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A laboratory scale piezoelectric array for underwater measurements of the fluctuating wall pressure beneath turbulent boundary layers

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Abstract
To capture the full spectrum of the fluctuating wall pressure beneath a turbulent boundary layer (TBL) provides a unique challenge in transducer design. This paper discusses the design, construction and testing of an array of surface-mounted piezoelectric ceramic elements with the goal of having both the spatial resolution and the frequency bandwidth to accurately sense the low-frequency, low-wavenumber events beneath a TBL at moderately low Reynolds numbers. The array is constructed from twenty 1.27 cm tall prismatic rods with 0.18 cm × 0.16 cm cross-section made of Navy type II piezoelectric ceramic material. Calibration was performed by comparing the response of a Navy H56 precision-calibrated hydrophone to the outputs of each element on the array for a given input from a Navy J9 projector. The elements show an average sensitivity of −184 dB (re: 1 V μPa⁻¹) and are assembled with a centre-to-centre spacing of 0.2 cm. Measurements of the fluctuating wall pressure below a 2d TBL with Reynolds numbers (based on momentum thickness) ranging from 2100 to 4300 show that the dimensions of the elements are between 64 and 107 viscous length units, respectively. A spatial and temporal footprint of the fluctuating wall pressure reveals convective speeds averaging 75% of the free stream velocity.

Keywords: turbulence, wall pressure

(Some figures may appear in colour only in the online journal)

1. Introduction and motivation

Turbulent boundary layers (TBL) that form on the hull of surface and underwater vessels can elevate the noise floor for underwater sonar systems. This is caused by the hydrodynamic footprint of the turbulent structures passing over the surface where the sonar system is placed. Because the sonar system is incapable of distinguishing the hydrodynamic pressure fluctuations from the acoustic signal, the overall signal-to-noise ratio (SNR) is reduced [1]. This reduction in the SNR remains a significant limitation to underwater vessels as it restricts the speed at which these vessels can travel while scanning their surroundings. Practical methods for reducing this source of flow noise are therefore of interest for defence and commercial underwater navigation and imaging.

A precursor to mitigating TBL-related noise is a detailed understanding of the spatial and temporal characteristics of the boundary layer turbulence as well as the pressure footprint that it furnishes at the fluid/structure interface. This fluctuating wall
pressure signature is strongly determined by the turbulence in the flow immediately adjacent and is difficult to predict given the broad range of flow scales in the boundary layer. Following the review by Hwang et al [2], several characteristic trends in the pressure power spectral density (PSD) are expected which are illustrated here in figure 1 using a Goody model. Foremost, the amplitude of the PSD shows three unique power law dependences with \( \omega \delta / U_c \) where \( U_c \) is the convective speed (assumed to be constant), \( \omega \) is the radian frequency and \( \delta \) is the boundary layer thickness. For \( \omega \delta / U_c \ll 1 \), this spectrum is expected to show a \( S_{PP} \propto (\omega \delta / U_c)^2 \) trend which transforms to \( S_{PP} \propto (\omega \delta / U_c)^{-0.7} \) to \(-1\) trend between \( 1 < \omega \delta / U_c < 0.3 \tau_u \delta / \nu \) (this intermediate trend is driven by the cascade of energy from the large-scale flow structures). At higher frequencies where \( \omega \delta / U_c > 0.3 \tau_u \delta / \nu \), the roll-off is more drastic, shown here to exhibit an \( S_{PP} \propto (\omega \delta / U_c)^{-5/2} \) trend which transforms to \( S_{PP} \propto (\omega \delta / U_c)^{-7} \) to \(-5\) trend. As for the smallest length scales of the fluctuating wall pressure, they are of order \( \lambda_{ss} = \nu / \tau_s [3] \) where \( \nu \) is the kinematic viscosity, \( \tau_s = \sqrt{\tau_w / \rho_f} \) is the friction velocity, \( \tau_w \) is the shear stress at the wall and \( \rho_f \) is the fluid density. And so, a first step to developing effective control strategies is to have an understanding of the parameters influencing the turbulence in the boundary layer. This can only be accomplished through accurate simulation (Reynolds number restricted) or direct measurement (both velocity and fluctuating wall pressure) and remains a difficult proposition despite over a century of study on this topic.

Several factors govern the fluctuating wall pressure signature beneath a turbulent boundary layer (pressure gradient, surface roughness, freestream turbulence) which are difficult to control under full-scale conditions. It is therefore desirable to study these problems in a laboratory scale environment where conditions can be more easily controlled and documented. However, a persistent difficulty with the smaller laboratory scale environment is the limited availability of hardware suitable for accurately resolving the full wavenumber–frequency spectrum of the pressure signatures at these scales.

Generally speaking, transducers and transducer arrays that are designed for measuring low-frequency fluctuations have large sensing elements. On the contrary, the events residing near the fluid structure interface beneath a TBL are small in scale, thus requiring that the sensor element be small, all the while maintaining a flat sensitivity response to low-frequency fluctuations. For a sensor array, the spatial separation between adjacent sensors is restricted by the size of the element, with larger elements biasing the measurements towards the characteristics of the larger and faster moving turbulent structures. It is believed that these large-scale events reside in the outer regions of the TBL. Therefore, sensors that are capable of only capturing the large-scale pressure fluctuations do little in the way of helping to understand the slower and smaller scale events found in the vicinity of the viscous sublayer near the fluid/structure interface. This is especially problematic if one aims at validating the accuracy of simulations using benchmark tests performed in a laboratory scale environment or to develop reliable scaling laws for predicting the behaviour of full-scale systems of practical engineering interest. Acoustic transducer arrays with larger and more conventional elements, therefore, do not have the correct spatial configuration to resolve the higher wavenumber events observed in laboratory scale TBL studies. This is unfortunately a persistent problem in under water environments since many techniques that have recently been used to make high-resolution measurements of turbulence, such as hot films and hot wires, cannot be used under water [4–6]. A further complication arises from the fact that the larger and more energetic structures in low Reynolds number flat-plate TBL flows \( (2 \times 10^4 < Re_\theta < 4 \times 10^4) \) reside primarily at lower frequencies \((3–50 \text{ Hz}) [7] \) which are simply due to the slower moving free stream velocity \( (U_\infty) \) or geometric constraints imposed by the scale and capabilities of the laboratory [2].

To accurately resolve the wide spectrum of scales observed in the fluctuating wall pressure beneath a TBL, two requirements must be addressed when designing the pressure sensing instrument: (1) the sensors must be small enough to resolve the small-scale structures that convect at low speeds and (2) the sensors must be able to sense the low-frequency, large-scale structures that not only contain the most energy but are also the source from which higher frequency fluctuations draw their energy. The dual requirement of small sensors and low-frequency response demand the development of a unique acoustic array in order to properly measure the fluctuating wall pressure beneath a TBL in a laboratory scale facility. Due to the physical limitations and lack of demand from instrument manufacturers, no commercially available elements or arrays are available to suit the conditions required of many laboratory scale studies. Therefore, the focus of this paper is on the design, fabrication and validation of a unique underwater pressure sensor array capable of resolving the critical structures in a low-speed TBL. This presents several key challenges in sensor design which are addressed here.

\[4 \text{ It is further noted that in order to properly resolve a phenomenon at any scale without aliasing, the sensor spacing must be at greatest half the wavelength of the smallest phenomenon being studied.}\]
Piezoelectric materials are commonly used to sense pressure variations because they possess a molecular structure that produces an electrical charge as a result of a mechanical load. For example, the strain-charge piezoelectric relations of equations (1) and (2) show that the electrical displacement, \( \vec{D} \), is proportional to the applied stress, \( \vec{T} \). The area integral relation \( \int \vec{D} \cdot d\vec{S} = Q \), where \( \vec{D} \) is spatially independent and \( \vec{n} \) is the unit vector normal to the surface, reveals that the charge \( Q \) on the surface \( A \) is proportional to the stress \( \varepsilon_{T} \). These materials have good sensitivity and are mechanically robust to permit immersion in water to great depths without altering sensitivity.

When designing a sensor using piezoelectric elements for conducting wall pressure studies, one should be aware that there are several mechanisms by which the pressure signature at the fluid/structure interface can be unintentionally filtered from the signal, thereby escaping detection and lowering the accuracy of the measurement. One potential filtering mechanism is caused by the capacitative nature of the piezoelectric ceramic and how the accumulated charge is sensed and converted into a voltage signal using preamplifiers. A second source of filtering occurs in underwater applications where it is common to coat the transducers in a protective layer that is acoustically impedance matched to water. This maximizes transmission of acoustic waves while acting as a low-pass filter for the hydrodynamic (evanescent) pressure waves and is proportional to the thickness of the layer. Therefore, to design an array capable of accurately resolving the full pressure spectrum at the fluid/structure interface, both of these effects must be minimized. In what follows, an overview of the sensor element’s design will be provided in section 2.1 with attention to how the element’s geometry affects the frequency response of the transducer. This is followed by a more comprehensive discussion of the filtering effects due to the transducer size in section 2.3 and due to elastomer coatings in section 2.4.

### 2.1. Sensing mechanism

One potential sensing configuration for piezoelectric materials is to use them as capacitors that accumulate a charge in proportion to the imposed stress (pressure) which can then be measured to estimate the imposed stress. If magnetic and pyroelectric effects are ignored, the following relations (written in Voigt notation) describe the coupled strain and electric displacement fields in a piezoelectric ceramic subjected to electromechanical loading [8]:

\[
\vec{S} = \sigma_{T} \vec{T} + \vec{d}^{T} \vec{E} \tag{1}
\]

\[
\vec{D} = \vec{d} \vec{T} + \varepsilon_{T} \vec{E}. \tag{2}
\]

Here \( \vec{S} \) is a vector of mechanical strains, \( \sigma_{T} \) is the constant electric field compliance matrix, \( T \) is the stress vector, \( \vec{d} \) and \( \vec{d}^{T} \) are the strain-charge form piezoelectric coupling matrix and its transpose, respectively, \( \vec{E} \) is the electric field strength vector, \( \vec{D} \) is the electric displacement vector and \( \varepsilon_{T} \) is the dielectric permittivity of the material with the stress held constant.

The coordinate systems for the two lengthwise expanding bars explored in this study are shown in figure 2. In keeping with convention, the direction in which the ceramic is polarized is always designated \( x_{3} \). In the case of the \( 31 \) element, the primary stress is \( T_{1} \), which acts in the \( x_{1} \) direction and is perpendicular to the poling direction. For the \( 33 \) element, the primary stress is \( T_{3} \) which acts parallel to the poling direction. In both cases, the loading occurs along the lengthwise coordinate. If the normal stresses shown in figure 2 are considered, the matrix product of \( \vec{d} \) and \( \vec{T} \) becomes

\[
\vec{d} \vec{T} = T_{1}d_{31} + T_{2}d_{32} + T_{3}d_{33}. \tag{3}
\]

For general Navy-type ceramics, \( |d_{31}| < |d_{33}/2| \) and the coefficients are of opposite sign so that both \( 33 \) and \( 31 \) mode operations benefit from having a highly compliant material surrounding the non-sensing sides [8]. This mechanical configuration drives stresses that are orthogonal to the length direction towards negligible levels. This results in a uniaxial stress state along the length of the material which increases the charge generated by a given pressure load. The inclusion of a compliant isolator also reduces mechanical cross-talk between elements and removes a path through which undesired mechanical noise may enter the system.

As for the frequency-dependent sensitivity of the bar elements, this can be determined following the analysis of Berlincourt [9] which assumes that the presence of standing waves in the piezoelectric element describes the motion of the bar. In doing so, a method for finding simple circuit equivalents for piezoelectric ceramic transducers can be deduced [9]. It is desirable for this application to have a small centre-to-centre spacing between elements, so we will choose the geometry of the element to have a lengthwise expanding bar with an electric field acting perpendicular to the direction of strain (\( 31 \) mode). This requires that the width, \( w \), and thickness, \( h \), of the bar be small relative to its length, \( l \), which is the case for the element shown in figure 2(a).

To begin, the electric field, \( E_{3} \), is assumed to be constant throughout the bar since the electrodes form equipotential surfaces and wavelengths of electromagnetic perturbations at the frequencies of interest are orders of magnitude larger than the dimensions of the sensing elements proposed herein. Further, fringing effects are neglected, so \( E_{1} \) and \( E_{2} \) are zero. All normal and shear stresses other than the normal stress \( T_{1} \) are also assumed to be zero, which is valid if there is a compliant material surrounding the element. Inserting these
simplifications into equations (1) and (2) gives

\[ S_1 = s_{11} T_1 + d_{31} E_3 \]

(3)

\[ D_3 = d_{31} T_1 + \epsilon_{31} E_3. \]

(4)

The standing wave motion within the bar is described using a time-varying harmonic function for its displacement, \( \xi(x,t) \),

\[ \xi(x,t) = (A \sin(kx) + B \cos(kx)) e^{j\omega t}, \]

(5)

where \( x \) represents \( x_1 \) in figure 2(a), \( k = \omega/e_b \) and \( c^F_b = 1/\sqrt{1/(c^F_{11} \cdot \rho)} \) is the speed of propagation in the bar for a constant electric field using \( \rho \) as the material density of the element. The electric field is assumed to be constant across the bar such that the voltage across the electrodes is given by

\[ V = \int_0^h E_3 \, dz = E_3 h. \]

(6)

If we use the above functional form for the displacement field and define the particle velocity, \( u \), and local inward force, \( F \), as \( u_l \) and \( F_l \) for the front of the bar at \( x = 0 \), and \( u_b \) and \( F_b \) for the back of the bar at \( x = l \), the constants \( A \) and \( B \) in (5) can be found as follows:

\[ u = \frac{\partial \xi}{\partial t} = j\omega (A \sin(kx) + B \cos(kx)) e^{j\omega t}. \]

(7)

\[ A = -\left( \frac{u_b}{j\omega \sin(kl)} + \frac{u_t}{j\omega \cot(kl)} \right) e^{-j\omega t}. \]

(8)

\[ B = \frac{u_t}{j\omega} e^{-j\omega t}. \]

(9)

The inward acting forces at the extremities of the bar can be related to the stress imposed on the surface of the bar through the relations \( T_l(x=0) = -F_l/(wh) \) and \( T_1(x=l) = -F_b/(wh) \). These relationships can then be substituted into equation (4) applied along with equations (6)–(9) in order to determine the force on each end as a function of the end velocities and the applied voltage:

\[ F_l = \frac{wh\rho c^F_b}{j} \left( \frac{u_b}{\sin(kl)} + u_t \cot(kl) \right) + \frac{wd_{31}}{s_{11}} V, \]

(10)

and

\[ F_b = \frac{wh\rho c^F_b}{j} \left( u_t \cot(kl) + u_t \cot(kl) \cos(kl) + u_t \sin(kl) \right) + \frac{wd_{31}}{s_{11}} V. \]

(11)

Having now considered trigonometric identities, equations (10) and (11) become

\[ F_l = \frac{Z_0}{j\sin(kl)} u_b + \frac{Z_0}{j\sin(kl)} u_t + jZ_0 \tan \left( \frac{kl}{2} \right) u_t + N V, \]

(12)

and

\[ F_b = \frac{Z_0}{j\sin(kl)} u_b + \frac{Z_0}{j\sin(kl)} u_t + jZ_0 \tan \left( \frac{kl}{2} \right) u_t + N V, \]

(13)

where \( Z_0 = wh\rho c^F_b \) represents the mechanical impedance of the bar and \( N = d_{31} w/s_{11}^2 \) is known as the ratio of electromechanical turns. These equations are satisfied for the circuit diagram shown in figure 3(a) using the following component values:

\[ Z_1 = jZ_0 \tan \left( \frac{kl}{2} \right), \quad C_0 = \frac{wh}{k} \frac{\epsilon^F_{11}}{\epsilon_{33}} (1 - k_{31}^2) \]

\[ Z_2 = -j \frac{Z_0}{\sin(kl)}, \quad k = \omega/e_b \]

\[ Z_3 = \rho w c^E_b, \quad k_{31} = \frac{d_{31}^2}{s_{11}^2 \epsilon_{33}}, \quad k_{33} = \frac{d_{33}^2}{s_{33}^2 \epsilon_{33}} \]

\[ c^E_b = \frac{1}{\sqrt{\rho c^E_b}}, \quad N = \frac{wd_{31}}{s_{11}}. \]

(14)

Having now shown the derivation for the circuit model of a 31 mode element (equations (3)–(14)), the equivalent circuit for a 33 mode element, which is found using analogous treatment, is given without proof using the circuit model in figure 3(b) along with the following component values:

\[ Z_1 = jZ_0 \tan \left( \frac{kl}{2} \right), \quad C_0 = \frac{wh}{k} \frac{\epsilon^F_{11}}{\epsilon_{33}} (1 - k_{33}^2) \]

\[ Z_2 = -j \frac{Z_0}{\sin(kl)}, \quad k = \omega/e_b \]

\[ Z_3 = \rho w c^E_b, \quad k_{31} = \frac{d_{31}^2}{s_{11}^2 \epsilon_{33}}, \quad k_{33} = \frac{d_{33}^2}{s_{33}^2 \epsilon_{33}} \]

\[ c^E_b = \frac{1}{\sqrt{\rho c^E_b}}, \quad N = \frac{wd_{31}}{s_{11}}. \]

(15)

The two primary differences between the models is the size of the capacitance, \( C_0 \), and the presence of an additional negative capacitor in series in the 33 model. Since the capacitance is proportional to the area of the electrodes, a higher capacitance leads to a higher charge being developed per unit strain input.
2.2. Predicted sensitivity with preamplification considerations

When predicting the electrical performance of a transducer, the input impedance of the preamplifier and how it couples to the element are a very important consideration. For many hydrophone applications, voltage preamplifiers are used. A basic inverting preamplifier is shown in figure 4(a), where the output voltage is $V_{\text{out}} = -V_{\text{in}} \times R_A / (R_A + Z_s)$. If the impedance of the resistor $R_A$ is significantly larger than the source impedance of the sensor, $Z_s$, then $V_{\text{out}} \approx -V_{\text{in}} \times R_A / R_A$. However, in the low-frequency limit, the source impedance of the transducer element is primarily due to the capacitive nature of the piezoceramic, which is $Z_s = 1/(j\omega C_0)$. Therefore, in the limit as $\omega \to 0$, $Z_s \to \infty$, so that a voltage amplifier would require an unrealistically high input resistor in order to amplify the signal. For example, the theoretical sensitivity, $M_\text{th}$, of a piezoceramic 33 or 31 length expander element coupled with a voltage preamplifier or a charge amplifier is shown in figure 5 [8].

![Figure 4. Basic preamplifier circuits: (a) voltage amplifier, (b) charge amplifier.](image)

![Figure 5. Analytical response of a 31 and 33 length expander to a finite-impedance voltage preamp.](image)

<table>
<thead>
<tr>
<th>Table 1. Material properties for Navy type II ceramic [8].</th>
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<tbody>
<tr>
<td>$d_{33}$</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
</tr>
<tr>
<td>$\rho_v$</td>
</tr>
<tr>
<td>$\rho$</td>
</tr>
</tbody>
</table>

The 31 operation using voltage preamplification is much less pronounced; however, the voltage sensitivity in this mode is very weak. And so, the highest sensitivity is achieved using a 31 length expander with a charge amplifier giving a predicted value of approximately $-188 \text{ dB}$ (re: $1 \text{ V} \mu\text{Pa}^{-1}$).

As a final note, it is important to clarify that charge amplifiers do not amplify charge as their name suggests, but rather, sense the charge accumulated across the electrodes of a sensor, and then output a voltage proportional to that charge. A basic circuit diagram for a charge amplifier with a charge source is shown in figure 4(b). The charge produced by a piezoceramic element ($Q_s$) is proportional to the strain induced. For an ideal charge amplifier, the output voltage is $V_{\text{out}} = Q_s / C_f$. In reality, there is a finite amount of current that leaks through capacitors, thus limiting the low-frequency response. Also, any charge accumulated on the cables prior to the amplifier will be amplified as well, so the cables should be isolated from vibrations and shielded. For this study, PCB 422E11 commercial charge amplifiers were used, which have a low-frequency cut-off of 5 Hz and a high-frequency cut-off of 110 kHz.

2.3. Filtering due to transducer size

It is known that the size of a transducer’s sensing element influences the range of frequencies that can be resolved. As for dynamic pressure sensing instruments, Corcos [10] used pressure correlations to estimate the attenuation of signals due to spatial averaging over finite-size transducers, which resulted in a theoretical correction to the variance and spectrum of the TBL for a transducer of a given geometry. This correction, known as the Corcos correction, compares the effects of turbulence on transducers of finite area to the mapping by linear operators of a random variable of several variables. This takes into account the spatial decay in the coherence of the signal as well as the auto-correlation of various sensor geometries. In [11], it was noted that the Corcos correction is very sensitive to the mean convection speed of the pressure signatures; to date, there is little consensus in the scientific community on
the mean convective speed of the fluctuating wall pressure beneath TBL flows and is found to range anywhere between 60% and 85% of the free stream velocity. However, it is believed that in many of the previous studies, the fine scale contributions to the pressure signal had been attenuated to such an extent as to have escaped detection completely and those contributions may be an appreciable fraction of the signal [11]. Since the Corcos correction is not linear with transducer size, results may be misleading if pressure intensities are extrapolated. Schewe [12] conducted several experiments with dimensionless pressure transducer diameters \( (d^+ = d_u \tau / \nu) \) ranging from 19 ≤ \( d^+ \) ≤ 333. Here \( d_u \) is the diameter of the sensing element, \( \nu \) is the kinematic viscosity of the fluid and \( \tau \) is the friction velocity. The results showed how high-amplitude, characteristic pressure structures within a flat-plate TBL comprised longitudinal wavelengths of \( \lambda^+ = 145 \) and convective at speeds of 0.53\( U_{\infty} \); this low convection velocity was obtained using conditional sampling techniques which does not reflect the average convective speed sensed by the entire fluctuating wall pressure signal. This suggests that a sensor element size of \( d^+ \leq 145/2 \) is required in order to accurately capture the fluctuating wall pressure beneath a TBL.

Table 2 lists the flow conditions (in terms of \( Re_\theta = U_{\infty} \theta / \nu \)) and sensor diameters (in terms of \( d^+ \)) of various TBL studies reported in the open literature. The transducer size for this study ranged from 64 ≤ \( d^+ \) ≤ 107 which is sufficient to resolve the majority of the energy in the wall pressure signal, especially where the more energetic hydrodynamic signatures are concerned5. It is important to note that the spatial resolution in this study is considerably better than that used in prior underwater studies where the disparity between acoustic wavelengths and turbulent length scales is much greater than that in air. The present experimental configuration also allows for greater control of, and access to, the flow than towed and surface pop-up tests which represent the majority of test configurations reported in the open literature for underwater TBL studies.

### 2.4. Filtering due to elastomer coatings

For underwater measurements, elastomer layers are usually required in order to protect and isolate the sensors from water. Typically these elastomers are acoustically impedance matched to the water so that attenuation of the acoustic waves, as they propagate through the material, is minimal. However, hydrodynamic fluctuations caused by the boundary layer turbulence are evanescent and so significant filtering can occur due to the natural wavenumber filtering properties of the elastomer. Correction methods have been proposed. In particular, Ko and Schloemer [16] showed how a transfer function could be estimated by matching the wavenumber characteristics at the fluid/structure interface to the turbulent wall pressure spectrum developed by Corcos [11]. This requires the elastomer to be backed by a plane rigid surface. Like the transducer size correction described in section 2.3, signal attenuation due to the elastomer coating has its greatest effect at high frequencies. Since the highest frequency content is saturated with noise and the elastomer coating employed in this study was only approximately 0.2 cm thick, no correction has been applied to the data presented here. This is justified by noting that the attenuation from a 0.32 cm thick elastomer coating has been estimated to be 1.5 dB at 500 Hz, 3.1 dB at 1000 Hz and 5.5 dB at 1500 Hz [2].

### 3. Experimental array

The experimental pressure array was designed and built in-house at the Applied Research Laboratories at the University of Texas at Austin (ARL:UT). As detailed in the previous section, in order to minimize filtering due to the spatial dimensions of the elements, a very small surface area interfacing with the TBL was required. The analysis in sections 2.1 and 2.2 provided details on how to estimate the pressure-to-voltage sensitivity for 31 and 33 mode piezoelectric elements using either voltage or charge preamplification. Section 2.2 specifically showed that charge preamplification is preferable because it minimizes the influence of source impedance on the element sensitivity in the frequencies of interest for this TBL study. When using a charge preamplifier, the signal strength is proportional to the capacitance of the element, thus indicating that 31 sensing is a preferable configuration (recall equations (14) and (15) and the sensitivity curves in figure 5). Further, the stress analysis in section 2.1 showed that higher signals can be obtained by surrounding individual elements in the array by a highly compliant spacing material. For these reasons, 31 mode elements with compliant spacing material were used since they provide a small sensing area while maximizing the signal strength. The materials, specific design geometry and calibration method for the constructed array are presented below.

#### 3.1. Materials and construction

Twenty elements were constructed from a preplated and poled 0.16 cm (0.065″) thick Navy type II piezoelectric ceramic plate cut into 0.18 cm (0.080″) × 1.27 cm (0.5″) prismatic rods. The electrical impedance of each element was tested to ensure that no cracks were created during the cutting process. The individual rods were then formed into an array and acoustically isolated from each other with thin layers of corprene. This configuration yielded an inter-element spacing of 0.2 cm. The array was then backed by a steel plate coated with Kapton tape (a non-conductive thin layer). This backing

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Table 2. Flow conditions and sensing element sizes for various studies.

<table>
<thead>
<tr>
<th>Author</th>
<th>Type</th>
<th>( Re_\theta )</th>
<th>( d^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blake [14]</td>
<td>Wind tunnel</td>
<td>8300–18 000</td>
<td>43–84</td>
</tr>
<tr>
<td>Keith, Cipolla and Furey [15]</td>
<td>Towed water</td>
<td>48 000–110 000</td>
<td>481–1106</td>
</tr>
<tr>
<td>Current study</td>
<td>Water tunnel</td>
<td>2100–4300</td>
<td>64–107</td>
</tr>
</tbody>
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5 The inner coordinates used to determine \( d^+ \) are obtained from LDV measurements and are discussed in further detail in section 4.1.
configuration minimized the displacement at the element base thereby maximizing the charge production for a given applied pressure. A slab of acoustic damping material called SADM (designed by Syntech Materials) was placed behind the steel plate to reduce acoustic reflections caused by the air cavity behind the array from contaminating the measurements. The 20-element array was then placed in a PVC housing and immersed in an elastomer, which waterproofed the sensors. As pointed out in section 2.4, the elastomer coating between the top of the elements and the TBL can filter the non-acoustic pressure signals. Because of this, care was taken to make this layer as thin as possible above the sensing face while ensuring complete encapsulation. The final elastomer thickness above the sensor was found to measure approximately 0.2 cm. The PVC housing served as a convenient way to seal the back of the array so that the preamplifiers could be placed close to the array without the need to coat them in the elastomer as well. A sketch of the general design is shown in figure 6(a) with a picture of the completed array mounted flush to the surface of the instrumentation window in figure 6(b).

3.2. Calibration setup and results compared to theoretical predictions

The assembled pressure array was calibrated at ARL:UT’s Lake Travis Test Station (LTTS). The acoustic calibration was performed by evaluating the experimental pressure array using a calibrated acoustic source and reference hydrophone in accordance with classical comparison calibration methods outlined by the Naval Research Laboratory (NRL) [17]. The source used was a Navy J9 projector designed to generate underwater signals from 40 Hz to 20 kHz. The precision hydrophone was a Navy H56 with a flat sensitivity response from 100 Hz to 65 kHz and a precision calibration from 10 to 100 Hz. The precision calibrations of the reference instruments were performed and certified by the Underwater Sound Reference Division (USRD) of the Naval Undersea Warfare Center (NUWC). The calibration curves for the reference instruments are not publicly available, but the LTTS at ARL routinely performs calibrations for underwater pressure sensors that are used by the US Navy.

The source, experimental array and calibrated hydrophone were lowered to a depth of 6.1 m (20 ft) under the LTTS. The projector was placed 0.67 m (2.2 ft) away from both the experimental array and reference hydrophone. Single-frequency tone bursts were sent at varying frequencies and the outputs of both the array and reference hydrophone were gated to prevent the capture of surface reflections. The signals received from the experimental hydrophone were then used to calibrate it against the reference hydrophone. The voltage sensitivity of each element of the array, $M_x$, is determined at each test frequency from the output voltage of the experimental array, $V_x$, and the output voltage and voltage sensitivity of the reference hydrophone, $V_r$ and $M_r$, respectively, according to the equation [17]

$$M_x = \frac{M_r V_x}{V_r}.$$  (16)

The calibration plots for the experimental array are shown in figure 7. The first solid line is the calibration for the first sensor, the dash-dot line is the average sensitivity between 50 and 500 Hz, and the dashed lines represent $\pm 3 \text{ dB(re: 1 V} \mu \text{Pa}^{-1})$ from the average. The remainder of the elements are shifted down and labelled respectively with 2 dB grid lines.

The sensor element number 16 was found to possess half the sensitivity of all other elements on the array and so was not used in any of the tests; the reduced sensitivity is likely due to a lead-wire coming off during assembly. A combination of the projector sensitivity and a violation of free-field conditions caused the source level to drop off significantly below 80 Hz, where the calibrations deviate from a flat response. Given the smoothness of the response above 100 Hz, and the knowledge that this type of hydrophone has a flat low-frequency response that is only limited by the preamplifier, the reported average sensitivity measured is expected to accurately reflect the sensitivity of the instrument across the frequency range of interest.
4. Evaluation

4.1. Testing facility

A Rolling Hills Research Corporation Eidetics Model 1520 flow visualization water tunnel was utilized to gage the new array’s ability to measure fluctuating wall pressures below a TBL. An instrumented insert and flat plate were added allowing for free stream speeds up to 1.64 m s\(^{-1}\) over the instrumented surface where the array was mounted. The pressure array was used in conjunction with PCB Model 422E11 charge amplifiers to achieve element sensitivities of \(-184\) dB (re: 1 V µPa\(^{-1}\)). The pressure data were acquired using a National Instruments PXI-4472 8-Channel Dynamic Signal Acquisition Module in a PXI-1042Q chassis with a PXI-8331 controller card. The PXI-4472 has a ±10 V input range, variable anti-aliasing filters and 24 bit resolution analogue to digital conversion with 110 dB dynamic range. A more detailed description of the experimental setup can be found in [7] and [18].

Figure 8 shows several velocity profiles taken by a laser Doppler velocimeter (LDV) for free stream speeds ranging between 0.89 and 1.64 m s\(^{-1}\) above the first element on the array. Additional profiles have been drawn corresponding to turbulent \((n = 6\) power law\) and laminar (parabolic approximation) states of a boundary layer flow. The subtle variation from the power law indicates a slightly adverse pressure gradient, which was expected, and compares well to the flat-plate study of [19]. Inner scaling variables were determined using the log-law expression in conjunction with

### Table 3. Outer scaling variables.

<table>
<thead>
<tr>
<th>(U_\infty) (m s(^{-1}))</th>
<th>(\delta) (cm)</th>
<th>(\theta) (cm)</th>
<th>(u_t) (m s(^{-1})) ((\times 10^{-2}))</th>
<th>(d^+)</th>
<th>(Re_\delta)</th>
<th>(Re_\theta)</th>
<th>(Re_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.89</td>
<td>1.9</td>
<td>0.22</td>
<td>3.74</td>
<td>64</td>
<td>18 000</td>
<td>2100</td>
<td>760</td>
</tr>
<tr>
<td>1.15</td>
<td>2.0</td>
<td>0.23</td>
<td>4.67</td>
<td>80</td>
<td>25 000</td>
<td>2900</td>
<td>1000</td>
</tr>
<tr>
<td>1.41</td>
<td>2.2</td>
<td>0.25</td>
<td>5.43</td>
<td>93</td>
<td>33 000</td>
<td>3700</td>
<td>1300</td>
</tr>
<tr>
<td>1.64</td>
<td>2.2</td>
<td>0.24</td>
<td>6.25</td>
<td>107</td>
<td>38 000</td>
<td>4300</td>
<td>1500</td>
</tr>
</tbody>
</table>
classical Karman and additive constants $\kappa = 0.41$ and $B = 5.2$, respectively. This provided estimates for $d^+\rangle$ which are tabulated in table 3 along with estimates for the outer coordinate scales.

4.2. Facility noise

The ambient noise and testing noise sources were determined via a facility noise test which consisted of the instruments in table 4 for all four flow conditions and for a zero flow state. It was determined that the leading noise sources during the tests were due to line noise and the water tunnel motor. High-frequency noise was present in all of the flow conditions and is shown to be primarily the result of the harmonics of the motor. Figures 9(a) and (b) show the PSD and the coherence between a hydrophone channel and the motor accelerometer, respectively [20].
Figure 11. Spacetime contour of the fluctuating wall pressure below a flat-plate TBL at $Re_\theta = 4300$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Transducer</th>
<th>Make</th>
<th>Model</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Microphone</td>
<td>PCB</td>
<td>337C10</td>
<td>Air</td>
</tr>
<tr>
<td>2</td>
<td>Accelerometer</td>
<td>PCB</td>
<td>353B17</td>
<td>Motor collar</td>
</tr>
<tr>
<td>3</td>
<td>Accelerometer</td>
<td>PCB</td>
<td>353B17</td>
<td>Top of instrumented insert</td>
</tr>
<tr>
<td>4</td>
<td>Hydrophone</td>
<td>B&amp;K</td>
<td>8103</td>
<td>Inside instrumented insert</td>
</tr>
<tr>
<td>5</td>
<td>Hydrophone</td>
<td>Custom</td>
<td>Custom</td>
<td>Test window</td>
</tr>
<tr>
<td>6</td>
<td>Hydrophone</td>
<td>Custom</td>
<td>Custom</td>
<td>Test window</td>
</tr>
<tr>
<td>7</td>
<td>Hydrophone</td>
<td>Custom</td>
<td>Custom</td>
<td>Test window</td>
</tr>
<tr>
<td>8</td>
<td>Hydrophone</td>
<td>Custom</td>
<td>Custom</td>
<td>Test window</td>
</tr>
</tbody>
</table>

Table 4. Facility noise configuration.

The noise floor of the facility was also established. Figure 10(a) shows the PSD of the first pressure array element with the water tunnel pump off (no water flowing over the array) and on (water flowing over the array). The harmonic peaks in the hydrophone signal when the pump is off identify the electrical noise floor of the system. These peaks coincide with the main electrical supply to the facility: 30, 60 and 90 Hz. When flow is present, the hydrodynamic hump produced by the formation of a TBL over the hydrophone array rises well above the electrical noise floor of the system.

4.3. Measurement: pressure spectrum

Figure 10(b) shows the PSDs of the pressure signals for the different flow conditions. The expected trends that the frequency and amplitude of the largest strength structures grow with increasing Reynolds number are observed. As expected from the discussion in section 1 and illustrated in figure 1, the spectrum shows a low-frequency energy production and then a straight roll-off to higher frequencies until the signal is lost in the noise floor of the facility.

4.4. Correlations

Figure 11 shows the raw pressure signal across the array. Streaks are observed passing over the array at a regular speed and correspond to the passage of turbulent structures along the surface of the flat plate. The normalized cross-correlation coefficients across the pressure array, shown in figure 10(c), show a smooth decay. A slight discrepancy in the variance across the array was noted, which can be accounted for by the fact that the pressure-to-charge sensitivity of the elements varies along the array. Further, since the thickness of the elastomer varies slightly over the array, the hydrodynamic sensitivity of the elements may be different from the acoustic sensitivity and may be more location dependent.

Normalized spacetime correlations across the array are presented in figure 10(d). All cases show a ridge with a convective velocity of $U_c \approx 0.75U_\infty$, which is similar to the flat-plate study in [21] and towed-array study in [15], but less than that found for the flat-plate study presented in [22]. Additional studies show identical convective trends for flow over a body of revolution, flat-plate and pipe-flow configurations [23]. In [24], average convective speeds are reported to range from 0.70$U_\infty$ to 0.80$U_\infty$. The cross correlations exhibit a characteristic decay in both time and space due to nonlinear interactions and dissipation of the turbulence.

5. Summary and conclusions

A custom piezoelectric ceramic pressure transducer array was designed and fabricated in an effort to resolve the full spectrum of the fluctuating wall pressure beneath a flat-plate TBL flow. This posed many challenges in transducer design given the low-frequency, low-wavenumber events residing in these types of flows. In order to overcome this, a $3f$ mode piezoelectric sensing element was used in conjunction with charge preamplification rather than a standard voltage preamplifier. This allowed for the ability to use small elements with a centre-to-centre spacing of 0.2 cm, while sensing down to infrasonic frequencies. The resulting streamwise dimension of the elements in viscous length units ranged from $64 \leq d^+ \leq 107$ for Reynolds numbers based on momentum thickness of 2100–4300, respectively.

A compromise of this particular design architecture is the sensitivity of the elements to structural vibrations and facility-related noise which are inherent to nearly all laboratory
environments. This ultimately masked the small-scale pressure signatures making it difficult to accurately resolve the spectral decay of the high-frequency flow events. Pressure spectrum models predict viscous effects to play a more significant role in attenuating energy at approximately \( \omega \delta / U_c \approx 0.3u_\tau \delta / \nu \). This relates to frequencies between 1.6 and 4.9 kHz across the range of Reynolds numbers tested in this study, whereas contaminating noise became significant around 300 Hz. This might appear to be problematic; however, the dominant hydrodynamic hump in the pressure spectra had decayed by nearly 30 dB at 300 Hz and so facility-induced noise did little in the way of affecting the quality of this study. Spacetime surveys of the fluctuating wall pressure revealed long, coherent streaks corresponding to the passage of large-scale coherent structures. The convective speed of these structures was relatively constant, around 0.75U_\infty, for the range of Reynolds numbers studied.

Acknowledgments

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