimation of true physical and petrophysical properties of multi-layer sequences.

Conventional well logs cannot resolve thin beds, very thin beds, or laminations because bed thickness is too small when compared to the vertical resolution of logs. Across this type of layers, conventional logs respond to effective-medium properties (macroscopic properties of the formation sensed by the logging tool source-sensor arrangement), average petrophysical properties and net-to-gross, and do not lend themselves to the detection of individual bed boundaries. Without bed boundaries, it is not possible to define multi-layer models. Even if bed boundaries were detected from other measurements (e.g. high-resolution logs, borehole images, or core data), deterministic inversion would be rendered highly non-unique, thereby leading to unstable and inaccurate results.

Different techniques are commonly used to interpret measurements acquired across thin beds, very thin beds, and laminations. The two most common approaches are (a) stochastic inversion to estimate global statistical properties of sedimentary units, and (b) calculation of effective properties via Thomas-Stieber’s approach (Thomas and Stieber, 1975), or by incorporating advanced measurements in the interpretation method such as NMR, multi-component resistivity, or borehole images.

Deterministic inversion has been previously used to refine petrophysical interpretations across thin beds. Fast inversion methods, in particular, have enabled practical field applications over long depth sections of logs. Wang et al. (2009), Zhang et al. (1995), and Zhang et al. (1999) proposed simulation and inversion methods for quick interpretation of array-induction resistivity (AIT) measurements. Mendoza et al. (2007) developed a fast method to simulate density, neutron and GR logs based on linear weighting functions (Flux Scattering Functions, or FSFs) calculated with Monte Carlo simulations of particle-level transport in homogenous media. Liu et al. (2007) developed a method for combined inversion of density and resistivity logs based on Mendoza et al.’s (2007) fast linear simulation method and Wang et al.’s (2009) fast nonlinear inversion methods.

The combined inversion developed in this paper is an extension of Liu et al.’s (2007) joint inversion method for density and resistivity logs in which we have also included GR logs as input data. Inversion of GR logs is necessary because of their low vertical resolution compared to density and resistivity logs, and because of the importance of GR logs in the calculation of volumetric concentration of shale, \( V_{sh} \).

Previously published projects on well-log inversion do not report examples of the integration of different field measurements for improved petrophysical evaluation. Examples are limited to single-measurement inversions of either synthetic or field data (Dyos, 1987; Freedman and Minerbo, 1989; Frenkel and Mezzatesta, 1998; Gao and Torres-Verdin, 2003; Hakvoort, 1999; Tabarovsky and Rabinovich, 1996; Woodhouse et al., 1984). The main objective of this paper is to test the combined inversion approach on challenging field examples that include core data for cross-validation of results. In addition, we seek to compile a set of best practices for the routine application of combined inversion of field logs.

The field cases analyzed in this paper correspond to turbidite sedimentary sequences in the deepwater Gulf of Mexico. Layer thicknesses range from thin laminations to very thick beds. This wide range of thickness variations prompts us to divide the field applications of combined inversion into two groups: (a) cases of combined inversion for medium, thick, and very thick layers, and (b) cases of laminar models for thin beds and laminations. In this paper, we focus our attention to the first group of applications. Sanchez-Ramirez (2009) describes examples of combined inversion for the second group of applications.

**METHOD**

The inversion method assumes a vertical wellbore penetrating a stack of horizontal layers, as well as axially symmetric mud-filtrate invasion. Moreover, we assume layer-by-layer isotropic electrical resistivity.

Combined inversion consists of four main sequential steps: (a) detection of bed boundaries, (b) separate inversion of density, GR, and resistivity logs, (c) forward simulation of inverted layer properties to calculate data misfits, and (d) calculation of petrophysical properties from inversion results.

**Bed Boundary Selection**

Bed boundaries are detected based on inflection points of a master log. In turn, inflection points are determined by

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1Mark of Schlumberger
calculating the variance of the master log within a sliding depth window, and by placing a bed boundary wherever the variance increases above a pre-specified threshold.

**Forward Simulation and Inversion**

After bed-boundary selection, we perform separate inversion of each measurement based on available raw data (mono-sensor data for density, standard resolution GR, and single-array borehole-corrected conductivity measurements for induction resistivity) to calculate layer-by-layer density, GR, and resistivity.

The inversion of density and GR logs is based on Mendoza et al.'s (2007) rapid linear iterative refinement simulation method for borehole nuclear measurements using approximate FSFs constructed for general density and GR tools (UT Longhorn Tools). The FSFs are averaged radially (one-dimensional, 1D FSF) for their implementation to invert density and GR logs into layer-by-layer values of bulk density and total GR.

Inversion of AIT induction resistivity measurements is performed with a fast nonlinear minimization procedure developed by Wang et al. (2009). Inputs to this inversion are raw sub-array resistivity measurements; outputs are layer-by-layer values of invaded-zone resistivity \( \rho_{inv} \), true resistivity \( \rho_t \), and radial length of mud-filtrate invasion.

Deterministic inversion is formulated as the minimization of the quadratic cost function

\[
C(\mathbf{x}) = \frac{1}{2} \left[ \mathbf{e}(\mathbf{x})^T \mathbf{e}(\mathbf{x}) + \lambda^2 \| \mathbf{x} - \mathbf{x}_0 \|^2 \right],
\]

(1)

where \( \mathbf{x} \) is a vector containing the unknown layer properties, \( \mathbf{e}(\mathbf{x}) \) is the vector of data misfits or data residuals, and \( \lambda \) is a stabilization parameter that biases the solution \( \mathbf{x} \) to be close to certain prior, \( \mathbf{x}_0 \) (Hansen, 1998). The stabilization (regularization) additive term in the cost function is necessary to reduce instability in the solution due to noisy and inadequate measurements. In our case, \( \mathbf{x}_0 \) is equal to a representative reading of well logs within the inverted depth interval, which can be the average, minimum, or maximum value depending on the behavior of well logs.

The vector of data residuals is written as

\[
\mathbf{e}(\mathbf{x}) = \mathbf{d}(\mathbf{x}) - \mathbf{d}^0,
\]

(2)

where \( \mathbf{d}(\mathbf{x}) \) is the vector of numerically simulated log values, and \( \mathbf{d}^0 \) is the vector of field measurements. Data misfits quantify and verify the consistency of inverted results with available measurements.

For GR and density log inversion, we assume linearity between measurements and layer properties, whereupon numerically simulated logs are given by

\[
\mathbf{d}(\mathbf{x}) = \mathbf{FSF}(\mathbf{x})
\]

(3)

where the matrix \( \mathbf{FSF} \) is constructed with 1D (radially averaged) FSFs. Substitution of equation (3) into (1) yields

\[
C(\mathbf{x}) = \frac{1}{2} \left[ \mathbf{FSF}(\mathbf{x} - \mathbf{d}^0)^T \mathbf{FSF}(\mathbf{x} - \mathbf{d}^0) + \lambda^2 \| \mathbf{x} - \mathbf{x}_0 \|^2 \right],
\]

(4)

which we minimize using Tikhonov and Arsenin's (1977) regularization method.

On the other hand, inversion of induction resistivity measurements into layer-by-layer resistivity values requires nonlinear minimization. Accordingly, we approach the minimization of resistivity measurements with linear iterative steps via the Gauss-Newton method together with an efficient approximation of Fréchet derivatives (Wang et al., 2009).

**Petrophysical Interpretation**

After completing the three separate inversions, results are combined using a shaly-sand model (i.e. Waxman-Smits, Juhasz, Dual-Water, etc.) to calculate layer-by-layer porosity and saturation. The latter calculations are subsequently compared to basic log interpretation using the same shaly-sand model and associated parameters \( (a, m, n, R_w, \text{etc.}) \).

For instance, Best et. al.'s (1980) dual-water equation is given by

\[
\frac{1}{R_f} = \phi_f S_{sw} \left[ \frac{1}{R_w} + \frac{S_{wb}}{S_{sw}} \left( \frac{1}{R_{wb}} - \frac{1}{R_w} \right) \right],
\]

(5)

where \( R_f \) is true formation resistivity, \( \phi_f \) is total porosity, \( S_{sw} \) is total water saturation, \( S_{wb} \) is bound-water saturation, \( R_w \) is connate water resistivity, \( R_{wb} \) is bound-water resistivity, \( a \) is the tortuosity factor, and \( m \) and \( n \) are the cementation (lithology) and saturation exponents, respectively.

Non-shale porosity \( (\phi_n) \) is calculated using the density composition equation for a unit rock volume, given by

\[
\rho_b = \rho_{sh} V_{sh} + \rho_{m} \left( 1 - V_{sh} - \phi_n \right) + \left[ \rho_{sw} S_{we} + \rho_{h} \left( 1 - S_{we} \right) \right] \phi_e,
\]

(6)

where \( \rho_b \) is bulk rock density, \( \rho_{sh} \) is density of wet shale, \( \rho_m \) is density of matrix, \( \rho_{sw} \) is density of water, \( \rho_{h} \) is density of hydrocarbon, \( V_{sh} \) is volumetric concentration of wet shale, and \( S_{we} \) is non-shale water saturation.

It follows that

\[
\phi_e = \frac{\rho_b - \rho_{sh} V_{sh} - \rho_{m} \left( 1 - V_{sh} \right)}{\rho_{sw} S_{we} + \rho_{h} \left( 1 - S_{we} \right) - \rho_{sw}}.
\]

(7)

The expressions used to calculate \( S_{we}, S_{wb} \) and \( \phi_e \) are given by

\[
S_{we} = \frac{S_{sw} - S_{wb}}{1 - S_{wb}},
\]

(8)
where \( \phi_{sh} \) is total porosity of wet shale. This value can be quantified in a nearby shale based on NMR logs, or calculated using the equation below that requires knowledge of both density of wet shale \( (\rho_{sh}) \) and density of shale’s dry constituent \( (\rho_{sh-dry}) \):

\[
\phi_{sh} = \frac{\rho_{sh} - \rho_{sh-dry}}{\rho_w - \rho_{sh-dry}} .
\]  

Volumetric concentration of wet shale \( (V_{sh}) \) is calculated using a linear shale index, given by

\[
V_{sh} = \frac{GR - GR_s}{GR_{sh} - GR_s} ,
\]  

where \( GR \) is total gamma-ray reading, \( GR_s \) is gamma-ray reading in clean sands, and \( GR_{sh} \) is gamma-ray reading in shales.

Hydrocarbon pore-volume \( (HPV) \) and net-to-gross ratio \( (NGR) \) are calculated with the algorithm

\[
\text{if } V_{sh} > V_{sh-cf} \text{ and } \phi_e < \phi_{e-cf} \text{ or } S_{we} > S_{we-cf} ,
\]

then \( \text{pay}_j = 0 \) else \( \text{pay}_j = 1 \),

\[
HPV = \sum_{j=1}^{n} \phi_e \left(1 - S_{we} \right) \Delta h_j \text{pay}_j ,
\]

\[
NGR = \left( \frac{\sum_{j=1}^{n} \Delta h_j \text{pay}_j}{\sum_{j=1}^{n} \Delta h_j} \right) ,
\]  

We converted inversion results to layer-by-layer petrophysical properties using the Dual-Water model with the parameters listed in Table 1. A dispersed-shaly-sand model such as dual-water is necessary for interpretation because of evidence of dispersed clay in sand units. For example, volumetric shale concentration measured with laser grain tests (LGTs) for Wells No. 2 and 3 in sandy medium and thick beds exhibits variations from 3% to 30%, and up to 70% in thin beds and laminations. Figure 3 shows an example of illite coating the surface of quartz grains in Well No. 3 at 10.6 ft. Alternatively, Well No. 1 could have been analyzed with Archie’s equation because sand units are relatively clean \( (V_{sh} \text{ less than 20%}) \).

Invasion was not considered in the interpretation of field cases because of the lack of measurable contrast between deep- and shallow-sensing apparent resistivities. This lack of radial resistivity contrast is due to the absence of free water and presence of OBM in the sand units under study.

FIELD CASES OF COMBINED INVERSION

We implemented the combined inversion of density, GR, and induction resistivity logs with data acquired in three wells within three different fields located in the deepwater Gulf of Mexico. Formations under consideration are unconsolidated to poorly consolidated shaly-sand sequences in turbidites depositional systems. Sedimentary units are channelized sand-sheet systems anywhere from mid-slope to basin-floor positions. Figure 2 shows the outcrop of a turbidite channel margin where the distribution and thickness of sand-shale layers is similar to that of the field examples discussed in this section.

Depth segments chosen for inversion are sequences of oil-bearing sands and shale layers. Whole and sidewall core samples were recovered for cross-verification of measurements and interpretations.
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Table 2 shows NGR and HPV calculations for the analyzed wells using four sets of logs for GR, bulk rock density ($\rho_b$), and true resistivity ($R_t$). The sets of logs comprise standard resolution (SR) logs [SR, SR, SR], high resolution (HR) logs [HR, HR, HR], SR GR with layer-by-layer density and resistivity estimated from separate inversion [SR, SI, SI], and combined results from separate inversions [SI, SI, SI]. Table 3 describes the agreement of interpretation results with core measurements using the above-described four sets of well logs. We quantify the agreement to core measurements with both the linear correlation coefficient ($R^2$) and the average relative error ($RE$), given by

$$R^2 = \frac{\sum_{j=1}^{n} x_{calc,j} - x_{core,j}}{\eta \sum_{j=1}^{n} x_{calc,j}} \left( \frac{\sum_{j=1}^{n} x_{calc,j}^2}{\sum_{j=1}^{n} x_{core,j}^2} \right)$$

and

$$RE = \frac{1}{\eta} \sum_{j=1}^{n} \left( \frac{x_{core,j} - x_{calc,j}}{x_{core,j}} \right)^2 \times 100\%,$$

respectively, where $x_{calc,j}$ designates the $j$-th core measurement, $x_{core,j}$ designates the $j$-th calculated property, $\eta$ are the number of core measurements, and $j$ is the index of the summation.

Tables 4 and 5 describe the percent changes of (a) hydrocarbon reserves calculations and (b) agreement with core data, respectively, with respect to SR results [SR, SR, SR].

Well No. 1

The analyzed depth interval is composed of three sand units separated by heterolithic, mud rich shaly intervals logged with CNT-LDT-SGT-AIT2, CMR-GR2 and OBMI-GR2. Core measurements indicate that the three sand units exhibit similar petrophysical properties; however, their log responses are different. Log differences are due to shoulder-bed effects because the sand units represent thick beds with thicknesses (1.75ft, 1.7ft and 3.8ft - 0.53m, 0.52m, and 1.16m) comparable to the vertical resolution of logging tools.

Figure 4 shows the corresponding inversion results and data misfits for this case. Well logs resolve the thicker lower sand unit but cannot resolve the petrophysical properties of the thinner upper layers. Figure 5 indicates that HR measurements are closer to inverted results than SR logs.

Figure 6 shows core data, as well as $V_{sh}$, $S_{w}$, $S_{wt}$ and $\phi$ calculated from both standard resolution measurements and inverted results using the dual-water model with the properties listed in Table 1. This figure indicates that interpretations performed with inverted results agree better with core measurements than those obtained from standard well-log interpretation using SR logs. Table 5 quantifies the corresponding percent improvement with respect to standard well-log interpretation as 5% on average for $R^2$ and 38% as the average reduction for $RE$.

In the laminated sections of the well (e.g. 72.5ft to 78ft), logs measure effective shale- and sand-layer properties, whereas bed boundaries cannot be detected from inflection

1LS stands for long spacing density.
2SS stands for short spacing density.

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Table 1: Summary of petrophysical parameters assumed for the dual-water petrophysical interpretation of Wells No. 1, 2 and 3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Well No. 1</th>
<th>Well No. 2</th>
<th>Well No. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{w} - \phi$</td>
<td></td>
<td>DualW.</td>
<td>DualW.</td>
<td>DualW.</td>
</tr>
<tr>
<td>$GR, \rho_b$</td>
<td>GAPI</td>
<td>18</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>$GR, \rho_b$</td>
<td>GAPI</td>
<td>86</td>
<td>95</td>
<td>125</td>
</tr>
<tr>
<td>$V_d$ model</td>
<td>Linear</td>
<td>Linear</td>
<td>Linear</td>
<td>Linear</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$K$</td>
<td>-</td>
<td>1.80</td>
<td>1.80</td>
<td>1.80</td>
</tr>
<tr>
<td>$n$</td>
<td>-</td>
<td>2.10</td>
<td>2.00</td>
<td>1.71</td>
</tr>
<tr>
<td>$R_s$</td>
<td>ohm.m</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>$R_{sh}$</td>
<td>ohm.m</td>
<td>0.06</td>
<td>0.06</td>
<td>0.035</td>
</tr>
<tr>
<td>$\phi_{irr}$</td>
<td>v/v</td>
<td>0.16</td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>gm/cc</td>
<td>2.65</td>
<td>2.65</td>
<td>2.65</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>gm/cc</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>gm/cc</td>
<td>0.852</td>
<td>0.852</td>
<td>0.800</td>
</tr>
<tr>
<td>$\rho_{sh}$</td>
<td>gm/cc</td>
<td>2.42</td>
<td>2.45</td>
<td>2.536</td>
</tr>
</tbody>
</table>

Cutoffs for Reserves Calculation

| $S_{water}$       | -        | 0.65       | 0.65       | 0.65       |
| $S_{oil}$         | -        | 0.15       | 0.15       | 0.15       |
| $V_{sh}$          | -        | 0.40       | 0.40       | 0.40       |

Inversion-Related Parameters

<table>
<thead>
<tr>
<th>LS used</th>
<th>SS used</th>
<th>LS FSF factor</th>
<th>GR FSF factor</th>
<th>Regularization GR</th>
<th>Regularization Den.</th>
<th>AIT sub-arrays</th>
<th>AIT type</th>
<th>AIT lower window</th>
<th>AIT central window</th>
<th>AIT upper window</th>
<th>AIT noise level</th>
<th>AIT iterations</th>
<th>AIT regular factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>2 to 8</td>
<td>B</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>0.0</td>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

$^{1}$LS stands for long spacing density.
$^{2}$SS stands for short spacing density.

$^{3}$Marks of Schlumberger
Sanchez-Ramirez et al.

**TABLE 2: Summary of calculated hydrocarbon reserves using inverted results, SR, and HR logs.**

<table>
<thead>
<tr>
<th>Type of Logs Used&lt;sup&gt;3&lt;/sup&gt;</th>
<th>Well No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GR )</td>
<td>( \rho_b )</td>
<td>( R_t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SR</td>
<td>SR</td>
<td>0.110</td>
<td>0.355</td>
<td>0.349</td>
</tr>
<tr>
<td>HR</td>
<td>HR</td>
<td>0.124</td>
<td>0.346</td>
<td>0.371</td>
</tr>
<tr>
<td>SR</td>
<td>SI</td>
<td>0.108</td>
<td>0.348</td>
<td>0.369</td>
</tr>
<tr>
<td>SI</td>
<td>SI</td>
<td>0.135</td>
<td>0.364</td>
<td>0.418</td>
</tr>
</tbody>
</table>

**HPV (hydrocarbon ft)**

<table>
<thead>
<tr>
<th>Type of Logs Used</th>
<th>Well No.</th>
<th>1.12</th>
<th>10.79</th>
<th>9.72</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>SR</td>
<td>1.12</td>
<td>10.79</td>
<td>9.72</td>
</tr>
<tr>
<td>HR</td>
<td>HR</td>
<td>1.35</td>
<td>10.97</td>
<td>10.43</td>
</tr>
<tr>
<td>SR</td>
<td>SI</td>
<td>1.41</td>
<td>11.27</td>
<td>10.53</td>
</tr>
<tr>
<td>SI</td>
<td>SI</td>
<td>1.73</td>
<td>11.77</td>
<td>11.65</td>
</tr>
</tbody>
</table>

<sup>3</sup>SR stands for standard resolution, HR for high resolution, and SI for separate inversion.

**TABLE 4: Percent changes of reserves calculations with respect to SR calculations.**

<table>
<thead>
<tr>
<th>Type of Logs Used</th>
<th>Well No.</th>
<th>Aver</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GR )</td>
<td>( \rho_b )</td>
<td>( R_t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SR</td>
<td>SR</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HR</td>
<td>HR</td>
<td>12.7</td>
<td>-2.5</td>
<td>6.3</td>
<td>5.5</td>
</tr>
<tr>
<td>SR</td>
<td>SI</td>
<td>-1.8</td>
<td>-2.0</td>
<td>5.7</td>
<td>0.6</td>
</tr>
<tr>
<td>SI</td>
<td>SI</td>
<td>22.7</td>
<td>2.5</td>
<td>19.8</td>
<td>15.0</td>
</tr>
</tbody>
</table>

**HPV Change Respect SR Logs (%)**

<table>
<thead>
<tr>
<th>Type of Logs Used</th>
<th>Well No.</th>
<th>Aver</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR</td>
<td>SR</td>
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<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>HR</td>
<td>HR</td>
<td>20.5</td>
<td>1.7</td>
<td>7.3</td>
<td>9.8</td>
</tr>
<tr>
<td>SR</td>
<td>SI</td>
<td>25.9</td>
<td>4.4</td>
<td>8.3</td>
<td>12.9</td>
</tr>
<tr>
<td>SI</td>
<td>SI</td>
<td>54.5</td>
<td>9.1</td>
<td>19.9</td>
<td>27.8</td>
</tr>
</tbody>
</table>

**TABLE 3: Summary of calculated correlation between core measurements and calculations from inverted results (SI), SR, and HR logs.**

<table>
<thead>
<tr>
<th>Type of Logs Used</th>
<th>Well No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GR )</td>
<td>( \rho_b )</td>
<td>( R_t )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SR</td>
<td>SR</td>
<td>0.708</td>
<td>0.761</td>
<td>0.413</td>
</tr>
<tr>
<td>HR</td>
<td>HR</td>
<td>0.696</td>
<td>0.782</td>
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</tr>
<tr>
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<td>SI</td>
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<tr>
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<td>SR</td>
<td>143.0</td>
<td>68.1</td>
<td>68.4</td>
</tr>
<tr>
<td>HR</td>
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<td>70.0</td>
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<td>143.0</td>
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<td>SI</td>
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<td>58.1</td>
<td>62.6</td>
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<tr>
<td>HR</td>
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<td>SR</td>
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<td>SI</td>
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</tr>
<tr>
<td>SR</td>
<td>SR</td>
<td>23.0</td>
<td>10.3</td>
<td>12.7</td>
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<td>HR</td>
<td>HR</td>
<td>23.2</td>
<td>12.6</td>
<td>14.9</td>
</tr>
<tr>
<td>SR</td>
<td>SI</td>
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<tr>
<td>SI</td>
<td>SI</td>
<td>23.0</td>
<td>10.2</td>
<td>11.4</td>
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<td>SR</td>
<td>0.675</td>
<td>0.583</td>
<td>0.243</td>
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<tr>
<td>HR</td>
<td>HR</td>
<td>0.698</td>
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<td>SR</td>
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<td>0.658</td>
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<td>SI</td>
<td>SI</td>
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<td>HR</td>
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<td>SI</td>
<td>SI</td>
<td>58.0</td>
<td>38.2</td>
<td>32.8</td>
</tr>
</tbody>
</table>
FIG 4. Inversion results for Well No. 1. Track 1: field GR, simulated GR, GR from inversion, bit size (BS), and caliper. Track 2: relative depth. Track 3: 10in, 20in, 30in, 60in, 90in two-foot resolution apparent resistivities; and $R_t$ from inversion. Track 4: neutron porosity, LS density, simulated LS density, and density from inversion. Track 5: data misfits for GR and density. Track 6: data misfits for induction sub-array resistivities. Track 7: core photograph.

FIG 5. Comparison of inversion results, SR, and HR logs for Well No. 1. Track 1: SR GR, HR GR, GR from inversion, BS, and caliper. Track 2: relative depth. Track 3: one-foot, two-foot, and four-foot 90in resistivities; and $R_t$ from inversion. Track 4: neutron porosity, SR density, HR density, and density from inversion. Track 5: core photograph.
points of logs. Moreover, if bed boundaries were selected from a borehole image or core data, deterministic inversion would be rendered highly non-unique (hence unstable).

Parenthetically, we performed the same analysis on well logs acquired in a side track of Well No. 1 obtaining practically equivalent results. However, core data were not available in that well for cross-validation and therefore inversion results are not reported here.

Well No. 2

The thickness of the analyzed section is 150ft (45m), which is composed of amalgamated and non-amalgamated sheet sands intercalated with heterolithic shaly sands. The formation was logged with PEX-AIT-CMR² and OBMI-DSI-GR². Figure 7 compares inversion results to core measurements. From Table 5, we observe that the percent match to core measurements improved 30% for $R^2$ and 17% for $RE$ relative to standard well-log interpretation. Likewise, HPV calculations increased by approximately 9% when inversion results were used in the interpretation (see Table 4).

Well No. 3

The thickness of the analyzed section is 260ft (79m). It consists of amalgamated and non-amalgamated sheet sands intercalated with heterolithic shaly sands that were logged with HAPS-HLDS-QAIT-SGT². Inversion results improve the agreement with core measurements (15% in terms of $R^2$ and 9% in terms of $RE$) compared to results obtained with standard well-log interpretation.

Figure 8 is a close-up view of one of the core sections together with inversion results. The sharpening of layers that results from inversion is very close to that of bed boundaries observed in the core photograph, whereby the pay flag calculated from inversion results is higher than the one calculated with standard well-log interpretation in this well.

For this example, the value of $n=1.71$ used for the shaly-sand model was obtained from SCAL and yields the best overall results in the analyzed 260ft section. It is important to emphasize that changing $n$ (e.g. from 1.71 to 2.00) does not affect the overall statistical results reported in Tables 4 and 5.

Summary of Inversion Results for Wells No. 1, 2, and 3

From Table 4 we conclude that hydrocarbon reserves calculated using inversion results are higher than those calculated with standard interpretation of SR or HR logs. On average, for the three wells inversion increases NGR by approximately 15%, and HPV by approximately 28% with respect to standard interpretation of SR logs. From Table 5 we conclude that the agreement of interpretations performed from inverted results to core measurements is higher than that of standard interpretations performed with SR or HR logs. On average, the agreement to core measurements for the analyzed field cases improved by 19% with respect to standard interpretations of SR logs, where 19% is the arithmetic mean of 16% improvement in terms of $R^2$, and 21% improvement in terms of $RE$.

The impact of GR inversion in the combined inversion can be assessed from results summarized in Tables 4 and 5. Specifically, the inclusion of GR inversion into the joint inversion of density and resistivity logs resulted in the following improvements/reductions: 14% improvement of NGR, 15% improvement of HPV, 8% improvement of $R^2$, and 15% reduction of $RE$.

RECOMMENDED PRACTICES

Although the application of deterministic inversion is relatively simple, experience with the field cases described above allows us to generalize recommendations for best interpretation practices and quality control. In what follows, we summarize specific recommendations for log operations, selection of input logs, selection/detection of bed boundaries, preparation of raw input data, selection of...
**FIG 6.** Comparison of core measurements against interpretations from inversion and SR logs for Well No. 1. Track 1: $V_{sh}$ calculated from SR GR and inverted GR, and core $V_{sh}$. Track 2: relative depth. Track 3: 10in and 90in deep two-foot resolution apparent resistivities, and $R_t$ from inversion. Track 4: neutron porosity, LS density, simulated LS density, and density from inversion. Track 5: data misfits for GR and density. Track 6: data misfits for induction sub-array resistivities. Track 7: $S_w$ and $S_o$ calculated from SR logs and inversions results, and core $S_w$. Track 8: $\phi_t$ and $\phi_e$ calculated from SR logs and inversions results, and core $\phi_t$. Track 9: the core photograph.

**FIG 7.** Comparison of core data against interpretation results from inversion and SR logs for Well No. 2. Refer to Figure 6 for the corresponding log track description.
of measurement sensors, use of regularization, stretching of sensitivity functions, and evaluation and inspection of data misfits.

Log Operations

Some aspects of log operations to be considered and evaluated for adequate inversion results are the following:

• Although deterministic inversion in this study was implemented using only three conventional logs (i.e. density, GR, and induction resistivity), additional information such as borehole images, high-resolution logs and core data is necessary for selection/detection of bed boundaries and for cross-validation of inversion results.

• Repeat log sections along inspected depth intervals are recommended for verification of both selection of bed boundaries and quality of input measurements. The down log could be used in place of a repeat log section for GR and resistivity data whenever operation constraints do not allow repeat log passes.

• Logging at low speeds improves the repeatability of density and GR logs. High-resolution logging speeds are recommended to secure acceptable data quality.

• Smooth wireline cable motion is necessary to avoid depth mismatches between logs.

• GR, density, and resistivity logs should be acquired with the same tool string to minimize depth mismatches.

Selection of Input Logs

In addition to the typical curves required for standard log interpretation, the data below is required for inversion:

• Density inversion requires unfiltered mono-sensor density measurements, which consist of short-spacing (SS) and long-spacing (LS) density logs. If mono-sensor data are not available, then approximate inversion can be performed with the compensated density provided by wireline service companies. This procedure assumes that LS and compensated densities are very similar, which is the case in the presence of small environmental corrections.

• Currently, software modules developed by UT-Austin’s Joint Industry Research Consortium on Formation Evaluation only include resistivity-inversion algorithms for array-induction resistivity measurements AIT-B or AIT-H (Barber and Rosthal, 1991). The inversion algorithm requires borehole-corrected in-phase signals to be used as input.

• GR inversion can be implemented with processed SR or raw GR logs as long as the corresponding FSF is adjusted for each case. The inversion examples described above

FIG 8. Comparison of core data against interpretation results from inversion, HR, and SR logs for the interval 90-135ft of Well No. 3. Track 1: Vs calculations. Track 2: relative depth and pay-flag calculations. Track 3: 90in deep one-foot and two-foot resolution apparent resistivities, and R from inversion. Track 4: SR, HR, and inverted density. Track 5: Sw results. Track 6: $\phi$ results. A core photograph is superimposed on all but the second track.
Field Examples of the Combined Petrophysical Inversion of Gamma-ray, Density, and Resistivity Logs acquired in Thinly-bedded Clastic Rock Formations

were implemented with FSFs for SR GR.

- The sampling rate of input well logs should be high enough to include at least a few measurements within every layer. If the minimum bed thickness is close to 0.7 ft (0.21m), then a sampling rate of 0.1-0.2 ft (0.03-0.06m) will lead to satisfactory results. Very coarse sampling rates can lead to ill-posed inversion that will require stabilization (regularization).

Detection of Bed Boundaries

As implemented in this paper, inversion assumes a subsurface model consisting of stacked horizontal beds. Selection of layer boundaries constitutes one of the most critical steps for inversion due to the high sensitivity of the results to bed-boundary locations. Based on the field cases described above, we found that the minimum bed thickness resolvable with combined inversion is about 0.7ft. The field example for Well No. 2 included a few thin beds of approximately 0.7ft thickness wherein inversion results were satisfactory. Additionally, we found that the best approach to select bed boundaries was to identify inflection points of the LS mono-sensor density log or of the standard compensated density. We prefer SR to HR density because the latter may lead to both very thin beds where inversion is not accurate or incorrect bed boundaries can be obtained due to irregular spikes and log fluctuations (see Figure 8 for an example of irregular spikes in HR logs).

Density logs are preferred for the selection of bed boundaries over resistivity and GR logs because density measurements have better vertical resolution, and because density logs have the largest impact on the calculation of porosity (see equation 7). Regarding the use of AIT data, AIT raw measurements should not be used for detecting bed boundaries because they are not depth-aligned, and computed resistivity channels (i.e. medium, deep, etc) are not recommended because they are already processed in a way that can bias the detection of bed boundaries.

The inflection-point approach to detect bed boundaries fails when layers are thinner than the vertical resolution of well logs. Figure 9 shows an example of this situation. The 0.5ft thin bed in the depth interval from 9.75ft to 10.25ft appears thicker than actual based on density and GR logs. For example, GR inflection points will indicate that the thickness of the layer is 1.5 ft, spanning the depth interval from 9.2ft to 10.7ft. The same figure illustrates the lack of depth-alignment of AIT raw data.

Another advantage of choosing the density log to detect bed boundaries is that density measurements are associated with symmetrical forms of FSFs (Mendoza et al., 2007). Measurements associated with non-symmetrical FSFs (e.g. neutron measurements) can bias the detection of bed boundaries. Although induction measurements should not be associated with linear weighting functions (such as nuclear FSFs), in general they exhibit a non-symmetrical behavior (e.g. AIT-H), hence should not be used to detect bed boundaries.

Core data, HR logs, and borehole images constitute an important source of information to confirm bed boundaries. However, they are not recommended for direct bed-boundary detection because in general they are not acquired with the same string of conventional logging tools and may exhibit significant depth mismatches. This behavior is especially true for whole core measurements that are retrieved from depth segments specified by drillers.

As emphasized earlier, bed-boundary detection is perhaps the most critical step in the combined inversion. To illustrate the importance of bed-boundary detection, Figure 10 shows the effect of shifting bed boundaries on GR inversion.
in Well No. 1. Bed boundaries were randomly shifted ±0.2 ft (0.06m) with respect to their originally detected location. We observe measurable effects on inverted values resulting from the shift of bed boundaries. It is expected that shifts in bed-boundary location will have more pronounced effects on inverted values of thin beds than of thick beds.

**Regularization of GR and Density Logs**

Regularization (stabilization) of inversion is usually required for thin and medium-thickness beds depending on both sampling rate and measurement noise. Figure 11 illustrates the effect of Tikhonov-type regularization with different values of $\lambda$ used in the inversion of GR. The plot in the left-hand panel shows GR inversion results without regularization ($\lambda=0$). GR inversion results are out of range (-4310 GAPI to 4117 GAPI) in the depth interval 50.7-54.3 ft, where bed thickness ranges from 0.6 ft to 1.12 ft. These incorrect layer values disappear after applying regularization with $\lambda=0.1$, as shown in the center panel. The right-hand panel indicates that a large value of regularization parameter, namely, $\lambda=10$, suppresses the enhancement of inversion, causing the GR layer values obtained from inversion to be equal to representative well-log values ($\lambda_0$ in equation 1).

**“Stretching” of Flux-Scattering Sensitivity Functions**

Density and GR FSFs for specific commercial tools are not available for inversion. To circumvent this limitation, we adjusted our “UT Longhorn Tool” FSFs to replicate FSFs of commercial tools. We found that one way to improve the match between field and numerically simulated logs was to “stretch” the FSF by a constant factor. Table 6 indicates that similar stretching factors were estimated for the field cases of study. If the factor is above 1.0, then the FSF is stretched and consequently the vertical resolution of inversion results decreases. On the other hand, if the factor is below 1.0, the FSF is compressed (“squeezed”) and consequently the vertical resolution of inversion results increases.

Figure 12 shows GR inversion results for Well No. 2 using stretching factors of 1.0 and 2.1. We observe that a stretching factor of 2.1 provides a better agreement between numerically simulated and field logs, whereas inversion results show an enhancement of layer properties with respect to original logs. Moreover, we remark that the vertical resolution of GR FSF stretched by a factor of 2.1 (which is 24.9 in (0.63m)) is in very good agreement with the reported

<table>
<thead>
<tr>
<th>Well No.</th>
<th>GR Tool</th>
<th>Density Tool</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>SGTL</td>
<td>LDT LS</td>
</tr>
<tr>
<td>1 ST1</td>
<td>2.10</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>2.16</td>
<td>1.30</td>
</tr>
<tr>
<td>3</td>
<td>2.04</td>
<td>1.10</td>
</tr>
</tbody>
</table>

*Additional well data used to calculate FSF stretching factors.*

**FIG 11.** Example of inversion of GR data without ($\lambda=0$, left-hand panel) and with regularization ($\lambda=0.1$, center panel, and 10, right-hand panel) within a depth section with medium-thickness layers that lead to ill-posed (unstable) inversion.

**FIG 12.** Example of GR inversion for Well No. 2 using the general FSF (left-hand panel) and the same general FSF stretched by a factor equal to 2.1 (right-hand panel).
24 in (0.61 m) of vertical resolution for SGT tools. Figure 13 shows the GR FSF stretched by a factor of 2.1 and illustrates the procedure used to estimate log vertical resolution based on a detection threshold of 90%.

The stretching of FSFs is not only due to differences in tool configuration (e.g. source-sensor distance and arrangement) but also to smoothing, resolution matching, and filtering of raw data. For example, the SS density available in field cases was depth- and resolution-matched to the LS density. This procedure causes the SS data to lose vertical resolution. Similarly, the GR data used for inversion of field cases was smoothed with a 3-point running filter given by the equation below

$$GR_i = 0.5 (0.5GR_{raw}^{i+1} + GR_i^{raw} + 0.5GR_{raw}^{i-1}),$$

where $GR_{raw}$ is the raw GR at a given depth $i$, $GR_i$ is the filtered or SR GR at the same depth $i$, with the subscripts $i+1$ and $i-1$ used to identify samples above and below, respectively, the current analyzed sample, $i$.

### Analysis of Data Misfits

Data misfits quantify the difference between measured and numerically simulated data logs after inversion. Misfit errors will be close to zero when inverted values are reliable and measurements are noise free. However, in the course of this study we found the confidence ranges for data misfits shown in Table 7. The lowest data misfits are expected for density-log inversion given that bed boundaries are selected from density logs. For the case of resistivity logs, data misfits can be as high as 50% when the resistivity contrast is very high.

Large data misfits can be due to four groups of possibilities. First, additional layers might be necessary to construct the multi-layer model. Second, there are several model assumptions that are not met, for example horizontal layers, sharp invasion fronts, linear inversion for GR and density logs, etc. Third, there are depth mismatches among well logs. Last but not least, data misfits could be due to adverse borehole environmental conditions (e.g. washouts, mudcake, etc.) or tool malfunctioning.

Although the addition of extra layers in general decreases the data misfit, extra layers should not be included in the multi-layer model if they are not validated with borehole images, core data, HR logs, repeat sections, downs logs, etc.

### Mono-Sensor Selection

Different combinations of sensors can be selected for density and resistivity log inversion depending on either the particular condition of the well or the quality of the data.

Ideally, one should use both SS and LS data for density inversion. However, in special cases one should neglect the SS and use only the LS data in the inversion. An example of this situation is when SS data are processed to match the resolution of LS data, or when SS data are affected by borehole environmental conditions. In the examples of inversion of field data described above, SS data were omitted because they were previously processed to match the resolution of LS data.

The selection of induction sub-arrays allows several combinations of input data for resistivity inversion. AIT inversion can be performed with only one sub-array, all available sub-arrays, or any combination of sub-arrays. At the outset, it is convenient to perform resistivity inversion with all the available measurements included, and to attempt to single out noisy measurements based on the behavior and relative value of data misfits. For instance, one should neglect shallow sub-arrays in the case of severe washouts, or deep sub-arrays in the case dipping layers.

GR and density-log inversions are implemented only in the vertical direction, whereas AIT inversion assumes 2D variations of properties, i.e. vertically and radially, to accommodate for the estimation of $R_t$, $R_{xo}$, and radius of invasion. Although, mud filtrate invasion is commonplace, it can be neglected in some cases. For instance, for the examples of inversion of field measurements described earlier, we neglected presence of invasion by explicitly requiring that $R_t = R_{xo}$ in the estimation because of the small

### Table 7: Typical data misfits encountered in field applications of combined inversion.

<table>
<thead>
<tr>
<th>Inversion</th>
<th>Range of Misfit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>0–10%</td>
</tr>
<tr>
<td>GR</td>
<td>0–20%</td>
</tr>
<tr>
<td>Resistivity (low contrast)</td>
<td>0–20%</td>
</tr>
<tr>
<td>Resistivity (high contrast)</td>
<td>0–50%</td>
</tr>
</tbody>
</table>

![FIG 13. The left panel shows the 2D GR FSF stretched by 2.1. The right panel shows that using the stretched 1D FSF and applying the 90% detection threshold definition, the calculated vertical resolution is equal to 24.9 in, which matches the vertical resolution reported for SGT tools (24 in).](image-url)
difference between the two resistivities (OBM filtrate invading oil-bearing sand formations at irreducible water saturation).

LOG QUALITY CONTROL WITH INVERSION

Inversion allows petrophysicists to perform quality control of well logs. Below, we summarize the benefits of inversion concerning quality control of logs which are input to the combined inversion:

- Integration of logs for better petrophysical assessment.
- Detection of depth mismatches among logs caused by variations of cable speed, wave-motion compensators, pull-and-stick movement, etc.
- Examination of raw data, usually bypassed by well-log analysts.
- Detection of adverse borehole environmental conditions or tool malfunctioning.

Figures 14, 15 and 16 show examples of log-quality problems detected through inversion and diagnosed by abnormal and local-depth variations of data misfit.

Depth matching of logs is necessary because inversion assumes a common set of bed boundaries for all the input logs. Figure 14 shows an example of depth mismatch due to the irregular motion of the tool string in Well No. 1.

Figure 15 shows an example of noisy raw resistivity data in Well No. 2. The abnormal spike in sub-array numbers 2 and 3 at 37ft could not be reproduced and correlated with other sub-array measurements. Therefore, it can be attributed to tool malfunctioning of sub-arrays 2 and 3 at that depth.

Figure 16 shows one more example of depth mismatch due to a change of logging speed in Well No. 2. Although log depth matching was not implemented in this well, data misfits associated with GR inversion were relatively high in the depth interval 108ft-112ft. The reason for this behavior was a change of logging speed at 149ft. Speed measurements are commonly referred to the zero-point of the tool, which in that case is at the bottom of the tool string. Thus, when the bottom of the tool was located at 149ft, the GR sensor was located exactly at 112ft. Although density and resistivity logs were also affected by the change of logging speed, the corresponding effect on petrophysical interpretation was negligible because at that point the sensors were located within a shaly interval of negligible petrophysical importance.

CONCLUSIONS

The field examples considered in this paper indicate that combined inversion of GR, density, and resistivity logs provides a better agreement between petrophysical interpretation and core measurements (i.e., porosity, saturation and volumetric shale concentration). It is possible to obtain either an increase or a reduction of HPV from inversion results depending on the particular distribution of shales and sands. For instance, deterministic inversion of thin sands in a shale-dominant reservoir will lead to an increase of sand volume and therefore an increase of hydrocarbon reserves. On the other hand, deterministic inversion of thin shales in a sand-dominant reservoir will lead to a decrease of sand volume and, therefore, a decrease of hydrocarbon reserves. For the field examples considered in this paper, the benefit of inversion results over standard interpretation of SR logs on average was (a) 19% better agreement to core measurements, and (b) 15% and 28% extra hydrocarbon reserves in terms of NGR and HPV, respectively.

Field examples of inversion indicated that the minimum bed thickness necessary for successful deterministic inversion was approximately 0.7 ft (0.21m). However, the minimum bed thickness could be shorter than 0.7 ft in cases of wells logged with high sampling rates and logging operations with good depth control (e.g., shallow wells, onshore wells, etc).

The most critical step for inversion was the detection of bed boundaries. Slight perturbations applied to bed boundaries caused significant changes of inverted results, especially across very thin beds. Field cases confirmed that inflection points of LS or compensated density measurements are the best choice for detecting bed boundaries because the density log exhibits the highest vertical resolution among triple-combo logs (GR, density, neutron, and resistivity), has the maximum effect on porosity calculations, and is associated with a spatially symmetric FSF. Borehole measurements such as micro-resistivity could be used to detect boundaries when the tool is mounted on the same density pad. The least desirable option for bed-boundary detection are GR and resistivity logs. Additionally, even though HR logs can detect thinner beds than SR logs, the latter are preferred over the former because they can lead to very thin layers where inversion is not reliable (due to high non-unicness) or cause false layer detection due to abnormal log fluctuations due to measurement noise and sudden variations of borehole environmental conditions.

Although the combined inversion method does not require high-resolution borehole images, core measurements, or repeat sections, this information is important to cross-validate both detected bed boundaries and petrophysical interpretations performed with inversion results.

The inclusion of GR logs in the combined inversion of density and resistivity is necessary because of the low vertical resolution of GR logs compared to that of density and resistivity logs. The examples of inversion undertaken in this paper indicate that at least half of the improvement in core matching and hydrocarbon reserves with respect to standard well-log interpretation is due to the enhancement of GR logs through inversion.

FSFs required for density and GR inversion should be reliable representations of actual measurement acquisition. The FSFs for “UT Longhorn Tools” were calculated for generic logging tools. As a result, they had to be adjusted to match the corresponding FSFs of commercial tools. We performed this adjustment by “stretching” the UT Longhorn FSFs by a certain factor until securing a good match between
Field Examples of the Combined Petrophysical Inversion of Gamma-ray, Density, and Resistivity Logs acquired in Thinly-bedded Clastic Rock Formations


FIG 15. Example of noisy data detected through resistivity inversion in Well No. 2. Tracks show sub-array field and numerically simulated resistivities for sub-arrays 2, 3, and 4. Numerical simulations agree very well with all sub-array field measurements except the prominent spike at 37ft included in sub-arrays 2 and 3.

FIG 16. Depth mismatch detected in Well No. 2 and originated by a change of logging speed at 149ft. This depth mismatch caused significant data misfits in the GR inversion between 108ft and 112ft. Track 1: field GR, simulated GR, GR from inversion, bit size (BS), and caliper. Track 2: 10in and 90in deep two-foot resolution apparent resistivities, and RT from inversion. Track 3: LS density, simulated density, and final density from inversion. Track 4: relative depth, cable speed (where ft/hr=0.3048m/hr), tension (where lbf=4.45N), and logging tool string. Track 5: data misfits for GR and density. Track 6: data misfits for induction sub-array resistivities.
numerically simulated and measured well logs. In so doing, we assumed an arrangement of sensors and sources similar to those of "UT Longhorn Tools" FSFs. The calculated stretching factors were as follows: 2.1 for SGT/HGNS GR; 1.10 and 3.7 for LS and SS PEX density measurements, respectively; and 1.3 and 3.9 for the LS and SS LDT density measurements, respectively.

Selection of sensors for inversion depends on borehole environmental conditions and log quality. For example, in one of the field cases considered, resistivity inversion results improved by neglecting deep sub-arrays that were highly affected by layer dip. In addition, noisy sub-arrays, borehole-affected mono-sensor density data, or shallow sub-arrays affected by severe washouts should not be included as input to the combined inversion.

Large data misfits could indicate the need for additional layers, or marginal quality of logs due to borehole environmental conditions or tool malfunctioning. They could also indicate violation of model assumptions implicit in the numerical simulation of well logs, such as non-horizontal layers, non-piston-like radial invasion, electrical anisotropy, etc. In general, our experience with combined inversion of well logs shows that the method is exceedingly valuable to diagnose measurement biases often unheeded by standard well-log interpretation.

**NOMENCLATURE AND MNEMONICS**

\[ a \] Archie’s tortuosity factor
\[ C(\mathbf{x}) \] cost function used for inversion
\[ V_{sh} \] volumetric concentration of shale, (frac.)
\[ V_{sh-cf} \] \( V_{sh} \) cutoff applied for reserves, (frac.)
\[ \mathbf{d}(\mathbf{x}) \] vector of numerically simulated logs
\[ \mathbf{ds} \] depth shifted logs
\[ \mathbf{e}(\mathbf{x}) \] vector of data misfits
\[ \mathbf{FSF} \] matrix constructed with 1D FSFs
\[ GR \] total gamma-ray measurement, (GAPI)
\[ GR_{raw} \] raw GR at a given depth \( i \), (GAPI)
\[ GR_{i} \] filtered or SR GR at a given depth \( i \), (GAPI)
\[ GR_{s} \] gamma-ray reading in clean sands, (GAPI)
\[ GR_{sh} \] gamma-ray reading in shales, (GAPI)
\[ HPV \] hydrocarbon pore volume, (hydrocarbon ft)
\[ HR \] high resolution logs
\[ i \] depth index
\[ j \] summation index
\[ m \] Archie’s cementation exponent
\[ n \] Archie’s saturation exponent
\[ NGR \] net-to-gross ratio, (frac.)
\[ NMR \] nuclear magnetic resonance
\[ pay \] pay flag
\[ RE \] average relative error, (%)
Field Examples of the Combined Petrophysical Inversion of Gamma-ray, Density, and Resistivity Logs acquired in Thinly-bedded Clastic Rock Formations

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