Multi-Point Measurements in an Axisymmetric Sudden Expansion,

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ABSTRACT

A combination of flying hot-wire and stationary hot wire measurements have been used to estimate the u-component of the streamwise fluctuating velocity field for the radial and azimuthal plane in an axisymmetric sudden expansion flow for downstream step heights ranging from $z/h = 3$ to 12. This is performed using the technique of Linear Stochastic Estimation (LSE). The facility contains an expansion ratio of 3:1, and measurements are taken at a Reynolds number of 54,000 based on centerline velocity and inlet pipe diameter. The well resolved two-point correlation measurements obtained from the flying hot wire experiment are used in conjunction with an azimuthal array of 15 single component stationary hot-wire probes at 2 radial locations of $r/R = 0.12$ and 0.57, normalized with the expansion pipe's radius, $R$. From these measurements, an experimental description of the axisymmetric sudden expansion is illustrated by combining the instantaneous information from the LSE, with the averaged low-dimensional picture from the Proper Orthogonal Decomposition (POD). This Complementary technique captures the unique low-dimensional unsteady behavior of the axisymmetric step flow.

KEYWORDS

POD, LSE, Complementary Technique, Turbulence, Sudden Expansion, Experimental, Eigenvalue, Correlation, Anemometry, Flying wire

INTRODUCTION

The Axisymmetric Sudden Expansion has provided a fascinating topic of study for several decades. It has been examined extensively in the propulsion industry because of its physical similarities to sudden dump combustors. Swirl is introduced upstream from the expansion, and the recirculation region is used as a flame holder. Other applications include HVAC and Automotive systems. From a scientific standpoint, sudden expansion geometries have provided insight into the fundamental behaviour of massively
separated turbulent flows.

Suddenly expanding flows are three dimensional in nature. As the flow enters the expansion, a free shear layer is formed, thus separating the recirculating region from the potential core. Within the recirculating region adverse pressure gradients induce reverse flow activity. It is here that much effort has been exhausted identifying the key events leading to the collapse of eddy structures along the wall region (most prominently being the decay of the Reynolds stresses), thereby defining the reattachment length. The 2-D slice in the $r, z$ plane has been studied extensively because of the flow’s azimuthal mean invariance. The more recent studies of Cole and Glauser (1998a,b), demonstrated qualitatively that the reattachment region is highly unsteady with a typical frequency similar to that of the structures shedding from the expansion lip. Rothe and Johnston (1976) had defined the reattachment region as the physical location at which the instantaneous velocity was either in forward or reverse motion 50% of the time. Cole and Glauser (1998b) captured this unsteady behaviour by studying the radial and streamwise plane in the axisymmetric sudden expansion. Further more, Eaton and Johnston (1981) established five principle parameters governing the reattachment length: 1) initial boundary layer state, 2) initial boundary layer thickness, 3) freestream turbulence, 4) pressure gradient, and 5) aspect ratio. The reader is referred to Eaton and Johnston (1981) for a compilation of results from Bradshaw and Wong (1972) and many others.

Recently, as modelling techniques have become more developed, attention has been given to the azimuthal dimension, and its role on the instantaneous behaviour of axisymmetric flows. Continued work of Glauser and George (1987) by Citriniti and George (2000) demonstrated the instantaneous behaviour in the axisymmetric jet of the $r, \theta$ plane at several downstream diameters. Taylor et al. (2001) reconstructed temporal images of the $r, \theta$ plane in the compressible jet.

For the present study, the work of Cole and Glauser (1998b) is extended to include the $r, \theta$ plane in order to demonstrate the azimuthal unsteadiness in the axisymmetric sudden expansion. A detailed understanding of the downstream events from this analysis could provide insight into the upstream events, because of the recirculating nature of sudden expansion flows.

**Proper Orthogonal Decomposition (POD)**

The ability to illustrate and characterize the unsteady behaviour of highly turbulent flows has been a complex subject for many branches within the fluid dynamics community. One of the more physical structure identification techniques, Proper Orthogonal Decomposition (POD), utilizes the two point statistics from several discrete spatial locations in order to segregate the various structures apparent in a flow regime. Several researchers have been applying this technique to the axisymmetric jet and 2D shear layer [v. Glauser and George (1987), Taylor et. al. (2001), Gamard and George(2002), Citriniti and George (2000), Delville et al (1999) and Ukeiley et al (2001)]. The sudden expansion is a natural extension to these studies because it is a conical flow, but with the additional complexity of being massively separated.

POD had been around for several years before its inception to the turbulence community in 1967. Others had introduced the methodologies independently, although it is Lumley (1967) who is recognized for applying this technique to turbulent flows. POD provides an unbiased technique for decomposing a random velocity field into coherent structures $\phi$ of finite energy. The structures are deterministic and are selected so that the largest projection of a random generalized function onto the coherent structure is maximized. Solving an integral eigenvalue problem determines the energy contribution of each candidate structure to the velocity field’s total spatial energy spectrum in Eqn. 1 as follows,

$$\iint R_g(\vec{x}, \vec{x}', t, t')\phi_j(\vec{x}', t')d\vec{x}'dt' = \lambda\phi_j(\vec{x}, t)$$

(1)
where \( R_{ij} \) is the kernel and is a weighted, symmetric spatial transformation of the Reynolds two-point correlation tensor, \( R_{ij}(\tilde{x}, \tilde{x}', t, t') = u_i(\tilde{x}, t)u_j(\tilde{x}', t') \) using the quadrature rule similar to that of Glauser and George (1987). The solution to Eqn. 1 yields an orthonormal and discrete sequence of eigenfunctions \( \phi^{(n)}_i \), corresponding to real eigenvalues \( \lambda^{(n)} \), as outlined under the orthogonality principles of Hilbert Schmidt theory. The sequence of solutions can also be used to reconstruct the original weighted kernel in Eqn. 2, to validate the energy distribution accounted for in the decomposition process.

\[
R_{ij}(\tilde{x}, \tilde{x}', t, t') = \sum_{n=1}^{\infty} \sum_{j} \lambda^{(n)} \phi^{(n)}_i(\tilde{x}, t) \phi^{(n)\ast}_j(\tilde{x}', t')
\]  

(2)

Here, in Eqn. 3, the orthonormal eigenfunction sequence can be utilized to express the original generalized ensemble, where from Eqn. 4, the coefficients \( a_n \) are random and uncorrelated,

\[
u_i(\tilde{x}, t) = \sum_{n=1}^{\infty} a_n \phi^{(n)}_i(\tilde{x}, t)
\]  

(3)

\[
a_n = \int_D u_i(\tilde{x}, t) \phi^{(n)\ast}_i(\tilde{x}, t) d\tilde{x} dt
\]  

(4)

For the present study, the axis-symmetry of the sudden expansion simplifies the POD technique by incorporating Fourier analysis, as can be done for any random generalized function that is stationary in time, or is homogenous or periodic. The later assumption is a proven case for axisymmetric flows. Therefore, the azimuthal spatial direction is described with azimuthal Fourier modes, and a new expression for the kernel is shown,

\[
B_{ij}(r, r', z_o, m, f) = \int R_{ij}(r, r', z_o, \theta, \tau) e^{i(2\pi r + m\theta)} d\theta d\tau
\]  

(5)

where \( B_{ij} \) is a weighted spatial transformation of the symmetric kernel tensor and is implemented into the original integral eigenvalue problem of Eqn. 1 as follows,

\[
\iint B_{ij}(r, r', z_o, m, f) \phi^{(n)}_j(r', m, f', z_o) r'dr' = \lambda^{(n)}(m, f) \phi^{(n)}_i(r, m, f, z_o)
\]  

(6)

For a more comprehensive review of the POD technique, see Berkooz et. al. (1993).

**Linear Stochastic Estimation (LSE)**

One of the experimental complexities of the POD technique is the amount of hardware required for reconstructing a time dependent picture of discrete energy modes through applications of Eqns. 3 and 4. Even a low dimensional time dependent picture of the velocity field could require hundreds of channels of instrumentation in order to resolve the large scale events in the flow field. Citriniti and George (2000) performed a 138 hot wire experiment on the turbulent jet. For such reasons, an estimation of the temporal events has been suggested [v. Bonnet et al (1994), Cole and Glauser (1998b), Taylor et. al. (2001)]. Adrian (1977) proposed, that the stochastic estimation of a random flow field could be performed on the bases of conditional averaging. Original techniques of stochastic estimation are performed by expanding the conditional average in a power series, and determining coefficients that would satisfy necessary and sufficient provisions. These provisions are satisfied by truncating the expansion such that the mean square error between the estimation and the conditional average are minimized. Tung and Adrian (1980) showed that by expanding the conditional average as high as the third and fourth order terms yielded nearly identical results as the first order term only. Also the error associated with the conditional averaging appeared to be of the same order as the non-linear coefficients. This first order estimation is expressed as:
where $A_{ik}$ is determined by:

$$u_j(x)u_k(x)A_{ik}(x') = u_j(x)u_i(x')$$  

Applications of Eqn. 8, requires the Reynolds stress, $u_j(x)u_k(x)$ and the two-point correlation $u_j(x)u_i(x')$, which is also utilized in the POD technique.

**Complementary Technique**

Bonnet et. al. (1994) implemented a technique that combines the instantaneous information from the LSE with the POD. Recently, the Complementary technique has been used by Taylor et. al. (2001) in the compressible jet. Both showed that one could selectively capture and demonstrate the temporal evolution of certain events in turbulent flows. The technique consists of substituting the estimated velocity field from Eqn. 7, into the velocity field reconstruction of Eqn. 4, which yields a temporal reconstruction of discrete energy modes. It is the motivation of this study to illustrate, via the complementary technique, a low dimensional picture of an axisymmetric sudden expansion flow with the primary focus on the time evolution of turbulent structures in the $r, \theta$ plane.

**EXPERIMENTAL FACILITY**

The axisymmetric sudden expansion facility located in the Experimental Research Laboratory at Clarkson University is shown in figure 1. An axial blower provides airflow to the facility. Air enters through an axisymmetric linear diffuser and several sections of fine mesh grid, essential for removing any swirling motions induced by the axial blower. The air then travels through a 5th order contraction, with a ratio of 14:1. Using a 3 in. diameter Plexiglas pipe, 12 ft. in length, ensures fully developed flow at the inlet to the expansion.

![Figure 1: Experimental Facility of the Axisymmetric Sudden Expansion](image)

The test section has an expansion ratio of 3:1 with a test length of 5 ft. Also an exit section, 4 ft. in length, contains a filtered end fabricated out of 0.25 in. diameter drinking straws sandwiched between two fine mesh grids. This prevents any downstream disturbances. The entire tunnel is aligned using a small
Before the experiment, a cross-wire probe was placed at the lip of the expansion to ensure no mean swirl, \( W = 0 \). All measurements were conducted at a Reynolds number of 54,000 based on bulk velocity and inlet pipe diameter. This pertains to a centerline velocity, \( U_{cl} \) of 10.35 m/s at the expansion inlet. Due to the directional ambiguity of hot wire anemometry, the flying wire technique is utilized to capture the two point statistics in regions where reverse flow and hot wire rectification errors are likely. Therefore, a small slot is milled along the bottom length of the test section so that an externally mounted flying traversing mechanism can support a rake of hot-wire instruments. This traversing mechanism is non-intrusive as it sits on the top surface of the bench, and runs parallel to the axis of the tunnel. The traverse is capable of accelerating a sled to a constant impulse velocity of 3m/s between \( z/h = 3 \) and 12. For a more detailed explanation of the flying wire technique and of the facility, see Cole and Glauser (1998a). Also, for a complete outline of the experimental procedure for the work presented in this paper, the reader is referred to Eaton (1999) and Tinney (2001).

**Flying and Stationary Hot Wire Measurements**

The present study is separated into two experiments, flying and stationary hot wire measurements. Both experiments employ 16 differential channels of A/D conversion, sampling at a frequency of 2kHz and low pass filtering at 820 Hz. All hot wires contain a sensing length of 1.5mm.

Eaton’s (1999) flying hot wire measurements utilized two rakes containing a total of 8 cross wires. This was essential for capturing the two-point statistics about 8 radial and 45 azimuthal spatial locations in order to construct the Reynolds Stress tensor expressed earlier. The \( <vw> \) correlations were unattainable given the conditions of the experiment.

Figure 2: a) 360 point grid density for creating Reynolds Stress two point correlation tensor. Darkened circles indicate stationary probe placement for LSE technique, b) Coordinate system of sudden expansion. From these measurements, a complete set of basis functions for the \( u \) and \( v \) components of the streamwise fluctuating velocity field are captured in the \( r, \theta \) plane for downstream distances from \( z/h = 3 \) to 12. The
grid density for these 360 points is shown in figure 2 with an azimuthal separation distance of 8°.

For the stationary hot wire measurements, a probe rake containing single component hot wires was used to capture the instantaneous \( u \)-component of the streamwise velocity field at 15 discrete locations. Two azimuthal arrays of probes depicted by the solid dots in figure 2, supported 4 and 11 probes at \( r_1/R = 0.12 \) and \( r_2/R = 0.57 \), respectively. These stationary measurements allowed for an instantaneous velocity reconstruction of the radial and azimuthal plane at several step heights using the LSE technique. Probe placement for this experiment was crucial in order to prevent large rectification errors brought upon by high turbulence intensities located outside of the potential core region of the flow. Two criteria where necessary for deciding the placement of these stationary probes, that is turbulence intensity and correlation length. Results from the stationary experiment were compared with the streamwise statistical profiles of Cole and Glauser (1998b). Errors where found to exist as high as 30% in the first and second order statistics. However, the objective of the present study is to illustrate the low dimensional unsteady behavior of the large scale eddy structures in sudden expansion flows, as opposed to focusing on small-scale events that may perhaps fall within the truncation error of the estimation technique of Adrian (1977).

**EXPERIMENTAL FINDINGS**

The resultant localized eigenvalue distribution of Eqn. 6, integrated over all 8 POD \((n)\) and 23 Azimuthal \((m)\) modes is shown in Figure 3a. A maximum occurs at \( z/h = 7 \) and 8, as one might expect from comparison of the experimental TKE of Cole and Glauser (1998a). It is here that the merging of the potential core shear layers collide and cause the largest production of TKE. These results are used to normalize the local energy contribution for each POD and azimuthal mode shown in successive figures. This energy distribution illustrates the inhomogeneous nature of the sudden expansion’s streamwise direction. Velocity spectra from the stationary experiment at \( r_1/R \) and \( r_2/R \) are shown in Figure 3b to illustrate the inhomogeneous nature of the flow’s radial direction. This latter illustration is obtained by averaging the spectra over the azimuthal plane at discrete radial and step height locations.

![Figure 3: a)Total eigenvalue energy distribution, b)Velocity spectra of stationary measurements, z/h=6](image)

The experimental results from the decomposition yield a noticeable dominance of the first POD mode for each azimuthal mode as shown in Figure 4. Also, from this eigenspectra, the majority of the azimuthal energy is shown to exist in the first few azimuthal Fourier modes. Here, one can see at \( z/h = 3 \), the majority of the flow field’s energy is dominated by the \( 0^{th} \) and \( 1^{st} \) azimuthal modes, with a relative contribution of 5% and 6.5%, respectively in the first POD mode. As one progresses downstream towards...
the reattachment region ($z/h = 8.9$), a similar amount of energy is seen in the first two azimuthal modes, where as at $z/h = 12$, a transition of energy is seen as the relative contribution switches to $10\%$ in the $0^{th}$ mode, and just over $6\%$ in the $1^{st}$ azimuthal mode. It is here that the flow is redeveloping into a fully turbulent pipe.

Recent findings in the axisymmetric jet of Taylor et al. (2001), and Gamard and George (2002), indicate an energy shift from higher azimuthal modes upstream near the jet lip, to the second azimuthal mode, far down stream. For the sudden expansion, the energy shift only exists between the first two azimuthal modes because of the outer wall’s entrapment of the flow. For the present study, only the results of the first four POD modes are illustrated, for several azimuthal modes at a given step height.

Thus, the focus of interest from these eigenspectra illustrations is the dominance in the low dimensional POD and Azimuthal modes. This provides insight into the tools necessary for adequately displaying and understanding important characteristics in sudden expansion flows.

**Temporal Estimation of the Sudden Expansion**

Before illustrating the temporal evolution of the discrete energy modes, an estimation of the flow
including all 8 POD and 23 azimuthal modes are shown. This validates the decomposition and reconstruction of the POD technique as it is applied to the \( r, \theta \) plane at \( z/h = 6 \), in the sudden expansion. As one can see, only very small discrepancies exist between the raw LSE and the full Complementary reconstruction, which are attributed to FFT round off errors.

Figure 5: \( u \)-component of the streamwise fluctuating velocity: a) LSE technique, b) Complementary technique with full POD and azimuthal mode reconstruction.

**Discrete Mode Reconstruction**

To fully appreciate the instantaneous nature of displaying select POD and Azimuthal modes, the reader is referred to [www.ecs.syr.edu/faculty/mglauser](http://www.ecs.syr.edu/faculty/mglauser) where movie sequences can be viewed. Otherwise, several independent snapshots containing a discrete reconstruction of the first POD mode \( (n=1) \) for azimuthal modes \( (m) \, 0,1,2,3,4 \) are illustrated in figure 6. Here, the same four time steps from figure 5 are used for comparative purposes with the discrete reconstructions located in the top left hand corners of figures 6a, b, c and d. These reconstructions of only certain azimuthal and POD modes in figure 6, are produced with only 30% of the local energy and explain any discrepancies in amplitude between figures 5 and 6. Similar results from Citriniti and George (2000) utilized 40% to 50% of the POD and azimuthal mode’s energy to capture the characteristic events in the shear layer of the incompressible jet. It is to the author’s knowledge that this is the first appearance of temporally phased images of discrete energy modes, coupled with an integrated image of discrete modes. As one can see, the contribution of the first POD mode with only the first five low dimensional azimuthal modes is able to reconstruct the dominant characteristics of the \( r, \theta \) velocity field. For each snapshot in time, the images are phased identically.

Although the POD technique yields dominance in the 0th and 1st azimuthal modes on average, one can see that instantaneously, other azimuthal modes, \( (m = 2,3,4 \text{ and others}) \) may appear to contribute more energy at one time setting. Figures 6a and 6d illustrate this phenomena when at \( t = 0.0045 \text{ s} \), the \( m = 0 \) and \( m = 4 \) modes dominate the flow fields energy contribution. However this changes at \( t = 0.0460 \text{ s} \), when the \( m = 3 \) mode, alone dictates the shape of the integrated picture in the top left corner of figure 6d.
CONCLUSION

In this paper, the instantaneous $u$-component of the streamwise fluctuating velocity of the $r$, $\theta$ plane in the axisymmetric sudden expansion was illustrated. It was shown that relatively little information was required in reconstructing the characteristics of this flow field, using only the low dimensional dominant modes provided by the POD technique. Also, we have shown the simplicity of using the LSE technique especially in regions where stationary hot wire measurements are unreliable for obtaining instantaneous velocities.

The flow unsteadiness in the azimuthal direction is clearly visible in the sudden expansion. These POD results have indicated an energy shift from the $m = 1$ azimuthal mode upstream near the expansion lip, to a more relaxed $m = 0$ azimuthal mode in the redevelopment region. The fact that the sudden expansion flow is clearly low dimensional bodes well for future implementation of different flow control strategies based on the POD/LSE methods. In particular the results presented here provide guidance on the relevant azimuthal Fourier modes and their respective phases, that should be excited (e.g., at the expansion lip) to effectively control structures in the flow.

References


