Inversion-based petrophysical interpretation of logging-while-drilling nuclear and resistivity measurements

Olabode Ijasan¹, Carlos Torres-Verdín¹, and William E. Preeg²

ABSTRACT

Interpretation of borehole measurements acquired in high-angle (HA) and horizontal (HZ) wells is challenging due to the significant influence of well trajectory and bed geometrical effects. Experience shows that accurate integrated interpretation of well logs acquired in HA/HZ wells requires explicit consideration of 3D measurement physics. The most reliable alternative for interpretation of well logs in HA/HZ wells is with inversion techniques that correct measurements for shoulder-bed, undulating well trajectory, and bed geometrical effects while taking advantage of high data resolution. We discovered an efficient layer-based inversion workflow for combined, quantitative petrophysical and compositional interpretation of logging-while-drilling sector-based nuclear (density, neutron porosity, photoelectric factor, gamma ray) and array propagation resistivity measurements acquired in HA/HZ wells. A challenging synthetic benchmark example confirmed improved formation evaluation with the layer-based inversion workflow across hydrocarbon-bearing zones in HA/HZ wells, where estimated hydrocarbon pore volume and porosity increased by 10% and 15%, respectively, with respect to conventional interpretation methods. Furthermore, application of the inversion-based method to a field example of HZ well across calcite-cemented siltstone layers confirmed its advantage over conventional interpretation techniques.

INTRODUCTION

High-angle (HA) and horizontal (HZ) wells increase exposure of hydrocarbon-bearing zones to the wellbore, thereby improving logging data quality and hydrocarbon flow area. Occasionally, imperfect HZ well placement gives rise to well trajectory undulations that traverse multiple bed boundaries, whereby borehole measurements respond to multiple layer properties at the same time. A detailed understanding of the 3D relationship between well-log measurement physics and bed geometrical effects is essential to develop new and improved formation evaluation methods in HA/HZ wells (Passey et al., 2005).

Logging-while-drilling (LWD) measurements usually consist of azimuthal image logs in multisector tool rotations. With adequate understanding of 3D tool responses and effective volume of investigation (EVOI), a wealth of information, including bed dip, azimuth, and true stratigraphic thickness (TST), can be interpreted from LWD borehole images. The EVOI of nuclear tools, parameterized as effective radial penetration length (EPL — often referred to as depth of image or depth of investigation), axial or vertical resolution, AR, and azimuthal aperture Δψ (Yin et al., 2008) can be estimated from spatial flux sensitivity functions (FSFs) (Mendoza et al., 2007; Ijasan et al., 2011). In vertical wells, the measurement value is jointly influenced by EPL and AR, whereas bed boundary detection is solely affected by AR. Assuming azimuthal homogeneity, Δψ has negligible influence in vertical wells even in the presence of invasion. On the other hand, in HA/HZ wells, the three EVOI parameters influence measurement value and boundary detection in different ways depending on the type of measurements (nuclear or resistivity), petrophysical properties, and bed geometrical properties, i.e., dip, stratigraphy, well trajectory, and tool orientation.

Inverse theory has proved valuable for interpretation of logs acquired in vertical wells, particularly across complex lithologies and thinly bedded formations. Sanchez-Ramirez et al. (2010) and Heidari et al. (2012) implement joint inversion of nuclear and resistivity wireline measurements for reduction of shoulder-bed effects and improved quantitative estimation of petrophysical properties. Mendoza et al. (2012) and Shetty et al. (2012) document the use of inversion techniques that explicitly consider spatial density responses for estimating formation bulk density ρb from sector-based LWD dual-detector and compensated density images. Additionally, Mendoza et al. (2010a) and Zhou et al. (2012) show...
that measurement integration is necessary for accurate quantitative petrophysical and geometrical interpretation in HA/HZ wells.

In this paper, we implement a nonlinear inversion workflow for combined, quantitative petrophysical and compositional interpretation of LWD sector-based nuclear (density, neutron porosity, photoelectric factor, natural gamma ray [GR]) and array propagation resistivity measurements in HA/HZ wells. We incorporate nuclear and resistivity models that reproduce 3D measurement responses in HA/HZ wells for explicit consideration of relationships characterizing measurement EVOI, bed geometrical effects, and well trajectory. Nuclear measurements are modeled with 3D Monte Carlo (MC)-derived FSFs (Mendoza et al., 2010b; Ijasan et al., 2013), and propagation resistivity responses are numerically simulated using the Schlumberger array resistivity compensation (ARC) forward model. By considering 3D tool responses in the inversion, we effectively reduce bed geometry and well trajectory effects on LWD measurements to obtain layer-by-layer petrophysical characteristics, while taking advantage of image data resolution in HA/HZ wells.

Different physical principles governing the various borehole measurements enforce separate tool responses in the subsurface (Ellis and Singer, 2007). For example, nuclear measurements respond to bulk petrophysical properties whereas the electrical resistivity response is directional in nature. This behavior calls for inversion for separate petrophysical properties within each measurement domain. In other words, we estimate layer-by-layer bulk density \( \rho_b \), neutron migration length \( L_{nm} \), photoelectric factor (PEF), volumetric concentration of shale \( C_{sh} \), and true electrical conductivity \( \sigma_t \), from the inversion of sector-based LWD density, neutron porosity, PEF, natural GR, and array propagation resistivity measurements, respectively. Subsequently, compositional or multimineral analysis is carried out on the inverted layer-by-layer petrophysical properties for the estimation of grain density \( \rho_g \), total porosity \( \phi_T \), water saturation \( S_w \), and hydrocarbon pore volume (HPV), assuming a prior knowledge of the GR-\( C_{sh} \) relationship and Archie’s parameters.

We document and verify the application of the inversion workflow with (1) a synthetic example in an 82° updip well across known formation composition, (2) a field example in an HA interval, and (3) a field example in an HZ interval.

### INVERSION-BASED INTERPRETATION WORKFLOW

The interpretation workflow consists of three main steps. First, we assume a formation geometrical model that is locally described by 2D structural properties obtained from consolidated geometrical interpretation of sector-based LWD gamma (density, PEF, GR) borehole images. Structural properties consist of bed boundaries, dips, and azimuths for describing the 2D layer-based formation model. In this model, we assume that LWD logs are acquired after minimal formation exposure to borehole environmental conditions, such that mud-filter invasion is negligible and formation petrophysical properties are

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**Table 1.** Depth-periodic layer properties, compositions, and thicknesses assumed in the synthetic example.

<table>
<thead>
<tr>
<th>Layer</th>
<th>TST (cm)</th>
<th>Volumetric concentrations (%)</th>
<th>Solids</th>
<th>Fluids</th>
</tr>
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<td></td>
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<tr>
<td></td>
<td></td>
<td>Calcite</td>
<td>Dolomite</td>
<td>Kaolinite</td>
</tr>
<tr>
<td>I</td>
<td>52.93</td>
<td>38</td>
<td>40</td>
<td>10</td>
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<tr>
<td>II</td>
<td>14.43</td>
<td>5</td>
<td>5</td>
<td>10</td>
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<tr>
<td>III</td>
<td>14.43</td>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>IV</td>
<td>14.43</td>
<td>38</td>
<td>40</td>
<td>10</td>
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<tr>
<td>V</td>
<td>5.28</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>VI</td>
<td>5.28</td>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>VII</td>
<td>5.28</td>
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<tr>
<td>XIII</td>
<td>52.93</td>
<td>38</td>
<td>40</td>
<td>10</td>
</tr>
</tbody>
</table>

**Gamma ray response (API)**

<table>
<thead>
<tr>
<th>Component density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component formula</td>
</tr>
</tbody>
</table>

**Component formula**

\( \text{CaCO}_3, \text{CaMg}_2(\text{CO}_3)_2, \text{Al}_2\text{Si}_4\text{O}_{10}(\text{OH})_8, \text{SiO}_2, \text{C}_8\text{H}_{18}, \text{H}_2\text{O} \)
approximately constant and isotropic within each layer structure. Next, we impose the formation geometrical model on nuclear and resistivity measurement domains for separate petrophysical inversion. Finally, a multimineral compositional solver is used to estimate layer-by-layer solid and fluid volumetric concentrations from inverted layer properties. Figure 1 describes the three steps of the inversion-based interpretation workflow.

### Consolidated gamma geometrical model

Layer geometrical properties, bed thickness, apparent dip, and apparent azimuth are calculated from nuclear gamma images, i.e., sector-based density, PEF, and GR (see step I in Figure 1). Initially, bed boundaries are detected along well depth and around borehole azimuth from short-spaced $\rho_{SS}$, long-spaced $\rho_{LS}$, or...
compensated $\rho_{CO}$ density images using a threshold variance algorithm (Uzoh et al., 2009). Selected bed boundaries, resulting from bed boundary planes intersecting a circular borehole, define the “gamma borehole” sinusoids (Yin et al., 2008). Next, a least-squares minimization method (Plumb and Luthi, 1989) is used to estimate bed boundary location, apparent dip, and apparent azimuth from the quadratic cost function written as

$$C(B_b, \theta_b, \beta_b) = \left\| B_b + \text{EPL} \tan \theta_b - A_{b,j} \right\|^2 + \frac{D}{2} + \text{EPL} 
\times \left\| \cos(\alpha_j - \beta_b) - \cos \beta_b \right\| \tan \theta_b,$$

(1)

where $j$ and $b$ designate the $j$th sector-based bin and $b$th bed boundary, respectively, $D$ is the borehole diameter, $\alpha_j$ is the vector of azimuthal bins in degrees, $B_b$ is the depth-coherent physical bed boundary location at $\alpha_j = 0^\circ$ (north) in measured depth, $\theta_b$ is the apparent bed dip, and $\beta_b$ is the apparent bed strike or azimuth; $A_{b,j}$ are the gamma borehole sinusoid locations detected using the threshold variance algorithm along borehole depth and around borehole azimuth. Note that $A_{b,j}$ includes the inherent EPL effect of sector-based measurements, such that equation 1 corrects for depth coherence and is readily differentiable to obtain analytical Jacobian gradients in conjunction with nonlinear least-squares minimization.

Additional sinusoids are detected from PEF and GR images. For single-curve PEF and GR logs, we assume that the detected boundaries are located at the intersection between the bed boundary plane and the hole axis; i.e., $\cos(\alpha_j - \beta_b) = 0$. This is because single curves cannot distinguish between updip or downdip bed boundaries. The complete set of $B_b$, $\theta_b$, and $\beta_b$, defining the layer-based geometry, is referred to as the consolidated gamma geometrical model. Additionally, particularly in HZ wells with undulating well trajectories, we refine the set of $B_b$, $\theta_b$, and $\beta_b$ by manually picking more sinusoids from sector-based measurements. Together with well-trajectory data or borehole true-vertical depth (TVD), we infer the 2D layer stratigraphic properties from the consolidated geometrical model (Griffiths, 2009).

**Measurement domains: Property parameterization and data space**

Measurement physics of well logs plays an important role in the development of interpretation techniques. Integrated interpretation demands explicit multiphysics characterization because different well logs measure or interact with different formation petrophysical properties.

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**Figure 3.** Layer-based inversion results for the synthetic example, showing inverted (a) bulk density $\rho_b$ with layer designations, (b) migration length $L_{m}$, (c) shale concentration $C_{sh}$, (d) photoelectric factor PEF, and (e) true conductivity $\sigma_t$ in solid blue lines. True synthetic model properties are identified with dashed red lines, respectively, and 95% inversion confidence intervals are shown with solid green error bars, respectively. (f) Cumulative plots of compositional volumetric concentrations $V_n$ obtained from SNUPAR-based solver and Archie’s saturation model. Confidence intervals or uncertainties in compositional volumes, per layer, are shown with black error bars.
properties and volumes. Neutron porosity $\phi_N$ is primarily governed by the hydrogen index (HI) and $L_m$; $\rho_b$ by electron density $\rho_e$; PEF by average atomic number; GR by naturally occurring radioactivity of thorium, uranium, and potassium; and apparent resistivity $R_{app}$ by $\sigma_i$ of the formation (Ellis and Singer, 2007). Consequently, in each measurement domain, we estimate layer-by-layer physical properties governing each measurement. For the examples discussed in this paper, linear GR-C$_{sh}$ relationships (often applied in sand-shale laminated formations) are assumed in the API GR measurement domain. The parameterization of formation properties involves $\rho_b$, $L_m$, PEF, C$_{sh}$, and $\sigma_i$ for characterization of density ($\rho_{SS}$,$\rho_{LS}$,$\rho_{CO}$), neutron detector count rate and porosity (ncps, fcps, $\phi_N$), PEF, natural GR, and apparent electrical conductivity ($1/R_{app}$) measurements, respectively. In this parameterization, the neutron porosity tool is calibrated with Schlumberger’s nuclear parameter calculator (SNUPAR, McKeon et al., 1989); i.e., for a given neutron source, $L_m$ is calculated in base-cases of water-filled limestone, sandstone, and dolomite porosity units.

Depending on the available measurements, nuclear data are assimilated into the interpretation in the following manner:

1) sector binning scheme, e.g., single curves or sector-based (quadrant or multiple bins)
2) detector data type, e.g., neutron, density dual-detector (ncps-fcps, $\rho_{SS}$,$\rho_{LS}$); compensated ($\rho_{CO}$,$\phi_N$); or single-detector lithology measurements (PEF, GR)

In resistivity data space, all array propagation channels of phase-shift, attenuation, 400 kHz and 2 MHz are used for $\sigma_i$ inversion, i.e., a total of 20 channels. Appendix A provides further details about the data space.

**Overview of the nonlinear inversion method**

In the second step of our workflow (step II in Figure 1), we impose the formation geometrical model on nuclear and resistivity measurements for separate nonlinear inversion within each measurement domain. Accordingly, the quadratic misfit cost function to be minimized is given by

$$C(p) = \|e(p)\|^2_2 + \lambda^2\|p - p^0\|^2_2.$$  \hspace{1cm} (2)
where \( \mathbf{p} = (\rho_b, L_m, \text{PEF}, C_{\text{sh}}, \sigma_t) \) is the vector of layer physical properties, \( \mathbf{p}^0 \) is the initial guess, \( \mathbf{e}(\mathbf{p}) = \mathbf{d}(\mathbf{p}) - \mathbf{d}_i \) is the vector of data misfit, where \( \mathbf{d}(\mathbf{p}) \) is numerically simulated data and \( \mathbf{d}_i \) designates the available measurements, and \( \lambda \) is a regularization (stabilization) parameter calculated with the generalized cross-validation (GCV) method (Hansen, 1998) and intended to provide selective weighting to the two additive terms included in the quadratic cost function. The ensuing expression for iterative nonlinear minimization of equation 2, using the regularized Levenberg-Marquardt method (Aster et al., 2005) becomes

\[
J^T J + \lambda^2 W_m^T W_m \Delta \mathbf{p} = -J^T \mathbf{e} + \lambda^2 W_m^T W_m \mathbf{p}^k,
\]

where \( \Delta \mathbf{p} = \mathbf{p}^{k+1} - \mathbf{p}^k \) and \( k \) designates the \( k \)th iteration, \( J \) is the Jacobian (sensitivity) matrix, and \( W_m \) is the model weighting matrix used to control the importance of each layer where entries of its diagonal are inversely proportional to relative layer thicknesses such that emphasis can be placed on thin beds to improve uniqueness across thinly laminated intervals; when \( W_m \) is the identity matrix, all layers are given equal importance. Convergence is achieved when \( d(\log ||\mathbf{e}(\mathbf{p})||^2)/dk = 0 \).

In the nuclear measurement domain, we construct \( J \) from 3D FSFs (Mendoza et al., 2012). For the case of density, PEF, and GR measurements, their FSFs are approximately constant for varying formation \( \rho_b \), PEF, and \( C_{\text{sh}} \), respectively (Mendoza et al., 2010b; Ijasan et al., 2011). This approximation is valid across bed boundaries in HA/HZ wells. Consequently, \( J \) for density, PEF, and GR measurements is assumed constant, whereby the separate inversion of density, PEF, and GR logs in equation 3 simplifies to linear minimization of the quadratic cost function in equation 2 (Heidari et al., 2012; Mendoza et al., 2012).

On the other hand, neutron porosity FSFs exhibit significant flux perturbations at bed boundaries, across varying formation HI, and in the presence of borehole environmental effects. We account for these flux perturbations by augmenting neutron FSFs with a recently developed semianalytical fast diffusion flux-difference (DFD) approximation (Ijasan et al., 2013), so that \( J \) for inverting neutron porosity consists of transport and diffusion parts.

In the resistivity measurement domain, \( J \) is numerically calculated from finite-difference approximations of partial derivatives. Appendix A provides further descriptions about the entries of \( J \) for inversion of nuclear and resistivity measurements.

### Compositional petrophysics

The final step in our interpretation workflow (step III in Figure 1) involves estimation of layer-by-layer \( \rho_g \), \( \phi_T \), and HPV. We invoke a

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**Figure 5.** Comparison of inversion-based and conventional interpretation of synthetic LWD nuclear and resistivity measurements acquired in 82° updip well. Cumulative plots of compositional volumetric concentrations \( V_n \) obtained from SNUPAR-based solver and Archie’s saturation model for (a) inversion-based interpretation and (b) conventional interpretation. Comparison of calculated (c) total porosity \( \phi_T \), (d) water saturation \( S_w \), and (e) grain density \( \rho_g \). (f) Difference in HPV between inversion-based and conventional interpretation.
SNUPAR-based compositional solver (Heidari et al., 2012) coupled with appropriate $S_w$ models (e.g., Archie or shaly sand models with known Archie’s parameters) and GR-$C_{th}$ relationships (e.g., linear; Larionov, 1969; Clavier et al., 1971) in the estimation of solid and fluid compositions from inverted layer-by-layer properties, $p(\rho_b, L_m, \text{PEF}, C_{th}, \sigma_t)$. The compositional solver takes advantage of the deterministic relationship between complex rock/fluid mixtures and nuclear physical properties, quantified by the SNUPAR program. Appendix B describes the formulations of the SNUPAR-based solver. Note that other commercial multiminerals solvers could be applied at this step of the interpretation workflow.

APPLICATION TO HA AND HZ WELLS

This section describes the application of the inversion-based interpretation workflow. Using a synthetic example in 82° updip well, we compare inversion-based interpretation to conventional interpretation. The analysis also considers the effects of bed-geometry uncertainty on the inverted layer-by-layer physical properties. Furthermore, we apply the workflow to LWD nuclear and resistivity measurements acquired in highly deviated and HZ well intervals of a hydrocarbon field located in West Africa.

Synthetic benchmark example

We assume a layered synthetic model, of depth-periodic compositions and varying TST, to describe shoulder-bed, EVOI, and bed dip effects on nuclear and resistivity measurements acquired in HA/HZ wells. For simplicity but without sacrifice of generality, we invoke Archie’s saturation model. Tables 1 and 2 describe layer compositions, petrophysical properties, and Archie’s parameters assumed in the synthetic example. With an updip well of 82° inclination and bed azimuths of 30°, borehole nuclear logs are numerically simulated using the FSF technique and generic LWD logging tools (Mendoza et al., 2007; Ijasan et al., 2011, 2013), and array propagation resistivity curves are simulated using the ARC forward model. In the data space, we assume 16-sector bins for sector-based density ($\rho_{SS}$, $\rho_{LS}$, $\rho_{CO}$) and GR measurements, quadrant-sector bins for neutron porosity and detector count rate ($\phi_N$, ncps, fcps) measurements, and single curves for PEF and array propagation resistivity channels. Measurements are sampled every 7.6 cm (0.25 ft) along the well trajectory. Additionally, 10% random noise is added to the synthetic measurements to mimic actual field logs. Figure 2 shows the numerically simulated synthetic data.

As discussed in the workflow, the formation geometrical model is obtained from bed boundary detection of the $\rho_{SS}$ image; GR image,
and PEF log. The geometrical model is then imposed on nuclear and resistivity measurement domains for layer-based separate inversion. Figure 3 shows layer-based physical properties ($\rho_b$, $L_m$, $C_{sh}$, PEF, $\sigma_t$) from inversion, together with solid and fluid concentrations from the compositional solver. Additionally, green error bars in Figure 3a–3e identify 95% inversion confidence intervals, estimated using the model covariance in equation A-1 of Appendix A. Error bars, quantifying nonuniqueness and stability of the inversion, are largest across the 5.28-cm (2.08-inch) beds of layers VI and VII (Table 1), especially for inverted $L_m$ (Figure 3b) and $\sigma_t$ (Figure 3e). This behavior is due to the fact that the consolidated geometrical model is below EVOI resolution of the neutron and resistivity logs.

Table 3. Multimineral and fluid component models assumed in the West Africa hydrocarbon field examples.

<table>
<thead>
<tr>
<th>Component formula</th>
<th>Limestone</th>
<th>Orthoclase feldspar</th>
<th>Sandstone</th>
<th>Hydrocarbon</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaCO$_3$</td>
<td></td>
<td></td>
<td></td>
<td>$C_8H_{18}$</td>
<td>$H_2O$</td>
</tr>
<tr>
<td>KAlSi$_3$O$_8$</td>
<td>2.71</td>
<td>2.52</td>
<td>2.65</td>
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<tr>
<td>SiO$_2$</td>
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<td>0</td>
</tr>
<tr>
<td>Component density ($g/cm^3$)</td>
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<td></td>
<td></td>
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<tr>
<td>Gamma-ray response (API)</td>
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<tr>
<td>Poupon-Leveaux Saturation Parameters</td>
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<tr>
<td>Constant $a$</td>
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<td>Exponent $m$</td>
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<td>Exponent $n$</td>
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<td></td>
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<tr>
<td>Connate water resistivity $R_w$ ($\Omega$m)</td>
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<td></td>
<td>8.27</td>
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<tr>
<td>Shale resistivity $R_{sh}$ ($\Omega$m)</td>
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<td></td>
<td>36.14</td>
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</tr>
<tr>
<td>Shale porosity $\phi_{sh}$ (%)</td>
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</tbody>
</table>
Such an EVOI effect manifests itself as nonuniqueness in inverted layer properties. Note that true model properties, identified with dashed red lines, are within the confidence intervals. Figure 3f shows layer-by-layer volumetric concentrations of solid and fluid compositions, where black error bars identify confidence intervals, per component per layer, propagated from inversion and SNUPAR-based solver. Volumetric concentrations in Figure 3f are in agreement with synthetic properties in Tables 1 and 2. To appraise data misfit, we compare numerically simulated LWD measurements across the solid and fluid compositions in Figure 3f to the original synthetic data in Figure 2. Figure 4 shows these comparisons where average misfits and misfit standard deviations are below 1.5% and 5%, respectively, for nuclear measurements and 1% and 6%, respectively, for apparent resistivity measurements.

Next, we perform the conventional interpretation approach applied in vertical wells on synthetic logs. Conventional interpretation involves multimineral analysis of depth-matched nuclear logs and 100-cm (40-inch) high-frequency phase-shift array resistivity (P40H) log, using the SNUPAR-based solver and Archie’s saturation model. Figure 5 shows comparisons of $\phi_T$, $S_w$, $\rho_g$, and HPV obtained from conventional and inversion-based interpretation. In Figure 5f, green- and black-shaded zones identify where conventional interpretation under- and overestimates HPV, respectively. We observe HPV differences, as much as 15% pore volume (originating from inaccurate interpretation of $\phi_T$ and $S_w$ by 15% and 10%, respectively), are particularly accentuated at bed boundaries and across thin layers V to VIII.

Consolidated geometrical interpretation of $B_b$, $\theta_b$, and $\beta_b$, described earlier, is a crucial step for the inversion-based method. By introducing 10 realizations of random perturbations in $B_b$, $\theta_b$, and $\beta_b$, we investigate effects of bed-geometry uncertainty on inverted $\rho_b$ and $L_m$ from dual-detector and compensated density and neutron porosity measurements. Figure 6 describes the perturbed geometrical model, and subsequently inverted $\rho_b$ and $L_m$. We observe, in Figure 6d–6f, that thin layers are very sensitive to perturbations in the geometrical model. These instability effects on inverted $\rho_b$ and $L_m$ are as high as 0.4 g/cm³ and 7 cm, respectively, corresponding to 23 and 15 pu in limestone units, for an average perturbation of $\pm 8.9$ cm in $B_b$, $\pm 2.5^\circ$ in $\theta_b$, and $\pm 1.5^\circ$ in $\beta_b$ in an 82° well. Such a behavior is qualitatively consistent with error bars in Figure 3 and is important learning for layer-based inversion in HA/HZ wells, thus emphasizing the need for accurate geometrical interpretation (Yin et al., 2006; Griffiths, 2009).

Figure 8. Layer-based inversion results of HA field example, showing inverted (a) bulk density $\rho_b$, (b) migration length $L_m$, (c) photoelectric factor PEF, (d) shale concentration $C_{sh}$, and (e) true conductivity $\sigma_t$ in solid blue lines. Predicted properties of SNUPAR-based solver and Poupon-Leveaux saturation model for output compositional volumes are identified with dashed red lines; 95% inversion confidence intervals are identified with green error bars.
Field case example in a highly deviated well

The case of study is a hydrocarbon field located in West Africa, consisting of alternating laminations of argillaceous limestone and calcite-cemented feldspathic siltstones of 0.3 to 0.6 m TST (1 to 2 ft) (Mendoza et al., 2012). Figure 7 summarizes the LWD nuclear and resistivity measurements acquired in an HA interval of the field example. The HA interval is below the free oil-water contact in the hydrocarbon column, thus primarily saturated with water such that bottom sector density ($\phi_D$) and neutron ($\phi_N$) porosity logs almost overlap across the interval (Figure 7d). Table 3 describes multimineral and fluid component models assumed in the West Africa hydrocarbon field. Borehole density images are acquired on a 16-sector binning scheme, where the well trajectory varies between 78° and 82° for the interval. We observe that $\rho_{SS}$ and $\rho_{LS}$ images, in Figure 7a and 7b, respectively, are marred by standoff at the up (U) sector bins.

Shetty et al. (2012) describe model-based inversion of $\rho_{SS}$ and $\rho_{LS}$ images for interpretation of formation density and borehole shape, where borehole standoff is an inverted property. In this paper, we focus on petrophysical and compositional interpretation whereby the $\rho_{CO}$ image, in Figure 7c, suffices for our inversion workflow, given that it uses $\rho_{SS}$ and $\rho_{LS}$ for compensation and correction for borehole standoff.

The gamma bed boundary sinusoids for the consolidated geometrical model are superposed on the $\rho_{CO}$ image. Bed boundaries sinusoids obtained from $\rho_{CO}$ image only are identified with white lines, and those obtained from $\rho_{CO}$ image or/and PEF and GR logs are identified with magenta and green lines, respectively. It is worth noting that the consolidated geometrical model characterizes local petrophysical boundaries that represent layers of distinct petrophysical properties and not just distinct bulk densities. For example, within XX50–XX53 m and XX85–XX90 m, the $\rho_{CO}$ image shows a relatively constant response, and the GR and PEF logs indicate variations in lithology.

In addition to $\rho_{CO}$, PEF, and GR measurements, the single curve $\phi_N$ and 20 channels of array propagation resistivity curves are available for inversion. It follows from Appendix A that $J_{sd}$ is implemented for estimating $C_{sh}$ and PEF, and $J_{CO}$ is implemented for inverting $\rho_b$ and $L_m$. In Figure 8, we show inverted layer-by-layer properties, $\rho_b$, $C_{sh}$, PEF, $\sigma_t$ and their confidence intervals. Accordingly, the dashed red lines represent predicted SNUPAR properties upon convergence of the multimineral analysis. An
interesting observation is that error bars on estimated $L_m$, PEF, and $C_{\theta}$ are relatively larger than error bars on estimated $\rho_b$ and $\sigma_t$. This behavior is primarily due to data resolution per measured depth, given that $\phi_N$, PEF, and GR are single curves, and $\rho_{CO}$ and $R_{app}$ consist of 16-sector binning curves and 20 channels, respectively. As discussed, the purpose of inversion is to correct the LWD measurement for well trajectory, shoulder-bed, and EVOI effects, such that the inverted parameters represent true bed petrophysical properties. True bed properties are then analyzed with the SNUPAR-based multimineral solver, assuming Poupon-Leveaux saturation model (Poupon and Leveaux, 1971; see Table 3), to obtain solid and fluid volumetric concentrations. Figure 9a describes estimated solid and fluid volumetric concentrations, and Figure 9b–9d shows estimated $\phi_T$, $S_w$, and $\rho_g$, respectively.

Field case example in an HZ well

Reliable interpretation of logs acquired in HZ wells has been challenging due to limited understanding of HZ well trajectory effects on log responses. Figure 10 shows well data and trajectory in an HZ interval of the field example. The $\rho_{CO}$ image, in Figure 10a, exhibits the classic “bull’s-eye” feature between ZY00 and ZY75 m, particularly characteristic of undulating HZ wells. Such a feature is produced as the borehole traverses in and out of the clean and silty limestone layers, in updip and downdip drilling directions, respectively. Figure 10a also shows the gamma borehole bed boundaries, where dashed white lines represent apparent borehole sinuosids for inferring stratigraphic properties, and solid white lines represent intersection of the well trajectory with stratigraphic boundaries. Figure 10f describes the well trajectory and 2D curtain section of the layer-based formation model obtained from structural interpretation of the $\rho_{CO}$ bull’s-eye feature. The formation model is then imposed on nuclear and resistivity domains for layer-based inversion in TVD.

Figure 11 shows the inverted properties across the HZ interval in layer-by-layer TVD. The petrophysical layers from Z65–Z66 m in TVD are sampled multiple times by the HZ section such that 95% confidence intervals (green error bars in Figure 11) within these layers are relatively smaller than for other layers. This is one advantage of HZ well acquisition, i.e., to improve logging data quality within zones of interest.
Nuclear and P40H resistivity logs, shown in Figure 10, are re-sampled in TVD and input directly to the SNUPAR-based solver for conventional analysis. Figure 12 compares estimated petrophysical properties obtained from conventional interpretation with inversion-based interpretation. Similar to observations in the synthetic example, conventional analysis overestimates and underestimates HPV (Figure 12d) in water and hydrocarbon saturated layers, respectively. Below Z65 m, $S_w$ (Figure 12d) estimated from conventional analysis is significantly influenced by shoulder-bed effects, so much that HPV is overestimated by 10%. This behavior is attributed to large EVOI of resistivity measurements. Note that the HZ interval spans a TST interval of about 4 m (13 ft), in comparison to approximately 2.0-m (6.6-ft) EPL of the resistivity response. This implies, as a consequence of the EVOI effect, that the resistivity measurement samples multiple layers simultaneously. Additionally, in the HZ section, layers are completely parallel to the borehole such that anisotropy influences the apparent resistivity measurements. As described in Appendix A, $J_\sigma$ is numerically derived for the well trajectory in Figure 10f such that anisotropy and polarization horns are inherently considered in the finite-difference approximations.

To verify our inversion results, we perform numerical simulations of LWD measurements across the layer-by-layer properties in Figure 12a using the well trajectory in Figure 10f. Figure 13 compares field and the numerically simulated LWD measurements. We observe good agreement between field and numerically simulated measurements, with average data misfits and misfit standard deviations below 0.5% and 5%, respectively, for nuclear measurements and 3% and 10%, respectively, for resistivity measurements.

**DISCUSSION**

The inversion-based interpretation workflow consists of three steps: (1) consolidated geometrical interpretation of LWD nuclear gamma (density, PEF, GR) measurements to construct a 2D layer-based formation model; (2) separate nonlinear inversion of LWD nuclear and resistivity measurements, within each measurement domain; and (3) layer-by-layer compositional interpretation of estimated properties.

The consolidated geometry describes bed boundaries that define layers of locally distinct petrophysical properties. This model is constructed by calculating the local variance of density, PEF, and natural
GR logs. As shown in equation 1, $B_b$ is depth corrected for EPL such that it is independent of the respective gamma measurement used for its detection. Density, PEF, and natural GR measurements are good candidates for geometrical interpretation because of their high AR, shallow EPL, and focused $\Delta \psi$. Furthermore, bed boundaries detected using high-resolution borehole images, e.g., FMI (formation micro-imager) measurements and resistivity polarization horns can be used to enhance the geometrical model. The accuracy in structural and stratigraphic interpretation of sector-based LWD measurements is very crucial. Yin et al. (2006) describe the sensitivity of absolute error in apparent dip to TST estimation, where a 1° error in dip can give rise to TST relative errors of 20% and 100% in HA and HZ wells, respectively. With the synthetic example, we showed that uncertainty in bed-geometry, especially across thinly bedded formations, could give rise to errors as high as 23 and 15 pu in the inversion of density and neutron porosity measurements, respectively.

Borehole measurements acquired in HA/HZ wells are significantly influenced by well deviation effects, such that estimated HPV from traditional analysis can give rise to errors as high as 15% and 10% in HA and HZ intervals, respectively. We verified that inversion corrects LWD measurements for well trajectory effects and improves formation evaluation in HA/HZ wells. The inversion-based workflow is fast and easy to implement with the measurement domain and property parameterization platform. It promotes efficient assimilation of other borehole measurements, such as acoustic, magnetic resonance, and geochemical logs. Additionally, the platform enforces stability of inversion calculations and explicit multiphysics characterization, within each domain, for subsequent estimation of solid and fluid compositions. Furthermore, in the resistivity domain, we observed that inversion of apparent conductivity measurements ($1/R_{app}$) is most stable in comparison to inversion of apparent resistivity ($R_{app}$) and its logarithm (log $R_{app}$). In the gradient-based search for a local minimum, kth iterations with nonpositive layer-by-layer $\sigma$ solutions are reset; i.e., $\sigma^k = \sigma^{k-1} \forall \sigma^k \leq 0$.

The physics-based SNUPAR multimineral solver, in comparison with probability-based commercial mineral-solvers, calculates nuclear measurement parameters using semianalytical multigroup relationships that describe nuclear transport phenomena. To improve convergence of the solver, properties of solid and fluid components can be enhanced using information from core and XRD (x-ray diffraction) measurements.

The objective of HA/HZ wells is to improve logging data resolution and increase exposure of hydrocarbon-bearing zones to the

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**Figure 12.** Comparison of inversion-based and conventional interpretation results of HZ field example. Cumulative plots of compositional volumetric concentrations $V_n$ obtained from SNUPAR-based solver and Poupon-Leveaux saturation model for (a) inversion-based interpretation and (b) conventional interpretation. Comparison of (c) total porosity $\phi_T$, (d) water saturation $S_w$, and (e) grain density $\rho_g$. (f) Difference in HPV between inversion-based and conventional interpretation. (g) Well trajectory across the HZ interval is identified with a solid red line, and the apparent bed orientation is identified with tadpolelike points, i.e., bed dip and azimuth.
wellbore. This important advantage is invoked by our inversion workflow. We observe from Figures 8, 9, 11, and 12 that data resolution in terms of HZ well sampling, borehole imaging, and multiarray channels can significantly improve confidence in estimated layer-by-layer petrophysical properties. Additionally, the inversion-based workflow is implemented for locally described 2D geometrical curtain sections. Interpreting extended logging depth intervals will require multiple runs of the inversion-based workflow with several localized curtain sections.

It is worth noting that separate inversion in the nuclear domain is about 11 times faster than in the resistivity domain. For example, the 150-m HZ logging interval discussed in Figures 10–13 covering a 3.5-m true vertical section required approximately 5 and 55 minutes (4 and 8 iterations) for misfit minimization in the nuclear and resistivity domains, respectively. This CPU time distribution arises because nuclear Jacobian matrices were constructed with 3D FSFs, where the conductivity Jacobian matrix required multiple forward simulations to approximate the associated partial derivatives.

**CONCLUSIONS**

We introduced an efficient inversion-based workflow for combined, quantitative interpretation of LWD nuclear and resistivity measurements acquired in HA/HZ wells. The workflow estimates layer-by-layer petrophysical properties ($\rho_b$, $L_m$, PEF, $C_{sh}$, $\sigma_t$), using separate nonlinear inversion of nuclear (dual-detector, compensated, single-detector; density, neutron porosity, PEF, GR) and resistivity (array propagation) measurements. Using a SNUPAR-based multiminerals solver, together with appropriate $S_w$ model and GR-$C_{sh}$ relationship, the inverted layer-by-layer petrophysical properties were subsequently used to estimate solid composition and hydrocarbon saturation.

Even though well deviation increases exposure of formation to wellbore, interpretation in HA/HZ wells is significantly hindered by well trajectory, 3D bed geometry, shoulder-bed, and EVOI effects. The main purpose of inversion is to correct and minimize these geometrical effects in LWD measurements, so that layer-by-layer properties are due to petrophysical effects only, while taking

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**Figure 13.** Qualitative comparison of forward simulated measurements, from inverted compositions, and HZ field data. (a) Field compensated density data $\rho_{co}$ and (b) forward simulated compensated density from inverted compositions. Field and forward simulated (c) neutron porosity $\phi_N$, (d) GR, and (e) PEF logs. Field and forward simulated logs are identified with solid red and dashed blue lines, respectively; (f) 2 MHz phase-shift array resistivity curves, field, and forward simulated curves are identified with solid and dashed colored lines, respectively.
advantage of high data resolution. The examples considered in this paper indicated that conventional analysis (developed for vertical wells) often yields inaccurate estimation of HPV in HA/HZ wells.

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SYMBOLS AND NOMENCLATURE

\[
\begin{align*}
\alpha_i & = \text{Vector of azimuthal bins (°)} \\
\beta_p & = \text{Apparent azimuth (°)} \\
\theta_p & = \text{Apparent dip (°)} \\
\rho_p & = \text{Bulk density (g/cm}^3\text{)} \\
\phi_D & = \text{Density porosity (v/v)} \\
\phi_N & = \text{Neutron near-to-far ratio porosity (v/v)} \\
\phi_{sh} & = \text{Shale porosity (v/v)} \\
\sigma_T & = \text{True conductivity (S/m)} \\
\phi_T & = \text{Total porosity (\%)} \\
\Delta\psi & = \text{Azimuthal aperture (°)}
\end{align*}
\]

Subscripts

\[
\begin{align*}
b & = \text{Bed boundary} \\
\text{CO} & = \text{Compensated} \\
dd & = \text{Dual-detector} \\
e & = \text{Electron} \\
g & = \text{Grain} \\
j & = \text{Sector-based bin} \\
\text{LS} & = \text{Long-space} \\
m & = \text{Measurement depth point} \\
n_p & = \text{Total number of layers} \\
n_{sp} & = \text{Total number of data points} \\
sd & = \text{Single-detector} \\
snu & = \text{SNUPAR} \\
\text{SS} & = \text{Short-space} \\
k & = \text{Nonlinear iteration} \\
T & = \text{Transpose}
\end{align*}
\]

Superscripts

\[
\begin{align*}
k & = \text{Nonlinear iteration} \\
T & = \text{Transpose}
\end{align*}
\]

APPENDIX A

NONLINEAR SEPARATE INVERSION IN LWD MEASUREMENT DOMAINS

We implement the Levenberg-Marquardt iterative nonlinear minimization method (Hansen, 1998; Aster et al., 2005) in each measurement domain, as shown in equation 3, such that 95% confidence intervals for each inverted petrophysical parameter are calculated with

\[
p \pm 1.96\sqrt{\|\mathbf{e}(p)\|_2^2 \cdot \text{diag}(\mathbf{J}^T\mathbf{J})^{-1}}, \quad (A-1)
\]

where the superscript \( T \) designates the matrix transpose.

Data and parameter space

In each measurement domain, the layer-by-layer inverted petrophysical property is given by

\[
p = [p_1, \ldots, p_b, \ldots, p_{n_p}]^T, \quad (A-2)
\]

whereas data are written as
where \( s \) and \( b \) designate the \( s \)th available data and \( b \)th layer, respectively; \( n_{dp} \) is the total number of available data points; and \( n_{j} \) is the total number of layers. In the nuclear domain, \( n_{dp} \) is \( J \times M \times n_{det} \), where \( n_{det} \) is the number of detectors, \( J \) is the number of azimuthal sector bins, and \( M \) is the number of sampled depth points. For example, \( J = 1 \) for single curves of PEF and GR logs, whereas \( J = 16 \) for 16-sector binning scheme. Similarly, \( n_{det} = 1 \) for \( \rho_{CO} \) and \( \phi_{N} \), whereas \( n_{det} = 2 \) for dual-detector density, neutron (ncps, fcps) measurements. In the resistivity domain, \( n_{dp} \) is \( 20 \times M \), i.e., four channels each (phase, attenuation, and dual frequencies) for five array (16-, 22-, 28-, 34-, and 40-inch) propagation measurements. In total, the number of measurement domains, \( n_{md} \), is five, i.e., density, neutron porosity, PEF, GR, and resistivity.

**Numerical construction of operator matrices**

In the nuclear measurement domain, entries of \( J \) are constructed directly from 3D FSFs, as follows:

\[
\begin{bmatrix}
FSF_{1,1,1}^{sd} & \cdots & FSF_{1,1,b}^{sd} & \cdots & FSF_{1,n_b}^{sd} \\
FSF_{1,1,1}^{sd} & \cdots & FSF_{1,m,b}^{sd} & \cdots & FSF_{1,m,n_b}^{sd} \\
FSF_{1,1,1}^{sd} & \cdots & FSF_{1,1,b}^{sd} & \cdots & FSF_{1,n_b,b}^{sd} \\
\vdots & \cdots & \vdots & \cdots & \vdots \\
FSF_{J,1,1}^{sd} & \cdots & FSF_{J,1,b}^{sd} & \cdots & FSF_{J,1,n_b}^{sd} \\
FSF_{J,m,1}^{sd} & \cdots & FSF_{J,m,b}^{sd} & \cdots & FSF_{J,m,n_b}^{sd} \\
FSF_{J,1,1}^{sd} & \cdots & FSF_{J,1,b}^{sd} & \cdots & FSF_{J,1,n_b}^{sd} \\
\vdots & \cdots & \vdots & \cdots & \vdots \\
FSF_{J,1,1}^{sd} & \cdots & FSF_{J,1,b}^{sd} & \cdots & FSF_{J,1,n_b}^{sd} \\
\end{bmatrix}
\]

\[ (A-4) \]

where superscript/subscript \( sd \) designates single-detector nuclear measurement (e.g., PEF and GR); subscripts \( j, m, \) and \( b \) designate the \( j \)th sector-based bin up to \( J \) azimuthal orientations, \( m \)th measurement point (along the well trajectory) up to \( M \) depth points, and \( b \)th layer up to \( n_{b} \) layers, respectively. In other words, \( FSF_{j,m,b}^{sd} \) is the sensitivity of a single-detector well log measurement at the \( m \)th depth point and \( j \)th azimuth to \( b \)th layer. It follows from equation \( A-4 \) that the Jacobian matrix for dual-detector measurements (gamma densities and neutron count rates) \( J_{dd}^{sd} \) is obtained from \( J_{d}^{sd} \) of short-spaced (SS)/near and long-spaced (LS)/far detectors for density/neutron measurements, i.e.,

\[
J_{dd}^{sd} = [J_{sd,SS/near}^{sd}, J_{sd,LS/far}^{sd}]^{T}. \tag{A-5}
\]

To account for flux perturbations in neutron porosity measurements acquired in HA/HZ wells, we implement a semianalytical transport-diffusion approximation (Ijasan et al., 2013) such that \( J_{dd}^{sd} \) for dual-detector neutron count rates, becomes

\[
J_{dd} = [J_{sd,near}^{sd}, J_{sd,far}^{sd}]^{T} + [J_{DFD,near}^{sd}, J_{DFD,far}^{sd}]^{T}, \tag{A-6}
\]

where \( J_{DFD} \) is the DFD approximation for neutron near and far detectors, accordingly. In equation \( A-6 \), the first term on the right-hand side describes the transport part and the second term describes the diffusion part of \( \phi_{N} \) measurement physics.

**Finite-difference approximations of operator matrices**

Entries of Jacobian matrices, \( J_{CO} \) and \( J_{\phi} \) for inversion of compensated nuclear (\( \rho_{CO}, \phi_{N} \)) and apparent electrical conductivity measurements (1/\( R_{app} \)), respectively, are obtained by finite-difference approximations of partial derivatives using rapid numerical simulations of well-log data, \( d(p) \). In the nuclear measurement domain, entries of \( J_{CO} \) are numerically calculated from equations \( A-5, A-6 \), and appropriate postprocessing algorithm, i.e., density spine- and rib compensation and neutron near-to-far ratio porosity, for \( \rho_{CO} \) and \( \phi_{N} \), respectively. Similarly, elements of \( J_{\phi} \) for inversion of apparent electrical conductivity measurements (1/\( R_{app} \)) are obtained from perturbations of \( d(p) \) due to perturbations in true layer conductivities, \( p(\sigma_{r}) \). It follows that effects of resistivity anisotropy and polarization horns are explicitly included in numerically simulated \( R_{app} \), and consequently in \( J_{\phi} \).

**APPENDIX B**

**SNUPAR-BASED COMPOSITIONAL SOLVER**

The SNUPAR-based compositional solver (Heidari et al., 2012) calculates solid and fluid volumetric concentrations from inverted layer-by-layer petrophysical properties \( p \) using Occam’s inversion method (Aster et al., 2005). The minimized quadratic cost function is expressed as

\[
C(x) = ||p_{smu}(x) - p||^{2} + \alpha^{2}||x - x_{0}||^{2}, \tag{B-1}
\]

where \( x = [V_1, \ldots, V_n, \ldots, V_N]^{T} \) is the vector of volumetric concentrations, \( V_n \) is the volumetric concentration of \( n \)th solid or fluid component for up to \( N \) components subject to \( 0 \leq V_n \leq 1 \) and \( \sum_{n=1}^{N} V_n = 1 \). \( p(\sigma_{r}) \) is obtained by layer-by-layer petrophysical parameters, \( U_{r} \) is volumetric PEF (a product of \( \rho_{p} \) and PEF), \( p_{smu}(x) \) is SNUPAR-calculated petrophysical properties given an arbitrary initial guess \( x_{0} \), of volumetric concentrations, and \( \alpha \) is regularization (stabilization) parameter calculated with the GCV method (Hansen, 1998). The ensuing constrained iterative nonlinear minimization function is given by

\[
(J_{smu}^{T} + \alpha^{2}I)x^{k+1} = J_{smu}^{T}\hat{p}(x^{k}) + \alpha^{2}x_{0}, \tag{B-2}
\]

where \( J_{smu} \) is a Jacobian matrix of approximate partial derivatives obtained by finite-difference SNUPAR calculations of \( p_{smu}(x) \), \( \hat{p}(x^{k}) \) is the modified misfit vector, \( k \) designates the \( k \)th iteration, and \( I \) is the identity matrix. The Jacobian matrix \( J_{smu} \) is derived using

\[
J_{smu} = \begin{bmatrix}
\frac{\partial \phi_{N}}{\partial x} & \frac{\partial (1/L_{app})}{\partial x} & \frac{\partial U_{r}}{\partial x} & \frac{\partial C_{nu}}{\partial x} & \frac{\partial \sigma_{r}}{\partial x} & 1
\end{bmatrix}^{T}, \tag{B-3}
\]

and the modified misfit vector \( \hat{p}(x^{k}) \) is expressed as...
\[
\hat{\mathbf{p}}(x^k) = \left[ \mathbf{p} - \mathbf{p}_{\text{sol}}(x^k) \right]_0 + \mathbf{J}_{\text{sol}} x^k, \tag{B-4}
\]

where the unity-vector and zero entries in equations B-3 and B-4, respectively, minimize the quadratic function \( r^2 = (\sum_{n=1}^{N} V_n - 1)^2 \) in a least-squares manner. Positivity constraints are enforced on the volumetric concentrations by resetting negative solutions to their previous values in the sequence of iterations and renormalizing them so that \( \sum_{n=1}^{N} V_n = 1 \). In equation B-3, the nuclear derivatives are numerically obtained with SNUPAR calculations and a GR-\( C_{ab} \) relationship (e.g., linear: Larionov, 1969; Stieber, 1970; Clavier et al., 1971), whereas the conductivity derivative is obtained with an appropriate \( S_{\text{e}} \) model (e.g., Archie, Poupon-Leveaux, etc.) assuming known values of Archie’s parameters, connate water resistivity \( R_{w} \), shale resistivity \( R_{sh} \), and shale porosity \( \phi_{sh} \). The constrained minimization formulation, equations B-1 to B-4, and the above-described procedure ensure positivity and unity summation of volumetric concentrations \( V_n \). Additionally, to improve uniqueness for any arbitrary \( x^k, N \leq n_{\text{max}} \). After convergence of the solver, \( V_n \) is recalculated as \( \frac{1}{\sum_{n=1}^{N} V_n} x_n^{k+1} \) from equation B-2 is output, and confidence intervals from the nonlinear petrophysical inversion (equations 3 and A-1) are propagated to estimate uncertainties for solid and fluid concentrations. For the cases studied in this paper, \( \left| \mathbf{p}_{\text{sol}}(x) - \mathbf{p} \right||^2 \leq 10^{-3} \) and \( \epsilon \leq 10^{-4} \) after convergence. Finally, \( \rho_{\text{nc}}, \phi_{\text{fr}, S_{\text{e}}}, \text{and HPV} \) are calculated from \( x_n^{k+1} \) and the mineral/fluid composition models.

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