Investigation of Tip Vortex Aperiodicity in Hover

Anand Karpatne Jayant Sirohi Swathi M. Mula Charles Tinney

Department of Aerospace Engineering and Engineering Mechanics,
The University of Texas at Austin, Austin, TX 78712, USA

Abstract

Previous research has indicated aperiodicity in the positions of tip vortices emanating from a rotor blade in hover. The objective of the current study is to develop an analysis of the tip vortex aperiodicity in hover and to validate it with measurements on a reduced-scale, 1m diameter, four-bladed rotor. A “vortex ring emitter model” was developed to study the statistics of the tip vortices emitted from a rotor during hover. The locations of tip vortex cores for wake ages ranging from 0° to 260° were measured on the reduced-scale rotor using a stereo PIV system. The blade loading for the reduced scaled rotor was $C_t/\sigma = 0.044$ and the blade rotational speed was 1520 RPM, which corresponds to a tip Reynolds number of 248,000. The 95 % confidence region for the position of tip vortex cores exhibited an anisotropic, aperiodic pattern, approximating an ellipse. Moreover, the principal axis of this ellipse appeared to align itself perpendicular to the slipstream boundary. The analytical model showed good correlation with experimental data in terms of the orientation and extent of the anisotropy. An empirical model for viscosity was also considered which helped model the core radius growth of the vortex ring with wake age.

Background and Motivation

The task of modeling unsteady flow over the vortex dominated wake of a helicopter has been a persistent challenge. Considerable effort has gone into capturing the interaction between vortices with each other and also with the rotor blade (BVI) [1]. It is, therefore, of great interest to investigate how these vortices align in the near wake and far wake of the rotor. Recent development in more advanced and non-intrusive optical planar measurement techniques, such as Particle Image Velocimetry (PIV), have provided a useful tool to more accurately and conveniently measure the trajectory of tip vortices. This technique was used by McAlister (2003) to study various aspects of tip vortex core characteristics including vortex core shape, diameter, circulation and swirl velocities at various wake ages[2]. From the images obtained, it was concluded that the core radius growth varied roughly as a function of square root of increasing wake age. In addition, centers of vorticity and swirl were identified for every wake age which were used to calculate the amount of circulation present in the core.

It has been previously observed that for a given wake age, tip vortex cores tend to exhibit random displacements (jitter) in their positions, shapes and size over a period of time [3, 4]. Stereo-PIV measurements on a full-scaled rotor indicated anisotropy in the vortex jitter [4]. This anisotropy was seen to be larger in the normal direction than in the span wise direction of the rotor blade. However, this observation was limited to a wake age between 0° to 30°. Preliminary experiments performed using stereo-PIV system by Mula et al.[5] on a reduced scale, 1m diameter, four-bladed rotor, confirmed the non-isotropic nature of vortex core aperiodicity for a wake age ranging from 0° to 260°. A very distinct preferred direction of this
jitter was observed and this appeared to be normal to the slip-stream boundary. This phenomenon of vortex jitter may further be studied to determine the mean position and other statistical characteristics of tip vortices. These statistical characteristics have a pronounced effect on helicopter performance, blade vibration, acoustics and brownout.

A number of vortex based techniques have been used in the past to model the wake of a rotor. One of the prevalent models is the “free-wake vortex” model which allows the vortex wake to freely convect downstream, obeying the physical laws of fluid dynamics. This method essentially involves discretizing the vortex wake into small filaments and then calculating the velocities induced by each filament on one another [6]. Although this technique provides an accurate estimate of the rotor wake, it is computationally expensive. However, an assumption of axisymmetric flow for a rotor in hover may greatly reduce the computation cost. One such model that uses this assumption is the vortex ring emitter model proposed by Brand et al.[7]. They used this technique to predict the dynamics of the wake development during the “vortex ring state” (VRS). This method provides a satisfactory estimate of the vortex wake for a rotorcraft in hover at low computation cost.

To get a more realistic vortex wake model, it is necessary to consider the effects of viscous diffusion. Several previous studies have investigated this issue. Ramasamy et al. [8] investigated the dependence of vortex core radius on filament strain and viscosity and came up with an analytical model to describe it. They further corroborated their experimental findings with their analytical viscous free vortex model. In addition, a comparative study of various viscous core growth modeling techniques was made and it was found that Iverson’s model was able to describe this phenomenon in the most comprehensive manner[9]. Bhagwat et al. [10] applied a simplified form of Squire’s viscosity model to come up with their free vortex wake viscous diffusion estimates.

In the current analysis, a comparative study between the vortex ring emitter model and experiments has been made in order to characterize vortex jitter in hover. Also, due to its ease of implementation, Squire’s viscous model is used to model the growth in vortex ring’s core radius due to viscous diffusion.

**Analytical Approach**

**Vortex Ring Emitter Model**

The methodology uses the underlying assumption of axisymmetric flow during hover. The analysis consists of emitting a toroidal vortex ring of circular cross section after every revolution of a rotor blade (Figure 1). This assumption of circular cross section would not hold good if the vortices interact with each other and deform/merge to lose their shape. However, owing to high induced velocities in the near wake, the emitted tip vortices rapidly translate down without getting enough time to interact with each other. This preserves the shape of the tip vortices and the assumption of circular cross section remains valid.

These vortex rings are allowed to convect downstream under the influence of induced velocities from rings emitted previously and their own self-induced velocities. These velocities are estimated by using the Biot-Savart law.

Figure 2 shows the top view of the vortex-ring geometry. Ring 1 has a radius ‘R’ and a circulation strength ‘Γ’. To calculate the velocity induced by ring 1 on ring 2, it is sufficient to calculate the induced velocity at just one point ‘p’ on the ring 2. This simplification results from the assumption...
of axisymmetric flow during hover. The point ‘\(p\)’ is chosen such that it lies on the y-axis and has the coordinates \((x_p, 0, z_p)\). Thus, the differential velocity induced by segment \(ds\) on Ring 1 at a point ‘\(p\)’ (Figure 2) is given by,

\[
dv = \frac{\Gamma}{4\pi} \left( \frac{ds \times r}{(r^2 + r_{\text{core}}^2)^{3/2}} \right)
\]

where \(r_{\text{core}}\) is the radius of the core of the toroidal ring and \(r\) is the distance of point ‘\(p\)’ from the element \(ds\). The position vector and the ring elemental vectors are given as \(r = (x_p - R \cos \theta, -R \sin \theta, z_p)\) and \(ds = (-R \sin \theta, R \cos \theta, 0)\).

The above equations are elliptic in nature and it is difficult to obtain a closed form analytical solution for them. Therefore, a numerical trapezoidal integration scheme is used with 1000 integration points. In this way, the radial and axial velocities induced by a ring on any other ring are obtained.

**Time stepping iterative scheme**

At every time step the total velocity induced on each ring due to the presence of all the other rings as well as by itself is calculated. This is responsible for the axial translation and radial expansion/contraction of the ring. It was observed that explicit time advancement was not a stable scheme for predicting the motion of vortex rings. Therefore, a trapezoidal predictor-corrector scheme was chosen for time stepping.

\[
r_i^{n+1} = r_i^n + \frac{\Delta t}{2} [V_{ix}(r_i^n, z_i^n) + V_{ix}(r_i^*, z_i^*)]
\]

\[
z_i^{n+1} = z_i^n + \frac{\Delta t}{2} [V_{iz}(r_i^n, z_i^n) + V_{iz}(r_i^*, z_i^*)]
\]

Here \(r_i^n\) and \(z_i^n\) represent the radial and axial position of the ring respectively at the \(n^{th}\) time step and \(\Delta t\) is the blade passage time. Equations (4a) and (4b) are used to predict an initial solution for radial and axial positions of the ring \((r_i^*, z_i^*)\) using a simple Euler scheme. Thereafter, velocities \(V_{ix}(r_i^*, z_i^*), V_{iz}(r_i^*, z_i^*)\) are calculated at this new position \((r_i^*, z_i^*)\). An average of the previously calculated and the new velocities is used to find the final new position of the ring (Equations (5a) and (5b)).

Moreover, all these rings induce a velocity on the rotor blade as well. The rotor blade was discretized into several segments (N) and the velocity induced by the rings was evaluated at each segment. This affects the local blade angle of attack,

\[
\alpha_i = \theta_0 - \tan^{-1} \left( \frac{V_{iz}}{\Omega r} \right)
\]

where \(\theta_0\) is the rotor blade pitch angle. Correspondingly, the local lift coefficient \(C_l(i)\) is calculated for
each $\alpha_i$ from a lookup table for airfoil data. The total thrust ‘$T$’ produced by the blade can then be given by,

$$T = \sum_{i=1}^{N_b} \frac{1}{2} \rho \left( V_{iz}^2 + (\Omega r(i))^2 \right) C_l(i) c(i) \frac{R}{N}$$  \hspace{1cm} (7)

Here ‘$i$’ is the blade segment number, $N_b$ is the total number of blades, $c(i)$ is the local chord length, $V_{iz}$ is the locally induced axial velocity and $\Omega r$ is the local radial velocity. This time varying lift is responsible for an adjustment in the vortex circulation strength for the next emitted vortex ring in each time step, given by the Kutta-Joukowski law,

$$T N_b R = \rho V_{tip} \Gamma$$ \hspace{1cm} (8)

where $V_{tip}$ is the tip velocity and $\Gamma$ is the circulation of the tip vortex ring that is being emitted. This iteration is continued at each time step throughout the entire process.

### Treatment of Core radius

The core radius of the tip vortex ring plays an important role in evaluating induced velocities (Equations (2),(3)), especially when calculating self-induced velocities. Therefore, the modeling of development of core radius becomes critical. Two factors have a profound effect on vortex core growth- ‘vortex filament strain’ and ‘viscous diffusion’. These two factors were studied independently and a combined core growth model was then created.

#### Vortex filament strain

The outer radius of any emitted vortex ring keeps changing with time due to the various radial velocities imparted on it. This leads to an increase/decrease in the length of the vortex ring filament. This effect is known as vortex filament strain. Assuming that the flow is inviscid, the total circulation of a vortex ring is conserved throughout its existence. Moreover, considering incompressible flow, the air density inside the vortex ring is also assumed to be constant, the conservation of circulation leads to the conservation of volume of the ring. Assuming that the outer radius of the vortex ring at the time of emission is the same as rotor radius $R_0$, and the initial core radius is $r_{core,0}$, the volume of the toroidal ring can be given as $V = 2\pi R_0 \pi r_{core,0}^2$. When the same ring convects downstream, at a particular wake age $\zeta$, the outer radius may be given as $R_\zeta$ and the core radius $r_{core,\zeta}$. In order to conserve the volume of the ring we can calculate the new core radius of the ring as

$$r_{core,\zeta} = r_{core,0} \sqrt{\frac{R_\zeta}{R_0}} \hspace{1cm} (9)$$

In this way, the core radius increase solely due to filament strain effects is evaluated.

#### Viscous diffusion

In reality, fluid flow in the wake of a rotorcraft is viscous in nature which leads to an expansion of the vortex core and diminishing vortex swirl velocities with time. Considering the fact that the vortex wake structure is highly turbulent, a simple laminar viscosity model (Lamb Oseen’s) would prove insufficient. Therefore, Squire’s model[11] was used to approximate the viscous effects. The total viscosity can be given as the sum of laminar and an averaged eddy viscosity,

$$\nu_{total} = \nu + a \frac{\Gamma}{2\pi}$$  \hspace{1cm} (10)

where $\nu$ is the laminar viscosity of the medium, $\nu_{total}$ is the total viscosity of the turbulent flow, $\Gamma$ is the vortex circulation and ‘$a$’ is an empirical constant obtained from experiments. The core radius growth is further approximated by

$$r_{core,\nu} = \sqrt{4\alpha_L \delta \nu t}$$ \hspace{1cm} (11)

where $\alpha_L = 1.25643$ is the Lamb’s constant, ‘$t$’ is the time elapsed since the formation of the tip vortex and $\delta$ is defined as the ratio between the apparent and actual viscosity and is the measure of turbulent viscosity in the medium. It has been previously found that $\delta$ was an experimental parameter that depends on the vortex Reynolds number, $Re_v = \Gamma/\nu$. Bhagwat et al. [10] used previous experimental data to come up with
an empirical scheme to directly estimate the value of $\delta$ for a given vortex Reynolds number. The current study uses the same methodology and the value of $\delta$ was approximated as 4, given the vortex Reynolds number of around 20000.

Considering an initial core radius ($r_{core,0}$) and replacing time ‘$t$’ by $\zeta/\Omega$, where $\zeta$ is the wake age and $\Omega$ is the angular velocity of the rotor, the viscous core growth with increasing wake age can be given as,

$$r_{core,\nu} = \sqrt{r_{core,0}^2 + 4\alpha L \delta \nu \frac{\zeta}{\Omega}}$$ (12)

**Combined core development**

The effects of both filament strain and viscous diffusion were included in the final core radius formulation. However, in order to couple the both diffusion and stretching effects on the core radius, three different combined growth models were studied.

In the first model, a simplified approach to calculating combined core growth was adopted. The change in radius $\Delta r_s$ and $\Delta r_{diff}$ due to strain and diffusion respectively was calculated separately (Equation (13a) and (13b)) and their effects were combined using Equation (14) to result in the final core radius.

$$\Delta r_s = r_{core,0} \sqrt{\frac{R_c}{R_0} - r_{core,0}}$$ (13a)

$$\Delta r_{diff} = \sqrt{r_{core,0}^2 + 4\alpha L \delta \nu \frac{\zeta}{\Omega}} - r_{core,0}$$ (13b)

$$r_{core,1} = r_{core,0} + \Delta r_s + \Delta r_{diff}$$ (14)

For the second model, it was assumed that both strain and diffusion occur independently and in a particular order. It was assumed that strain takes place followed by viscous diffusion. Therefore, the initial core radius $r_{core,0}$ in Equation (12) is now replaced by an effective stretching term as shown in Equation (15). These two methods are simple substitutes to the more complicated coupled problem of viscous diffusion and core stretching which assume that the phenomena are independent of each other.

$$r_{core,2} = \sqrt{r_{core,0}^2 \frac{R_c}{R_0} + 4\alpha L \delta \nu \frac{\zeta}{\Omega}}$$ (15)

A third model is constructed by modifying the effective time taken to reach a particular core radius (due to strain) in the viscous diffusion equation (12). For a vortex ring with positive filament strain (smaller core radius), viscous diffusion will not be able to produce the same core growth when compared to a vortex filament with no strain, for the same passage of time. The effective time can thus be given as,

$$t_{eff} = \frac{t}{1 + \epsilon}$$ (16)

where $t$ is the actual time elapsed ($\zeta/\Omega$), $\epsilon$ is the filament strain given by $(R_c - R_0)/R_0$ and $t_{eff}$ is the effective time elapsed for viscous diffusion. Therefore, Equation (16) can be modified as

$$t_{eff} = \frac{\zeta}{\Omega} \frac{R_0}{R_c}$$ (17)

Using Equation (17) and (12), the modified viscous core radius formulation can be given as,

$$r_{core,3} = \sqrt{r_{core,0}^2 + 4\alpha L \delta \nu \frac{\zeta}{\Omega} \frac{R_0}{R_c}}$$ (18)

where $R_0$ is the initial outer radius of the toroidal ring and $R$ is the outer radius of the ring at a wake age of $\zeta$. We can see that if vortex stretching takes place, $R > R_0$, the viscous core radius growth will be slower. In contrast, in case of vortex filament contraction, $R < R_0$, the viscous core radius growth will be enhanced. For the current analysis, all the three models were studied but the first model was chosen for core growth modeling, owing to its simplicity and ease of implementation.

**Experimental Setup**

A reduced-scale rotor test stand (Figure 3) was used for measuring the vortex trajectory from a four-bladed rotor at various wake ages and fixed collective pitch angles. The rotor is driven by a 9kW electric motor powered by a 10kW (max) Lambda TKE ESS 50-200 programmable DC power supply that outputs
up to 50V at 200A. The motor is capable of a maximum rotational speed of 8000RPM (133Hz) and a maximum torque of 10Nm. Two optical encoders are fixed to the motor and allow both 1/rev and 60/rev positioning of the rotor to be phase aligned with other laboratory instruments. These optical encoders also allow the speed of the rotor to be adjusted through a NI-PXI system. For the current study, a custom

Fabricated, fully articulated, four-bladed hub assembly was mounted on the rotor shaft. The rotor was driven directly by the electric motor, without a transmission. All four rotor blades comprised carbon fiber NACA 0012 profiles with constant chord lengths of 58.5mm and no twist. The blades had square tips and a small (less than 1.5 % of the chord) blunt trailing edge. The diameter of the entire assembly including blades, rotor hub and blade grips was measured to be 1009.65mm. For the current discussion, a collective pitch angle around $7.2^\circ$ was investigated at a rotor speed of $\omega = 1520$RPM (25.33Hz), corresponding to $Re_c = 248,000$.

The rotor was carefully tracked before the experiment. Measurements of the flow along a 2-D slice through the rotor slipstream $(r, z)$ were performed using a 2-camera, 3-component (stereo) PIV system by LaVision which includes two 2M pixel CCD cameras with 14bit resolution and 7Hz(double frame mode)

Figures and Diagrams

Figure 3: Rotor test stand with rotor installed in the anechoic chamber [5]

Results and Discussions

Thrust Estimation

After around 60 rings are shed, the value of vortex circulation strength and the thrust generated by the rotor stabilized (Figure 5). However, minor fluctuations in the instantaneous thrust of the order of <1% was still present in the system. The thrust produced by the rotor was normalized by the average thrust produced, after 250 rings were shed.

Slipstream Boundary

As the tip vortex positions are calculated at each revolution, it was possible to observe the development...
Figure 4: 95% confidence intervals for tip vortex core positions for various wake ages $\psi$ (ranging from 100° to 170°). Each red dot indicates instantaneous tip vortex core positions for a particular wake age.

Figure 5: Instantaneous Thrust (Analytical) generated by the rotor, normalized by the average thrust after 250 rings were generated.

Figure 6: Analytical predictions for Tip vortex core positions (indicated by blue dots) normalized by rotor diameter.

Core Radius Growth

The core radius growth with increasing wake age was also obtained. In Figure 7, core radius growth is plotted in the absence of viscosity, by only considering vortex filament strain effects. It is observed that the rate of increase of core radius slows down with increasing wake age. This can be explained in terms of slipstream boundary development. In the initial wake ages, the slipstream boundary begins to contract rapidly which results in a sharp increase in core radius. However, with increase in wake age, the vortices begin to roll up and slipstream boundary begins to diffuse and lose its shape. This results in a slight expansion in the radius of the vortex ring which causes the core radius to contract at higher wake ages.

In Figure 8 the effects of both the filament strain and viscosity are included using three core radius modeling approaches as discussed earlier. In this case, the increase in core radius due to filament strain is coupled with an increase due to viscous diffusion,
thereby resulting in an increased growth with wake age. It can be seen from the figure that the values of core radii estimated from Model 1 and 2 are in fairly good agreement especially at earlier wake ages. For larger wake ages, it was seen that the core radius estimated from Model 3 was close to the values estimated from the other two models.

In order to understand the effect of the experimental parameter, \( \delta \), core growth was modeled for various values of \( \delta \) as shown in Figure 9. It was seen that with increasing the value of \( \delta \), the rate of increase of core radius with wake age also increases. Therefore, it was important to choose the correct value of delta that corresponds to the conditions of the experiment. In the current analysis a value of \( \delta = 4 \) was found based on the scheme proposed by Bhagwat et al. [10].

The effect of core radius increase on induced swirl velocity is shown in Figure 10. It was seen that with increase in core radius, the magnitude of swirl velocity of the vortex decreases. However, the variation in velocity is more visible near the core center. With increase in distance from the core center, the difference in swirl velocities for various core radii diminishes.

Vortex Jitter Characterization

One of the most important results obtained from the analytical model was its ability to predict the vortex jitter phenomenon. The position of every vortex ring
core corresponding to various wake ages was calculated at every time step and a 95% confidence region for those positions was obtained. It was observed that the positions of these tip vortex cores exhibit aperiodicity and the direction of this jitter (alignment of the major axis of the 95% confidence ellipse) is roughly perpendicular to the orientation of the slip-stream boundary at that position. These observations were then correlated with experimental “jitter” characterization (Figure 11). The direction of jitter (orientation of major axis of ellipse) was estimated for wake ages of 90°, 180°, and 270° for both the analysis and the experiment. The angle that the major axis makes with the ellipse was calculated and results from analysis and experiments were compared (Figure 12). It can be seen from this figure, that the analytical estimates for vortex jitter orientation angles were in good correlation with experimental results. Using a linear fit, the slope of the line shown in this figure was found to be 0.89 which suggests a highly linear correlation between the analysis and experiments. It was not possible to correlate the analysis with experimental results for any other wake age because the vortex ring emitter model could only capture wake dynamics occurring in increments of 90° wake ages for a four bladed rotor. This stems from the underlying assumption that only one vortex ring is emitted per rotor blade per revolution.

Summary and Conclusions

It was observed that the vortex ring emitter model proved to be a simple and inexpensive tool to analyze complex tip vortex dynamics for a rotorcraft in
An effort was made to model the vortex wake in hover and estimate the position of the slipstream boundary. The estimated thrust appeared to stabilize after around 60 rings were shed. However, minor thrust fluctuations (<1%) persisted in the system. Besides considering filament strain, a viscous growth was also considered while modeling the vortex rings and various methods were comparatively studied to predict the core radius growth. A monotonic increase in core radius was observed with increasing wake age. Also, it was observed from the experiments that the aperiodicity in the tip vortex positions at a particular wake age was anisotropic and there was a preferred direction of jitter. Analytical predictions supported these experimental findings and showed an extremely linear correlation (with 10% error) for three different wake ages (90°, 180°, and 270°). All these observations confirmed that this method could provide a simplified and computationally inexpensive approximation to the “Free Vortex method” for hover considerations.

One of the limitations of this method was its inability to capture changes in vortex dynamics at wake ages which are not a multiple of 90°. Furthermore, it was assumed that the tip vortex cores maintain their circular cross section and do not merge with each other. This observation holds good in the near wake. However, far wake vortex tumbling/merging phenomena were not captured by this model. Future work would include correlating the blade loading and other performance characteristics obtained by the Blade Element Momentum Theory (BEMT) and the current model. Moreover, a vortex blob technique could be used to model the vortex ring. This would help capture the changes in shape of the vortices as they interact/merge with each other.

Acknowledgment

The work is supported by Army/Navy/NASA’s Vertical Lift Research Center of Excellence (VLRCOE) led by the University of Maryland with Dr. Mike Rutkowski as Technical Monitor with Grant No. W911W6-11-2-0012. The authors would also like to thank Mr. Mark Dreier, Staff Engineer, Bell Helicopter Textron Inc., for his constant guidance and support towards the development and testing of the ‘Vortex Ring Emitter’ model.

References

